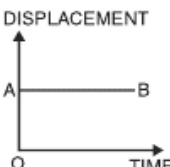



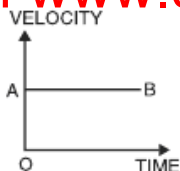
UNIT II

KINEMATICS

- **One Dimensional Motion.** The motion of an object is said to be one dimensional motion if only one out of the three coordinates specifying the position of the object changes with respect to times.
(an object moves along any of the three axes X, Y or Z).
- **Two dimensional motion.** The motion of an object is said to be two dimensional motion if two out of the three coordinates specifying the position of the object change with respect to time. (the object moves in a plane.)
- **Three dimensional motion.** The motion of an object is said to be three dimensional motion if all the three coordinates specifying the position of the object change with respect to time. (the object moves in space.)
- **Speed.** The speed of an object is defined as the ratio of distance covered and time taken i.e. $\text{speed} = \text{distance travelled} / (\text{time taken})$. Speed is a scalar quantity. It can only be zero or positive.
- **Instantaneous velocity.** The velocity of an object at a given instant of time is called its instantaneous velocity. When a body is moving with uniform velocity, its instantaneous velocity = average velocity = uniform velocity.
- **Graphs and Nature**

S.No.	Type of motion	Graph	Features of graph
●	For a stationary body, the time displacement graph is a st. line parallel to time axis.		The slope of st. line represents instantaneous velocity zero slope \rightarrow zero velocity.
●	When a body, is moving with a constant velocity, then time displacement graph will be a st. line inclined to time axis.		Constant slope. Magnitude of velocity is constant.

- When a body is moving with a constant velocity, the velocity time graph is a st-line parallel to time axis.



The slope of this graph represents the instantaneous acceleration zero acceleration.

- Relative velocity.** The relative velocity of one object w.r.t another is the velocity with which one object moves w.r.t another object. If \vec{v}_A and \vec{v}_B are the velocity of two objects A and B, and θ is the angle between them, then relative velocity of object A w.r.t B is given by

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \text{ where, } v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos \theta}$$

$$\text{and} \quad \tan \beta = \frac{v_B \sin \theta}{v_A - v_A \cos \theta}$$

Here, β is the angle which \vec{v}_{AB} makes with the direction of \vec{v}_A .

- Acceleration.** The acceleration of an object is defined as the ratio of change of velocity of the object, and time taken *i.e.*, Acceleration = change in velocity/time taken. Acceleration is a vector quantity. Acceleration is positive, if the velocity is increasing and is negative if velocity is decreasing. The negative acceleration is called **retardation** or **deceleration**.
- Instantaneous acceleration.** The acceleration of an object at a given instant is called its instantaneous acceleration.

$$\text{Instantaneous acceleration, } a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

- Formulae for uniformly accelerated motion along a straight line.**

For accelerated motion

$$1. \quad v = u + at$$

$$2. \quad s = ut + \frac{1}{2}at^2$$

$$3. \quad v^2 = u^2 + 2as$$

$$4. \quad D_n = v + \frac{a}{2}(2n-1)$$

For Retarded motion

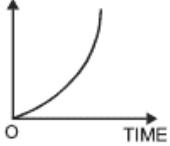

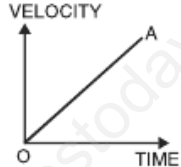
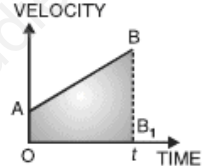
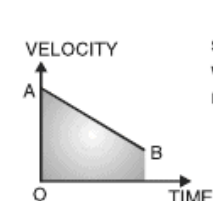
$$v = u - at$$

