UNIT – VI

GRAVITATION

- **Newton's law of gravitation.** It states that the gravitational force of attraction acting between two bodies of the universe is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them, *i.e.*, $F = G\frac{m_1 m_2}{r^2}$; where G is the universal gravitational constant.

  The value of $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}

- **Gravity :** It is the force of attraction exerted by earth towards its centre on a body lying on or near the surface of earth.

- **Acceleration due to gravity (g).** It is defined as the acceleration set up in a body while falling freely under the effect of gravity alone. It is a vector quantity.

$$g = \frac{GM}{R^2}$$

  where M and R are the mass and radius of the earth.

- **Variation of acceleration due to gravity.**

  (i) Effect of altitude, $g' = \frac{g R^2}{(R+h)^2}$ and $g' = g\left(1 - \frac{2h}{R}\right)$

  The first relation is valid when h is comparable with R and the second relation is valid when $h << R$. The value of g decreases with increase in h.

  (ii) **Effect of depth,** $g' = g\left(1 - \frac{d}{R}\right)$

  The acceleration due to gravity decreases with increase in depth d and becomes zero at the centre of earth.

- **Gravitational field.** It is the space around a material body in which its gravitational pull can be experienced by other bodies.

  The intensity of gravitational field at a point at a distance r from the centre of the body of mass M is given by $I = GM/r^2$ = g (acceleration due to gravity).

- **Gravitational potential.** The gravitational potential at a point in a gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration. Gravitational potential at a point, $V = \frac{\text{work done}(W)}{\text{test mass}(m_0)} = \frac{GM}{r}$

- Gravitational potential energy U = gravitational potential × mass of body

  $$= -\frac{GM}{r} \times m$$

  Gravitational intensity (I) is related to gravitational potential (V) at a point by the relation, $-\frac{dv}{dr}$

- **Satellite.** *A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.*

  (i) Orbital speed of a satellite when it is revolving around earth at height h is given by

  $$v_0 = R\sqrt{\frac{g}{R+h}}$$

  When the satellite is orbiting close to the surface of earth, *i.e.*, $h << R$, then

  $$v_0 = R\sqrt{\frac{g}{R}} = \sqrt{g\,R}$$

  (ii) Time period of satellite *(T)*. It is the time taken by the satellite to complete one revolution around the earth.

  $$T = \frac{2\pi(R+h)}{v_0} = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$$

(iii) Height of satellite above the earth's surface :

$$h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$$

(iv) Total energy of satellite, E = P.E. + K.E.

$$E = -\frac{GM\,m}{(R+h)} + \frac{1}{2}mv_0^2 = -\frac{GM\,m}{(R+h)} + \frac{1}{2}m\left(\frac{GM}{R+h}\right) = -\frac{GM\,m}{2(R+h)}$$

If the satellite is orbiting close to earth, then $r = R$. Now total energy of satellite.

$$E = -\frac{GM\,m}{2R}$$

(v) **Binding energy of satellite.** $= -E = \dfrac{GM\,m}{2r}$

- **Escape speed.** The escape speed on earth is defined as the minimum speed with which a body has to be projected vertically upwards from the surface of earth so that it just crosses the gravitational field of earth. Escape velocity v_e is given by,

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2\,g\,R}$$

For earth, the value of escape speed is 11.2 kms^{-1}.

VERY SHORT ANSWER TYPE QUESTION (1 MARK EACH)

Q 1. The mass of moon is nearly 10% of the mass of the earth. What will be the gravitational force of the earth on the moon, in comparison to the gravitational force of the moon on the earth?

Q 2. Why does one feel giddy while moving on a merry round?

Q 3. Name two factors which determine whether a planet would have atmosphere or not.

Q 4. The force of gravity due to earth on a body is proportional to its mass, then why does a heavy body not fall faster than a lighter body?

Q 5. The force of attraction due to a hollow spherical shell of uniform density on a point mass situated inside is zero, so can a body be shielded from gravitational influence?

