

Chapter: - Trigonometric functions

Q1. The angle of a triangle are in A.P. such that the greater is 5 times the least. Find the angle in radian.

Ans. $\frac{\pi}{9}, \frac{\pi}{3}, \frac{5\pi}{9}$.

Q2. Find the magnitude in radian and degree of the interior of a regular pentagon. **Ans.** $\frac{3\pi}{5}, 108^\circ$.

Q3. If $\cos x = \frac{-3}{5}$ and $\pi < x < \frac{3\pi}{2}$, find the value of other five trigonometric functions and hence evaluate $\frac{\cos ec x + \cot x}{\sec x - \tan x}$.

Ans. $\sin x = \frac{-4}{5}, \tan x = \frac{4}{3}, \cot x = \frac{3}{4}, \sec x = \frac{-5}{3}, \cos ec x = \frac{-5}{4}, \frac{1}{6}$.

Q4. Find the value of (i) $\cot 15^\circ$ (ii) $\cot 105^\circ$ (iii) $\tan 7\frac{1}{2}^\circ$ (iv) $\sin 22\frac{1}{2}^\circ$ (v) $\sin 18^\circ$ (vi) $\cos 36^\circ$ (vii) $\sin 54^\circ$

Ans. (i) $(2 + \sqrt{3})(ii)(\sqrt{3} - 2)(iii)(\sqrt{6} - \sqrt{4} - \sqrt{3} + \sqrt{2})(iv)(\frac{1}{2}\sqrt{2 - \sqrt{2}})(v)(\frac{\sqrt{5} - 1}{4})(vi)(\frac{\sqrt{5} + 1}{4})(vii)(\frac{\sqrt{5} + 1}{4})$

Q5. If $\cos x = \frac{-1}{3}$ where x lies in IIIrd quadrant then find the value of $\sin \frac{x}{2}, \cos \frac{x}{2}, \tan \frac{x}{2}$ **Ans.** $\frac{\sqrt{2}}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, -\sqrt{2}$

Q6. If $\tan x + \sin x = m, \tan x - \sin x = n$, to show that:- $m^2 - n^2 = 4\sqrt{mn}$

Q7. If A, B, C and D are the angle of a cyclic quadrilateral then prove that $\cos A + \cos B + \cos C + \cos D = 0$.

Q8. Find the maximum and minimum value of $\sin x + \cos x$. **Ans.** $\sqrt{2}, -\sqrt{2}$

Q9. If three angles of A, B and C are in A.P. then prove that: - $\cot B = \frac{\sin A - \sin C}{\cos C - \cos A}$

Q10. To prove that the following identities:-

$$(i) \frac{\cos(2\pi + \theta) \cos ec(2\pi + \theta) \tan(\frac{\pi}{2} + \theta)}{\sec(\frac{\pi}{2} + \theta) \cos \theta \cot(\pi + \theta)} = 1 \quad (ii) \sin \frac{x}{2} \sin \frac{7x}{2} + \sin \frac{3x}{2} \sin \frac{11x}{2} = \sin 2x \sin 5x$$

$$(iii) \frac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\frac{A+B}{2}\right) \cot\left(\frac{A-B}{2}\right) \quad (iv) \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$$

$$(v) \sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16} \quad (vi) (1 + \cos \frac{\pi}{8})(1 + \cos \frac{3\pi}{8})(1 + \cos \frac{5\pi}{8})(1 + \cos \frac{7\pi}{8}) = \frac{1}{8}$$

$$(vii) \frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A} \quad (viii) \frac{\tan 5A + \tan 3A}{\tan 5A - \tan 3A} = 4 \cos 2A \cos 4A$$

$$(ix) \sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x \quad (x) \frac{\tan 3x}{\tan x} \text{ never lies between } 1/3 \text{ and } 3.$$

$$(xi) \cos 2x = 2 \sin^2 y + 4 \cos(x+y) \sin x \sin y + \cos 2(x+y) \quad (xii) \sqrt{3} \cos ec 20^\circ - \sec 20^\circ = 4$$

$$(xiii) \cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A+B) = \sin^2(A+B)$$

$$(xiv) \cos A \cos 2A \cos 2^2 A \cos 2^3 A \cos 2^4 A \dots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$$