$$s = ut - \frac{1}{2}at^2$$

$$v^2 = u^2 - 2as$$

$$D_n = u - \frac{a}{2}(2n-1)$$

● **Graphs and nature**

S.No.	Type of motion	Graph	Features of graph
		DISPLACEMENT	
●	When a body is moving with a constant acceleration the time-displacement graph is a curve with bend upwards.		The slope of time-displacement curve (instantaneous velocity) increases with time.
		DISPLACEMENT	
●	When a body is moving with a constant retardation, the time-displacement graph is a curve with bend downwards.		(i) The slope of time-displacement curve (instantaneous velocity) decreases with time.
●	When a body is moving with a constant acceleration and its initial velocity is zero, the velocity-time graph is an oblique st line, passing through origin.		Greater will be the slope of st. line greater will be the instantaneous acceleration.
●	When a body is moving with a constant acceleration and its initial velocity is not zero, the velocity-time graph is an oblique st-line not passing through origin.		The area enclosed by the velocity-time graph with time axis represents the distance travelled by the body.
●	When a body is moving with a constant retardation and its initial velocity is not zero, the velocity-time graph is an oblique st. line not passing through origin.		slope represents acceleration which is negative i.e., retardation.

- **Scalars.** The quantities which have magnitudes and unit only but no direction. For example, mass, length, time, speed, work, temperature etc.
- **Vector.** The quantities which have magnitudes unit as well as direction and obeys vector laws of addition, multiplication etc. For example, displacement, velocity, acceleration, force, momentum etc.

S.No.	Resultant	Properties and results
●	Addition of vectors	<p>(i) Only vectors of same nature can be added.</p> <p>(ii) The addition of two vectors \vec{A} and \vec{B} is a resultant \vec{R}, where $R = (A^2 + B^2 + 2AB \cos\theta)^{1/2}$ and $\tan \beta = \frac{B \sin \theta}{A + B \cos \theta}$ where θ is the angle between \vec{A} and \vec{B} and β is the angle which \vec{R} makes with the direction of \vec{A}</p> <p>(iii) Vector addition is commutative i.e. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$</p> <p>(iv) Vector addition is associative i.e. $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$</p> <p>(v) R is maximum if $\theta = 0^\circ$ and is minimum if $\theta = 180^\circ$</p>

● **Subtraction of vectors.**

S.No.	Resultant	Properties and results
●	Subtraction of two vectors	<p>(i) Only vectors of same nature can be subtracted.</p> <p>(ii) Subtraction of \vec{B} from \vec{A} = vector addition of \vec{A} and $(-\vec{B})$ i.e. $\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \vec{R}$ where $R = \sqrt{A^2 + B^2 + 2AB \cos(180^\circ - \theta)}$ and $\tan \beta = \frac{B \sin(180^\circ - \theta)}{A + B \cos(180^\circ - \theta)}$ where θ is the angle between \vec{A} and \vec{B}; and β is the angle which \vec{R} makes with the direction of \vec{A}.</p> <p>(iii) Vector subtraction of two vectors is not commutative i.e. $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$</p> <p>(iv) Vector subtraction is not associative i.e. $\vec{A} - (\vec{B} - \vec{C}) \neq (\vec{A} - \vec{B}) - \vec{C}$</p>

- **Rectangular components of a vector in a plane.** If \vec{A} makes an angle θ with x-axis and \vec{A}_x and \vec{A}_y be the rectangular components of \vec{A} along x-axis and y-axis respectively, then $\vec{A} = \vec{A}_x + \vec{A}_y = A_x \hat{i} + A_y \hat{j}$

Here $A_x = A \cos \theta$ and $A_y = A \sin \theta$

and $A = (A_x^2 + A_y^2)^{1/2}$ and $\tan \theta = A_y / A_x$

- The dot product of two vectors \vec{A} and \vec{B} , represented by $\vec{A} \cdot \vec{B}$ is a scalar, which is equal to the product of the magnitudes of \vec{A} and \vec{B} and the cosine of the smaller angle between them.

If θ is the smaller angle between \vec{A} and \vec{B} , then $\vec{A} \cdot \vec{B} = AB \cos \theta$

$$(i) \quad \hat{i} \cdot \hat{i} = 1 = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$(ii) \quad \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

$$(iii) \quad \text{if } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \quad \text{and} \quad \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

- The vector product or cross product of two vectors \vec{A} and \vec{B} is represented as $\vec{A} \times \vec{B}$

If θ is the smaller angle between \vec{A} and \vec{B} , then $\vec{A} \times \vec{B} = \vec{C} = AB \sin \theta \hat{n}$ where \hat{n} is a unit vector perpendicular to the plane containing \vec{A} and \vec{B}