Q 6. The gravitational force between two bodies in 1 N if the distance between them is doubled, what will be the force between them?

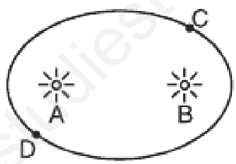
Q 7. A body of mass 5 kg is taken to the centre of the earth. What will be its (i) mass (ii) weight there.

Q 8. Why is gravitational potential energy negative?

Q 9. A satellite revolves close to the surface of a planet. How is its orbital velocity related with escape velocity of that planet.

Q 10. Does the escape velocity of a body from the earth depend on (i) mass of the body (ii) direction of projection

Q 11. Identify the position of sun in the following diagram if the linear speed of the planet is greater at C than at D.



Q 12. A satellite does not require any fuel to orbit the earth. Why?

Q 13. A satellite of small mass burns during its desent and not during ascent. Why?

Q 14. Is it possible to place an artificial satellite in an orbit so that it is always visible over New Delhi?

Q 15. If the density of a planet is doubled without any change in its radius, how does 'g' change on the planet.

Q 16. Mark the direction of gravitational intensity at (i) centre of a hemispherical shell of uniform mass density (ii) any arbitrary point on the upper surface of hemisphere.

Q 17. Why an astronaut in an orbiting space craft is not in zero gravity although weight less?

Q 18. Write one important use of (i) geostationary satellite (ii) polar satellite.

Q 19. A binary star system consists of two stars A and B which have time periods T_A and T_B, radius R_A and R_B and masses m_A and m_B which of the three quantities are same for the stars. Justify.

Q 20. The time period of the satellite of the earth is 5 hr. If the separation between earth and satellite is increased to 4 times the previous value, then what will be the new time period of satellite.

Q 21. The distance of Pluto from the sun is 40 times the distance of earth if the masses of earth and Pluto he equal, what will be ratio of gravitational forces of sun on these planets.

Q 22. If suddenly the gravitational force of attraction between earth and satellite become zero, what would happen to the satellite?

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

Q 1. If the radius of the earth were to decrease by 1%, keeping its mass same, how will the acceleration due to gravity change?

Q 2. If 'g' be the acceleration due to gravity on earth's surface. Calculate the gain in potential energy of an object of mass m raised from the surface of earth to a height equal to the radius or earth in term of 'g'.

Q 3. A satellite is moving round the earth with velocity v_0 what should be the minimum percentage increase in its velocity so that the satellite escapes.

Q 4. Two planets of radii r_1 and r_2 are made from the same material. Calculate the ratio of the acceleration due to gravity on the surface of the planets.

Q 5. If earth has a mass 9 times and radius 4 times than that of a planet 'P'. Calculate the escape velocity at the planet 'P' if its value on earth is 11.2 kms^{-1}

Q. 6. At what height from the surface of the earth will the value of 'g' be reduced by 36% of its value at the surface of earth.

Q. 7. At what depth is the value of 'g' same as at a height of 40 km from the surface of earth.

Q. 8. The mean orbital radius of the earth around the sun is 1.5×10^8 km. Calculate mass of the sun if $G = 6.67 \times 10^{-11}$ N m^2/kg^{-2}?

Q. 9. Draw graphs showing the variation of acceleration due to gravity with (i) height (ii) depth from the surface of earth.

Q. 10. Which planet of the solar system has the greatest gravitational field strength? What is the gravitational field strength of a planet where the weight of a 60 kg astronaut is 300 N.

Q. 11. Two satellites are at different heights from the surface of earth which would have greater velocity. Compare the speeds of two satellites of masses m and 4m and radii 2R and R respectively.

Q. 12. What is (i) inertial mass, (ii) gravitational mass. Are the two different?

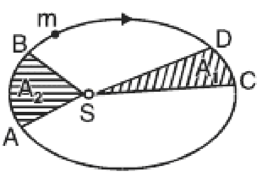
Q. 13. Why the space rockets are generally launched west to East?