- **Right handed screw rule.** It states that if a right handed screw placed with its axis perpendicular to the plane containing the two vectors \vec{A} and \vec{B} is rotated from the direction of \vec{A} to the direction of \vec{B} through smaller angle, then the sense of the advancement of the tip of the screw gives the direction of $(\vec{A} \times \vec{B})$

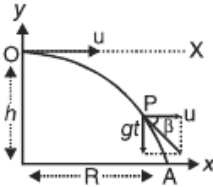
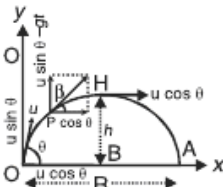
For unit vectors : (i) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$ and

$$(ii) \quad \hat{i}, \hat{j} \text{ and } \hat{k} : \hat{i} \times \hat{j} = \hat{k}; \hat{j} \times \hat{k} = \hat{i}; \hat{k} \times \hat{i} = \hat{j}$$

$$\text{If } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \quad \text{and} \quad \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- **Projectile.** Projectile is the name given to a body which is thrown with some initial velocity with the horizontal direction and then it is allowed to move under the effect of gravity alone.

S.No.	Projectile projected with velocity u	Horizontal projection	Projectile with angular projection
●	Path of projectile is a parabola		
●	Velocity of projectile at any instant t	$v = \sqrt{u^2 + g^2 t^2}$	$v = \sqrt{u^2 + g^2 t^2 - 2ugt \sin \theta}$
●	Direction of the velocity \vec{v}	$\beta = \tan^{-1} \left(\frac{gt}{u} \right)$	$\beta = \tan^{-1} \left(\tan \theta - \frac{gt}{u \cos \theta} \right)$
	with the horizontal direction		
●	Horizontal range	$R = u \sqrt{\frac{2h}{g}}$	$R = \frac{u^2 \sin 2\theta}{g}$
●	Time of flight	$t = \sqrt{\frac{2h}{g}}$	$T = \frac{2u \sin \theta}{g}$
●	Maximum height	$H = h$	$H = \frac{u^2 \sin^2 \theta}{2g}$

1 MARK QUESTIONS

- Why can speed of a particle not be negative?
- Is it possible in straight line motion a particle have zero speed and a non zero velocity?
- Suggest a situation in which an object is accelerated and have constant speed.
- Two balls of different masses are thrown vertically upward with same initial velocity. Maximum heights attained by them are h_1 and h_2 respectively what is h_1/h_2 ?
- A car moving with velocity of 50 kmh^{-1} on a straight road is ahead of a jeep moving with velocity 75 kmh^{-1} . How would the relative velocity be altered if jeep is ahead of car?
- Which of the two-linear velocity or the linear acceleration gives the direction of motion of a body?

ANSWERS

1. Because speed is distance travelled per second and distance is never negative.
2. No. it is not possible.
3. Uniform circular motion.
4. Same height $\therefore h_1/h_2 = 1$
5. No change
6. Linear velocity

1 MARK

7. Will the displacement of a particle change on changing the position of origin of the coordinate system?
8. If the instantaneous velocity of a particle is zero, will its instantaneous acceleration be necessarily zero?
9. Can a body subjected to a uniform acceleration always move in the straight line?
10. Write an example of zero vector.
11. State the essential condition for the addition of vectors.
12. When is the magnitude of $(\vec{A} + \vec{B})$ equal to the magnitude of $(\vec{A} - \vec{B})$?
13. What is the maximum number of component into which a vector can be resolved?
14. A body projected horizontally moves with the same horizontal velocity although it moves under gravity. Why?
15. What is the angle between velocity and acceleration at the highest point of a projectile motion?