Q. 14. Explain why a tennis ball bounces higher on hills than in plane?

Q. 15. The gravitational force on the earth due to the sun is greater than moon. However tidal effect due to the moon's pull is greater than the tidal effect due to sun. Why?

Q. 16. The mass of moon is $\frac{M}{81}$ (where M is mass of earth). Find the distance of the point where the gravitational field due to earth and moon cancel each other. Given distance of moon from earth is 60 R, where R is radius of earth.

Q. 17. The figure shows elliptical orbit of a planet m about the sun S. The shaded area of SCD is twice the shaded area SAB. If t_1 is the time for the planet to move from D to C and t_2 is time to move from A to B, what is the relation between t_1 and t_2?



Q. 18. Calculate the energy required to move a body of mass m from an orbit of radius 2R to 3R.

Q. 19. A man can jump 1.5 m high on earth. Calculate the height he may be able to jump on a planet whose density is one fourth that of the earth and whose radius is one third of the earth.

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

Q 1. Define gravitational potential at a point in the gravitational field. Obtain a relation for it. What is the position at which it is (i) maximum (ii) minimum.

Q 2. Find the potential energy of a system of four particles, each of mass m, placed at the vertices of a square of side. Also obtain the potential at the centre of the square.

Q 3. Three mass points each of mass m are placed at the vertices of an equilateral triangle of side l. What is the gravitational field and potential at the centroid of the triangle due to the three masses.

Q 4. Briefly explain the principle of launching an artificial satellite. Explain the use of multistage rockets in launching a satellite.

Q 5. In a two stage launch of a satellite, the first stage brings the satellite to a height of 150 km and the 2nd stage gives it the necessary critical speed to put it in a circular orbit. Which stage requires more expenditure of fuel? Given mass of earth = 6.0×10^{24} kg, radius of earth = 6400 km

Q 6. The escape velocity of a projectile on earth's surface is 11.2 kms^{-1} A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Q 7. A satellite orbits the earth at a height 'R' from the surface. How much energy must be expended to rocket the satellite out of earth's gravitational influence?

Q 9. Deduce the law of gravitation from Kepler's laws of planetary motion.

Q 10. Mention at least three conditions under which weight of a person can become zero.

LONG ANSWER TYPE QUESTIONS (5 MARKS)

Q 1. What is acceleration due to gravity?

Obtain relations to show how the value of 'g' changes with (i) attitude (ii) depth

Q 2. Define escape velocity obtain an expression for escape velocity of a body from the surface of earth? Does the escape velocity depend on (i) location from where it is projected (ii) the height of the location from where the body is launched.

Q 3. State Kepler's three laws of planetary motion. Prove the second and third law.

Q 4. Derive expression for the orbital velocity of a satellite and its time period.

What is a geostationary satellite. Obtain the expression for the height of the geostationary satellite.

Q 5. State universal law of gravitation. Explain briefly how Newton discovered the universal law of gravitation.

Q 6. Define the term gravitational potential energy. Is it a scalar or vector? Derive an expression for the gravitational potential energy at a point in the gravitational field of earth.

NUMERICALS

Q 1. The mass of planet Jupiter is 1.9×10^{27} kg and that of the sun is 1.99×10^{30}kg. The mean distance of Jupiter from the Sun is 7.8×10^{11}m. Calculate gravitational force which sun exerts on Jupiter, and the speed of Jupiter.

Q 2. A mass 'M' is broken into two parts of masses m_1 and m_2. How are m_1 and m_2 related so that force of gravitational attraction between the two parts is maximum.

Q 3. If the radius of earth shrinks by 2%, mass remaining constant. How would the value of acceleration due to gravity change ?

Q 4. A body hanging from a spring stretches it by 1 cm at the earth's surface. How much will the same body stretch at a place 1600 k/m above the earth's surface? Radius of earth 6400 km.

Q 5. Imagine a tunnel dug along a diameter of the earth. Show that a particle dropped from one end of the tunnel executes simple harmonic motion. What is the time period of this motion?