ANSWERS

7. Will not change.
8. No, (highest point of vertical upward motion under gravity)
9. No example. Projectile motion.
10. The velocity vectors of a stationary object is a zero vectors.
11. They must represent the physical quantities of same nature.
12. When \vec{A} is perpendicular to \vec{B} .
13. Infinite.
14. Because horizontal component of gravity is zero along horizontal direction.
15. 90°

1 MARK

16. When does (i) height attained by a projectile maximum? (ii) horizontal range is maximum?
17. What is the angle between velocity vector and acceleration vector in uniform circular motion?
18. A particle is in clockwise uniform circular motion the direction of its acceleration is radially inward. If sense of rotation or particle is anticlockwise then what is the direction of its acceleration?
19. A train is moving on a straight track with acceleration a . A passenger drops a stone. What is the acceleration of stone with respect to passenger?
20. What is the average value of acceleration vector in uniform circular motion over one cycle?
21. Does a vector quantity depends upon frame of reference chosen?
22. What is the angular velocity of the hour hand of a clock?
23. What is the source of centripetal acceleration for earth to go round the sun?

ANSWERS

16. height is maximum at $\theta = 90$
Range is maximum at $\theta = 45$.
17. 90°
18. Radial inward.
19. $\sqrt{a^2 + g^2}$ where g = Acceleration due to gravity.
20. Null vector.
21. No.
22. $\omega = \frac{2\pi}{12} = \frac{\pi}{6} \text{ rad h}^{-1}$
23. Gravitation force of sun.

1 MARK

24. What is the unit vector perpendicular to the plane of vectors \vec{A} and \vec{B} ?
25. What is the angle between $(\vec{A} + \vec{B})$ and $(\vec{A} \times \vec{B})$?

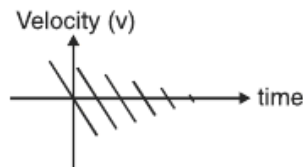
ANSWERS

24. $\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$
25. 90°

2 MARKS

1. What are positive and negative acceleration in straight line motion?
2. Can a body have zero velocity and still be accelerating? If yes gives any situation.

3. The displacement of a body is proportional to t^3 , where t is time elapsed. What is the nature of acceleration- time graph of the body?
5. Suggest a suitable physical situation for the following graph.



6. An object is in uniform motion along a straight line, what will be position time graph for the motion of the object if

(i) x_0 = positive, v = negative $|v|$ is constant

(ii) both x_0 and v are negative $|v|$ is constant

where x_0 is position at $t = 0$

ANSWERS

1. If speed of an object increases with time, its acceleration is positive. (Acceleration is in the direction of motion) and if speed of an object decreases with time its acceleration is negative (Acceleration is opposite to the direction of motion).
2. Yes, at the highest point of vertical upward motion under gravity.
3. as $s \propto t^3 \Rightarrow s = kt^3$

$$\text{velocity } V = \frac{ds}{dt} = 3kt^2$$

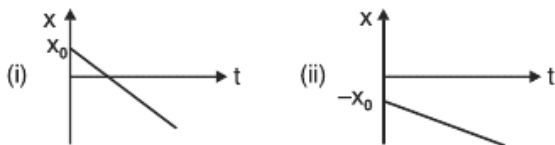
$$\text{acceleration } a = \frac{dv}{dt} = 6kt$$

$$\text{i.e } a \propto t$$

\Rightarrow motion is uniform, accelerated motion. $a - t$ graph is straight-line.

5. A ball thrown up with some initial velocity rebounding from the floor with reduced speed after each hit.

6.



7. A vector \vec{a} is turned through a small angle θ without a change in its length. What are $|\Delta \vec{a}|$ and Δa ?
8. What will be the effect on horizontal range of a projectile when its initial velocity is doubled keeping angle of projection same?
9. The greatest height to which a man can throw a stone is h . What will be the greatest distance upto which he can throw the stone?
10. A person sitting in a train moving at constant velocity throws a ball vertically upwards. How will the ball appear to move to an observer.
 - (i) Sitting inside the train
 - (ii) Standing outside the train
11. A gunman always keep his gun slightly tilted above the line of sight while shooting. Why?

ANSWERS

$$7. \quad |\Delta \vec{a}| = a \theta \quad \Delta a = 0$$

$$8. \quad R = \frac{u^2 \sin 2\theta}{g} \Rightarrow R \propto u^2$$

Range becomes four times.

$$9. \quad \text{Maximum height ; } H = \frac{u^2 \sin^2 \theta}{2g}$$

$$\Rightarrow H_{\max} = \frac{u^2}{2g} = h \text{ (at } \theta = 90^\circ \text{)}$$

$$\text{Max. Range } R_{\max} = \frac{u^2}{g} = 2h$$

10. (i) Vertical straight line motion
(ii) Parabolic path.
11. Because bullet follow parabolic trajectory under constant downward acceleration.