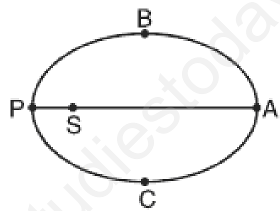
Q 6. The gravitational field intensity at a point 10,000 km from the centre of the earth is 4.8 N kg^{-1}. Calculate gravitational potential at that point.

Q 7. A geostationary satellite orbits the earth at a height of nearly 36000 km. What is the potential due to earth's gravity at the site of this satellite (take the potential energy at ∞ to be zero). Mass of earth is 6×10^{24} kg, radius of earth is 6400 km.

Q 8. How much faster than the present speed should the earth rotate so that bodies lying on the equator may fly off into space.

Q 9. The distance of Neptune and Saturn from the sun is nearly 10^{13}m and 10^{12}m respectively. Assuming that they move in circular orbits, then what will be the ratio of their periods.

Q 10. Let the speed of the planet at perihelion P in fig be v_p and Sun planet distance SP be r_p Relate (r_p, v_p) to the corresponding quantities at the aphelion (r_A, v_A). Will the planet take equal times to traverse BAC and CPB?



ANSWER FOR VERY SHORT QUESTIONS (1 MARK)

1. Both forces will be equal in magnitude as gravitational force is a mutual force between the two bodies.

2. When moving in a merry go round, our weight appears to decrease when we move down and increases when we move up, this change in weight makes us feel giddy.

3. (i) Value of acceleration due to gravity (ii) surface temperature of planet.

4. $\because F = \frac{GMm}{R^2}$ $F \propto m$ but $g = \frac{Gm}{R^2}$ and does not depend on 'm' hence they bodies fall with same 'g'.

5. No, the gravitational force is independent of intervening medium.

6. $F = 1$ $F' = \frac{F}{4}$

7. Mass does not change.

8. Because it arises due to attractive force of gravitation.

9. $\nu_e = \sqrt{2}\ \nu_o \quad \because \quad \nu_e = \sqrt{\dfrac{2\,GM}{R}} \quad \nu_o = \sqrt{\dfrac{GM}{R}}$ when $r = R$

11. No, $\nu_e = \sqrt{\dfrac{2\,GM}{R}}$

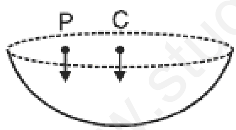
12. Sun should be at B as speed of planet is greater when it is closer to sun.

13. The gravitational force between satellite and earth provides the necessary centripetal force for the satellite to orbit the earth.

14. The speed of satellite during descent is much larger than during ascent, and so heat produced is large.

15. No, A satellite will be always visible only if it revolves in the equatorial plane, but New Delhi does not lie in the region of equitorial plane.

16. 'g' gets doubled as $g \propto \rho$ (density)

17. 

In both cases it will be downward

18. The astronaut is in the gravitational field of the earth and experiences gravity. However, the gravity is used in providing necessary centripetal force, so is in a state of free fall towards the earth.

19. Geostationary satellite are used for tele communication and polar satellite for remote rensing.

20. Angular velocity of binary stars are same is $w_A = w_B$,

$$\frac{2\pi}{T_A} = \frac{2\pi}{T_B} \quad \Rightarrow \quad T_A = T_B$$

21. $\dfrac{T_2^2}{T_1^2} = \left(\dfrac{R_2}{R_1}\right)^3 \Rightarrow T_2^2 = 64 \times 25 \Rightarrow T_2 = 40$ hr

22. $\frac{F_{es}}{F_{ps}} = \left(\frac{r_p}{r_e}\right)^2 = \left(\frac{40\, r_e}{r_e}\right)^2 = 1600:1$

23. The satellite will move tangentially to the original orbit with a velocity with which it was revolving.

SHORT ANSWER (2 MARKS)

1. $g = \frac{GM}{R^2}$ If R decreases by 1% it becomes $\frac{99}{100}R$

$$\therefore g' = \frac{GM}{(.99\,R)^2} = 1.02\frac{GM}{R^2} = (1+0.02)\frac{GM}{R^2}$$

$\therefore$ g' increases by 0.02 $\frac{GM}{R^2}$, therefore increases by 2%.

2. Gain in PE $= -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{2R} = \frac{g\,R^2\,m}{2\,R} = \frac{1}{2}mg\ R$

3. The maximum orbital velocity of a satellite orbiting near its surface is $\nu_o = \sqrt{gR} = \frac{\nu_e}{\sqrt{2}}$

For the satellite to escape gravitational pull the velocity must become v_e

But $\nu_e = \sqrt{2}\nu_o = 1.414\,\nu_o = (1+0.414)\nu_o$

This means that it has to increases 0.414 in 1 or 41.4%

$\therefore$ The minimum increase required, as the velocity of satellite is maximum when it is near the earth.

4. $g = \frac{GM}{r^2} = \frac{G\rho\frac{4}{3}\pi r^3}{r^2} = \frac{4\pi}{3}G\rho r$

$g \propto r$

$\therefore \frac{g_1}{g_2} = \frac{r_1}{r_2}$

5. $\nu_e = \sqrt{\frac{2GM}{R_e}}$ $\nu_p = \sqrt{\frac{2\,GM_p}{R_p}}$ $M_p = \frac{M}{9}, R_p = \frac{R_e}{4}$

$$\therefore\ \nu_p = \sqrt{2\,G\frac{M}{9}\times\frac{4}{R_e}} = \frac{2}{3}\sqrt{\frac{2GM}{R_e}} = \frac{2}{3}\times 11.2 = \frac{22.4}{3}$$

$= 7.47$ km/sec

6. $g' = 64\%$ of $g = \frac{64}{100}g$

$$g' = g\frac{R^2}{(R+h)^2} = \frac{64}{100}g$$

$$\therefore\ \frac{R}{R+h} = \frac{8}{10}$$

$$h = \frac{R}{4} = 1600\text{km}$$

7. $g_d = g_h$

$$g\left(1-\frac{d}{R}\right) = g\left(1-\frac{2h}{R}\right)$$

$d = 2h = 2 \times 40 = 80$ km

8. $R = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m

$T = 365$ days $= 365 \times 24 \times 3600$ s

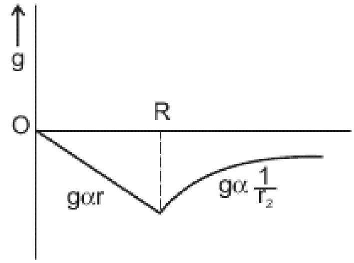
Centripetal force = gravitational force

$$\frac{m\nu^2}{R} = \frac{GMm}{R^2} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = \frac{GMm}{R^2}$$

$$M_s = \frac{4\pi^2R^3}{G\,T^2} = \frac{4\times 9.87\times(1.5\times 10^{11})^3}{6.64\times 10^{-11}\times(365\times 24\times 3600)^2}$$

$M_s = 2.01 \times 10^{30}$ kg

9.



$g \propto \frac{1}{r^2}$ above surface of earth; r = R + h

$g \propto r$ below surface of earth; r = R – d

g is max for r = R on surface of earth.

10. Jupiter has maximum gravitational field strength gravitational field strength

$$= \frac{F}{m} = \frac{300}{60}$$

$$= 5 \text{ N kg}^{-1}$$

11. $$\nu_0 = \sqrt{\frac{GM}{(R+h)}}$$

∴ Velocity of satellite closer to the earth's surface will be greater

$$\nu_{01} = \sqrt{\frac{GM}{2R}} \qquad \nu_{02} = \sqrt{\frac{GM}{R}}$$

$$\therefore \quad \frac{\nu_{01}}{\nu_{02}} = \left(\frac{1}{\sqrt{2}}\right),$$ where M is mass of the planet,

v_0 is independent of mass of the satellite.