2 MARKS

12. Is the acceleration of a particle in circular motion not always towards the centre. Explain.

Ans. No, acceleration is towards the centre only in case of uniform circular motion.

3 MARKS

1. Derive the relation

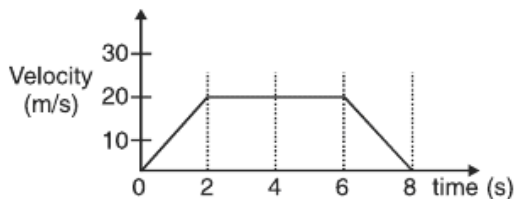
$$S_{nth} = u + \frac{a}{2}(2n-1)$$

where S_{nth} = distance travelled in n^{th} second

a = Uniform acceleration

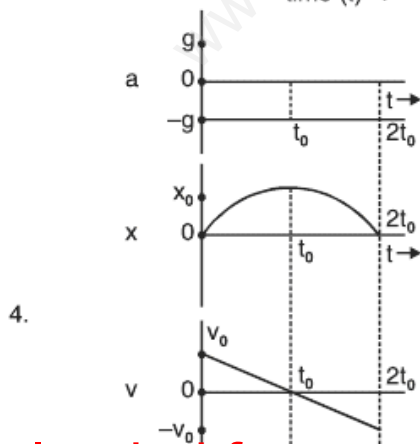
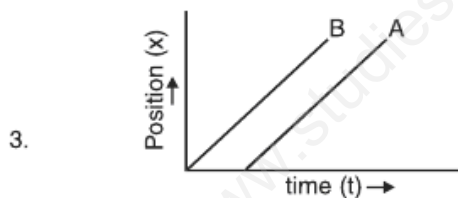
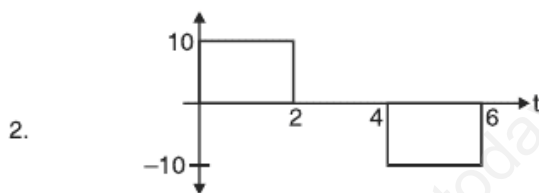
u = Initial speed

2. The velocity time graph for a particle is shown in figure. Draw acceleration time graph from it.



3. Draw position-time graphs of two objects, A and B moving along a straight line, when their relative velocity is
- (i) zero
4. For an object projected upward with a velocity v_0 , which comes back to the same point after some time, draw
- (i) Acceleration-time graph (ii) Position-time graph
- (iii) Velocity-time graph

ANSWERS



$2t_0 \rightarrow$ total time of the Journey

$x_0 \rightarrow$ highest position.

3 MARKS

5. Two vectors \vec{A} and \vec{B} are inclined to each other at an angle θ . Using triangle law of vector addition, find the magnitude and direction of their resultant.
6. Establish the following vector inequalities :

$$(i) \quad |\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$$

$$(ii) \quad |\vec{a} + \vec{b}| \geq |\vec{a}| - |\vec{b}|$$

7. A body is projected at an angle θ with the horizontal. Derive an expression for its horizontal range. Show that there are two angles θ_1 and θ_2 projections for the same horizontal range. such that $\theta_1 + \theta_2 = 90^\circ$
8. Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum range.
9. Show that there are two values of time for which a projectile is at the same height. Also show that the sum of these two times is equal to the time of flight.
10. Derive the relation between linear velocity and angular velocity in a uniform circular motion

5 MARKS

1. Derive the following equations of motion for an object moving with constant acceleration along a straight line using graphical method.

$$(i) \quad v = u + at$$

$$(ii) \quad S = ut + \frac{1}{2}at^2$$

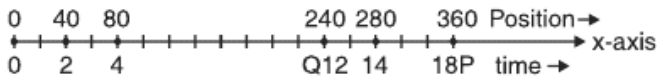
$$(iii) \quad v^2 = u^2 + 2as$$

Where symbols have usual meanings.