12. Inertial mass is the measure of inertia of the body $= m_i = \frac{F}{a}$

Gravitational mass of a body determine the gravitational pull between earth and the body.

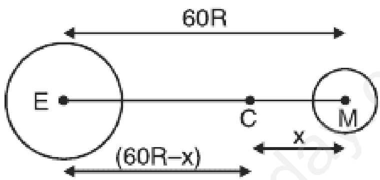
$$m_g = \frac{FR^2}{GM}$$

Both inertial mass and gravitational mass are not different but are equivalent.

13. Since the earth revolves from west to east, so when the rocket is launched from west to east the relative velocity of the rocket increases which helps it to rise without much consumption of fuel.

14. The value of 'g' on hills is less than at the plane, so the weight of tennis ball on the hills is lesser force than at planes that is why the earth attract the ball on hills with lesser force than at planes. Hence the ball bounces higher.

15. The tidal effect depends inversely on the cube of the distance, while gravitational force depends on the square of the distance.

16. 

Gravitational field at C due to earth

= Gravitational field at C due to earth moon

$$\frac{GM}{(60R-x)^2} = \frac{GM/81}{x^2}$$

$$81x^2 = (60R - x)^2$$

$$9x = 60\ R - x$$

$$x = 6\ R$$

17. According to Kepler's IInd law areal velocity for the planet is constant

$$\therefore \quad \frac{A_1}{t_1} = \frac{A_2}{t_2} \quad A_1 = 2A_2$$

$$\therefore \quad \frac{2A_2}{t_1} = \frac{A_2}{t_2}$$

$$t_1 = 2t_2$$

18. Gravitational P.E of mass m in orbit of radius R = $U = -\frac{GMm}{R}$

$$\therefore \quad U_i = -\frac{GMm}{2R}$$

$$U_f = -\frac{GMm}{3R}$$

$$\Delta U = U_f - U_i = GMm\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= \frac{GMm}{6R}$$

19. $g = \frac{4}{3}\pi G R \rho$

$$g' = \frac{4}{3}\pi G R' \rho'$$

The gain in P.E at the highest point will be same in both cases. Hence

mg'h' = mgh

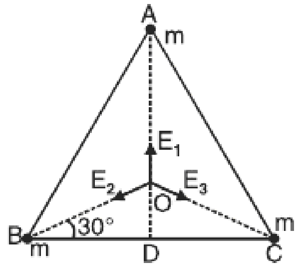
$$h' = \frac{mgh}{mg'} = \frac{m \times \frac{4}{3}\pi G R \rho h}{m \frac{4}{3}\pi G R' \rho'}$$

$$= \frac{R\rho h}{R'\rho'} = \frac{3R' \times 4\rho' \times 1.5}{R' \times \rho'}$$

= 18 m

ANSWER FOR 3 MARKS QUESTIONS

3.



$$E_1 = \frac{GM}{(OA)^2}$$

$$E_2 = \frac{GM}{(OB)^2}$$

$$E_3 = \frac{GM}{(OC)^2}$$

From Δ ODB $\text{Cos } 30° = \frac{BD}{OB} = \frac{\ell/2}{OB}$

$$OB = \frac{\ell/2}{\text{Cos}\,30°} = \frac{\ell}{2}\frac{2}{\sqrt{3}} = \ell/\sqrt{3}$$

Gravitational field at O due to m at A, B and C is say $\vec{E_1}, \vec{E_2}$ & $\vec{E_3}$

$$E = \sqrt{E_2^2 + E_3^2 + 2E_2E_3\text{Cos}120°}$$

$$= \sqrt{\frac{(GM3)^2}{l^2} + \left(\frac{3Gm}{l}\right)^2 + 2\left(\frac{3GM}{l}\right)\left(\frac{3GM}{l}\right)\left(-\frac{1}{2}\right)}$$

$$= \frac{3GM}{l} \text{ along OD}$$

$\vec{E}$ is equal and opposite to $\vec{E_1}$

$\therefore$ net gravitational field = zero

As gravitational potential is scalar

$$V = V_1 + V_2 + V_3$$

$$= -\frac{GM}{OA} - \frac{GM}{OB} - \frac{GM}{OC}$$

$$V = -\frac{3GM}{l/\sqrt{3}} = -3\sqrt{3}\frac{Gm}{l}$$

5. Work done on satellite in first stage = W_1 = PE at 150 km – PE at the surface

$$W_1 = -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$

$$= \frac{GMmh}{R(R+h)}$$

Work done on satellite in 2nd stage = W_2 = energy required to give orbital velocity v_o

$$= \frac{1}{2}m\nu_0^2 = \frac{1}{2}\left(\frac{GMm}{R+h}\right)$$

$$\frac{W_1}{W_2} = \frac{2h}{R} = \frac{2 \times 150}{6400} = \frac{3}{64} < 1$$

$\therefore$ $W_2 > W_1$ so second stage requires more energy

6. v_e = 11.2 kms^{-1}, velocity of projection = v = $3v_e$ Let m be the mass of projectile and v_o the velocity after it escapes gravitational pull.

By law of conservation of energy

$$\frac{1}{2}m\nu_0^2 = \frac{1}{2}m\nu^2 - \frac{1}{2}m\nu_e^2$$

$$\nu_0 = \sqrt{\nu^2 - \nu_e^2} = \sqrt{9\nu_e^2 - \nu_e^2} = \sqrt{8\nu_e^2}$$

$$= 22.4\sqrt{2}$$

$$= 31.68 \text{ km s}^{-1}$$

7. The energy required to pull the satellite from earth influence should be equal to the total energy with which it is revolving around the earth.

The K.E. of satellite $= \frac{1}{2}m\nu^2 = \frac{1}{2}m\frac{GM}{R+h}$ $\quad \therefore \nu = \sqrt{\frac{GM}{R+h}}$

The P.E of satellite $= -\frac{GMm}{R+h}$

$\therefore$ T.E. $= \frac{1}{2}\frac{mGM}{(R+h)} - \frac{GMm}{(R+h)} = -\frac{1}{2}\frac{GMm}{(R+h)}$

$\therefore$ Energy required will be $\left(+\frac{1}{2}\frac{GMm}{(R+h)}\right)$

9. Suppose a planet of mass m, moves around the sun in a circular orbit of radius 'r' with velocity v.

Then centripetal force $F = \frac{m\nu^2}{r}$

But $= \nu = \frac{2\pi r}{T}$

$F = m\frac{4\pi^2 r^2}{rT^2} = \frac{mr4\pi^2}{T^2}$

According to Kepler's IIIrd law

$T^2 \alpha r^3$

$T^2 = k r^3$

$\therefore F = \frac{mr\,4\pi^2}{k\,r^3} = \frac{4\pi^2 m}{kr^2} = \frac{4\pi^2}{k}\frac{m}{r^2}$

The force between planet and sun must be mutual, so must be proportional to mass of sun.

$\frac{4\pi^2}{k} \alpha m \quad \Rightarrow \quad \frac{4\pi^2}{k} = GM$

$\therefore F = \frac{GMm}{r^2}$ This is Newton's law of gravitation

10. (i) When the person is at centre of earth.

(ii) When the person is at the null points in space (at these points the gravitational forces due to different masses cancel each other)

(iii) when a person is standing in a freely falling lift.

(iv) When a person is inside a space craft which is orbiting around the earth.