2. A projectile is projected horizontally with a velocity u . Show that its trajectory is parabolic. And obtain expression for
 - (i) time of flight
 - (ii) horizontal range
 - (iii) velocity at any instant.
3. Define centripetal acceleration. Derive an expression for the centripetal acceleration of a particle moving with constant speed v along a circular path of radius r .

NUMERICALS

1. The V-t graphs of two objects make angle 30° and 60° with the time axis. Find the ratio of their accelerations.
2. When the angle between two vectors of equal magnitudes is $2\pi/3$, prove that the magnitude of the resultant is equal to either.
3. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, find a vector having the same magnitude as \vec{B} and parallel to \vec{A} .
4. What is the angle made by vector $\vec{A} = 2\hat{i} + 2\hat{j}$ with x-axis?
5. What is the vector sum of n coplanar forces, each of magnitude F , if each force makes an angle of $\frac{2\pi}{n}$ with the preceding one?
6. A car is moving along x-axis. As shown in figure it moves from O to P in 18 seconds and return from P to Q in 6 second. What are the average velocity and average speed of the car in going from
 - (i) O to P
 - (ii) from O to P and back to Q.



$$1. \frac{a_1}{a_2} = \frac{\tan 30}{\tan 60} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3} \Rightarrow 1 : 3$$

$$\begin{aligned} 2. R &= (P^2 + Q^2 + 2PQ \cos 0)^{1/2} \\ &= \left(P^2 + P^2 + 2P \cdot P \cos \frac{2\pi}{3} \right)^{1/2} \\ &= \left[2P^2 + 2P^2 \left(\frac{-1}{2} \right) \right]^{1/2} = P \end{aligned}$$

$$3. |\vec{A}| = \sqrt{3^2 + 4^2} = 5$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{3\hat{i} + 4\hat{j}}{5}$$

$$\text{also } |\vec{B}| = \sqrt{7^2 + 24^2} = 25$$

$$\begin{aligned} \text{desired vector} &= |\vec{B}| \hat{A} = 25 \frac{(3\hat{i} + 4\hat{j})}{5} \\ &= 5(3\hat{i} + 4\hat{j}) = 15\hat{i} + 20\hat{j} \end{aligned}$$

$$4. \theta = \tan^{-1} \frac{2}{2} = 45^\circ$$

5. Resultant force is zero.

6. (i) O to P Average velocity = 20 ms^{-1}

$$\text{average speed} = 20 \text{ ms}^{-1}$$

(ii) o to P and back to Q

$$\text{Average velocity} = 10 \text{ ms}^{-1}$$

$$\text{Average speed} = 20 \text{ ms}^{-1}$$

NUMERICALS

8. On a 60 km straight road, a bus travels the first 30 km with a uniform speed of 30 kmh^{-1} . How fast must the bus travel the next 30 km so as to have average speed of 40 kmh^{-1} for the entire trip?
9. The displacement x of a particle varies with time as $x = 4t^2 - 15t + 25$.
Find the position, velocity and acceleration of the particle at $t = 0$.
10. A driver take 0.20 second to apply the breaks (reaction time). If he is driving car at a speed of 54 kmh^{-1} and the breaks cause a deceleration of 6.0 ms^{-2} . Find the distance travelled by car after he sees the need to put the breaks.
11. A body covers 12 m in 2nd second and 20 m in 4th second. How much distance will it cover in 4 seconds after the 5th second.
12. A ball thrown vertically upwards with a speed of 19.6 ms^{-1} from the top of a tower returns to the earth in 6s. Find the height of the tower ($g = 9.8 \text{ m/s}^2$)
13. Two town A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed do the buses ply of the road?

ANSWER

$$8. \quad V_{\text{avg}} = \frac{S_1 + S_2}{t_1 + t_2} = \frac{S + S}{S \left(\frac{1}{V_1} + \frac{1}{V_2} \right)} = \frac{2 V_1 V_2}{V_1 + V_2}$$

$$\text{or} \quad 40 = \frac{2 \times 30 \times v_2}{30 + v_2} \Rightarrow v_2 = 60 \text{ kmh}^{-1}$$