ANSWER FOR NUMERICALS

1. $F = \frac{GMm}{r^2}$

$$= \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 1.9 \times 10^{27}}{\left(7.8 \times 10^{11}\right)^2}$$

$F = 4.1 \times 10^{23}$ N

$$\therefore F = \frac{m\nu^2}{r} \quad \Rightarrow \quad \nu = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{GMm}{r^2} \times \frac{r}{m}}$$

$$\nu = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 1.9 \times 10^{30}}{7.8 \times 10^{11}}}$$

$v = 1.3 \times 10^4$ m s^{-1}

2. Let $m_1 = m$ then $m_2 = M - m$

Force between them when they are separated by distance 'r'

$$F = \frac{Gm(M-m)}{r^2} = \frac{G}{r^2}\left(Mm - m^2\right)$$

For F to be maximum, differentiate F w.r.t m and equate to zero

$$\frac{dF}{dm} = \frac{G}{r^2}(M - 2m) = 0$$

$M = 2m$; $m = \frac{M}{2}$

$\therefore m_1 = m_2 = \frac{M}{2}$

3. increases by 4%

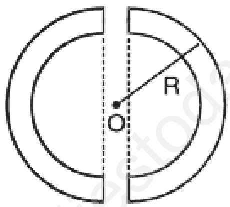
4. In equilibrium mg = kx, $g = \frac{GM}{R^2}$

at height h mg' = kx', $g' = \frac{GM}{(R+h)^2}$

$$\frac{g'}{g} = \frac{x'}{x} = \frac{R^2}{(R+h)^2}$$

$$\frac{x'}{x} = \frac{(6400)^2}{(6400+1600)^2} = \frac{16}{25} \qquad \therefore\ x' = \frac{16}{25} \times 1\,\text{cm} = 0.64\ \text{cm}$$

5.



The acceleration due to gravity at a depth below the earth's surface is given by

$$g_d = g\left(1 - \frac{d}{R}\right) = g\left(\frac{R-d}{R}\right)$$

$= \frac{g}{R}y$ where y is distance from centre of earth

$g_d \propto y$

As acceleration is proportional to displacement and is directed towards mean position, the motion would be S.H.M

$$T = \text{Time period} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$= 2\pi\sqrt{\frac{y}{g_d}}$$

$$= 2\pi\sqrt{\frac{R}{g}}$$

6. Gravitational intensity = $E = \frac{GM}{R^2}$

Gravitational potential $V = -\frac{GM}{R}$

$\therefore \frac{V}{E} = -R$

or, $V = -E \times R$

or $V = -4.8 \times 10{,}000 \times 10^3 = -4.8 \times 10^7$ J kg^{-1}

7. U = Potential at height $h = -\frac{GM}{R+h}$

$$U = -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6 + 36 \times 10^6} = -9.44 \times 10^6 \text{ J/kp}$$

8. The speed of earth = $\nu = \omega R = \frac{2\pi}{T} \times R$

at present

$$\nu = \frac{2\pi \times 6400 \times 10^3}{24 \times 3600}$$

The gravitational force should be equal to the centripetal force so that centrifugal force, given by $\frac{mv'^2}{R}$

$$F = \frac{GMm}{R^2} = \frac{mv'^2}{R}$$

$$v' = \sqrt{\frac{GM}{R}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6400 \times 10^3}}$$

$\therefore \frac{\nu'}{\nu} = 17$. $\therefore$ The velocity should become 17 times the present velocity

9. By kepler's III[rd] law

$$\left(\frac{T_n}{T_s}\right)^2 = \left(\frac{R_n}{R_s}\right)^3$$

$$\frac{T_n}{T_5} = \left(\frac{R_n}{R_s}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2}$$

$$= 10\sqrt{10} = 10 \times 3.16 = 31.6$$

$\therefore \; T_n : T_s = 36.6 : 1$

10. The magnitude of angular momentum at P is $L_p = m_p \, r_p \, v_p$

Similarly magnitude of angular momentum at A is $L_A = m_A r_A v_A$

From conservation of angular momentum

$m_p r_p v_p = m_A v_A r_A$

$$\frac{\nu_P}{\nu_A} = \frac{r_A}{r_P}$$

$\because \; r_A > r_P, \; \therefore \nu_P > \nu_A$

area bounded by SB & SC(SBAC > SBPC)

$\because$ By 2[nd] law equal areas are swept in equal intervals of time. Time taken to transverse BAC > time taken to traverse CPB