9. position $x = 25 \text{ m}$

$$\text{velocity } v = \frac{dx}{dt} \quad v = -15 \text{ ms}^{-1}$$

$$\text{acceleration } a = \frac{dv}{dt} = -8 \text{ ms}^{-2}$$

10. (distance covered during 0.20 s) +

(distance covered until rest)

$$= (15 \times 0.20) + [18.75] = 21.75 \text{ m}$$

11. $S_{2\text{nd}} = u + \frac{a}{2}(2 \times 2 - 1) \Rightarrow 4 + \frac{3}{2}a = 12$

$$S_{4\text{th}} = u + \frac{a}{2}(2 \times 4 - 1) \Rightarrow 4 + \frac{7}{2}a = 20$$

$$\Rightarrow u = 6\text{ms}^{-1} \quad \text{and} \quad a = 4\text{ms}^{-1}$$

$$\text{According to question} = 5g - 5_s \left(s = ut + \frac{1}{2}at^2 \right)$$

$$= 136 \text{ m}$$

12. using $s = ut + \frac{1}{2}at^2$

$$-h = 19.6 \times 6 + \frac{1}{2} \times (-9.8) \times 62$$

$$h = 58.8 \text{ m}$$

13. $V = 40 \text{ kmh}^{-1}$ and $T = 9 \text{ min}$

NUMERICALS

14. A motorboat is racing towards north at 25 kmh^{-1} and the water current in that region is 10 kmh^{-1} in the direction of 60° east of south. Find the resultant velocity of the boat.
15. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft position 10 second apart is 30° , what is the speed of the aircraft?
16. A boat is moving with a velocity $(3\hat{i} - 4\hat{j})$ with respect to ground. The water in river is flowing with a velocity $(-3\hat{i} - 4\hat{j})$ with respect to ground.

What is the relative velocity of boat with respect to river?

17. A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 ms^{-1} . Neglecting air resistance, find the time taken by the stone to reach the ground and the speed with which it hits the ground ($g = 9.8 \text{ ms}^{-2}$)
18. A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting the angle of projection, can one hope to hit the target 5 km away? Assume that the muzzle speed to be fixed and neglect air resistance.

ANSWER

14. $V = 21.8 \text{ kmh}^{-1}$

angle with north $\theta = 23.4^\circ$

15. Speed = 182.2 ms^{-1}

16. $\vec{v}_{\text{BW}} = \vec{v}_B - \vec{v}_W$

$$= 6\hat{i} + 8\hat{j}$$

17. time = 10 seconds

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{15^2 + 98^2} = 99.1 \text{ ms}^{-1}$$

18. Maximum Range = 3.46 km

So it is not possible.

NUMERICALS

19. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone?
20. A cyclist is riding with a speed of 27 kmh^{-1} . As he approaches a circular turn on the road of radius 30 m, he applies brakes and reduces his speed at the constant rate 0.5 ms^{-2} . What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

ANSWERS

$$19. \quad w = \frac{88}{25} \text{ rad s}^{-1} \quad w = \frac{2\pi}{T} = \frac{2\pi N}{t}$$

$$a = 991.2 \text{ cms}^{-2}$$

$$20. \quad a_c = \frac{v^2}{r} = 0.7 \text{ ms}^{-2}$$

$$a_T = 0.5 \text{ ms}^{-2}$$

$$a = \sqrt{a_c^2 + a_T^2} = 0.86 \text{ ms}^{-2}$$

If θ is the angle between the net acceleration and the velocity of the cyclist, then

$$\theta = \tan^{-1} \frac{a_c}{a_T} = \tan^{-1}(1.4) = 54^\circ 28'$$

NUMERICAL

21. If the magnitude of two vectors are 3 and 4 and their scalar product is 6, find angle between them.
22. Find the value of λ so that the vector $\vec{A} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{B} = 4\hat{i} - 2\hat{j} - 2\hat{k}$ are perpendicular to each other.
23. If \hat{i} and \hat{j} are unit vectors along X and y-axis respectively, then what is the magnitude and direction of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$.

ANSWERS

$$21. \quad \vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\text{or } 6 = (3 \times 4) \cos \theta$$

$$\text{or } \theta = 60^\circ$$

$$22. \quad \therefore \vec{A} \perp \vec{B} \Rightarrow \vec{A} \cdot \vec{B} = 0$$