Q11. Find the principal solution of the following functions:-

$$(i) \sin x = \frac{-1}{2}, \text{ Ans. } \frac{7\pi}{6}, \frac{11\pi}{6} \quad (ii) \sec x = 2, \text{ Ans. } \frac{\pi}{3}, \frac{5\pi}{3} \quad (iii) \tan x = -1, \text{ Ans. } \frac{3\pi}{4}, \frac{7\pi}{4}$$

Q12. If $\sin A + \sin B = a$ and $\cos A + \cos B = b$, then show that:-

$$(i) \cos(A+B) = \frac{b^2 - a^2}{b^2 + a^2} \quad (ii) \sin(A+B) = \frac{2ab}{b^2 + a^2}$$

Q13. If $A+B=45^\circ$, prove that:- (i) $(1+\tan A)(1+\tan B)=2$ (ii) $(\cot A-1)(\cot B-1)=2$

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Q14. Solve for x: (i) $7\cos^2x + 3\sin^2x = 4$. **Ans.** $x = n\pi \pm \frac{\pi}{3} \forall n \in \mathbb{Z}$

(ii) $\sin x + \sqrt{3} \cos x = 1$, **Ans.** $x = (4n+1)\frac{\pi}{2} \text{ or } (12m-1)\frac{\pi}{6} \forall m, n \in \mathbb{Z}$

(iii) $\cos x + \cos 2x + \cos 3x = 0$ **Ans.** $x = (2n+1)\frac{\pi}{4} \text{ or } 2m\pi \pm \frac{2\pi}{3} \forall m, n \in \mathbb{Z}$

(iv) $\sin x + \cos x = 1$ **Ans.** $x = (2n\pi + \frac{\pi}{2}) \text{ or } 2m\pi \forall m, n \in \mathbb{Z}$

(v) $\tan x + \tan 2x + \sqrt{3} \tan x \tan 2x = \sqrt{3}$ **Ans.** $x = \frac{1}{3}(n\pi + \frac{\pi}{3}) \forall n \in \mathbb{Z}$

Q15. Find the period and draw the graph of the following functions:-

- (i) $\sin x$ (ii) $\cos x$ (iii) $\tan x$ **Ans.** (i) 2π (ii) 2π (iii) π

Q16. If $A+B+C=\pi$ then prove the following identities:-

(i) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ (ii) $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

Q17. If the angle of Δ are in the ratio 1: 2:3, prove that the sides of Δ are in the ratio 1: $\sqrt{3}$: 2.

Q18. If $\sin^2 A + \sin^2 B = \sin^2 C$, prove that the Δ is right angled.

Q19. In ΔABC , If (i) $a=2$, $b=3$ and $\sin A = \frac{2}{3}$ Find $\angle B$ (ii) $a=18$, $b=24$, $c=30$ and $\angle C = 90^\circ$ Find $\sin A$, $\sin B$,

sin C (iii) $a=18$, $b=24$ and $c=30$, find $\cos A$, $\cos B$ and $\cos C$ **Ans.** (i) $\frac{\pi}{2}$, (ii) $\frac{3}{5}, \frac{4}{5}, 1$ (iii) $\frac{4}{5}, \frac{3}{5}, 0$

Q20. In ΔABC , if $\angle C = 60^\circ$, then prove that: $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$

Q21. In ΔABC , Prove the following:- (i) $(b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot C = 0$

(ii) $\frac{a^2 - c^2}{b^2} = \frac{\sin(A-C)}{\sin(A+C)}$ (iii) $b \cos B + c \cos C = a \cos(B-C)$ (iv) $a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) = 0$

(v) $\frac{a^2 + b^2}{a^2 + c^2} = \frac{1 + \cos(A-B)\cos C}{1 + \cos(A-C)\cos B}$ (vi) $\sin\left(\frac{B-C}{2}\right) = \frac{b-c}{a} \cos\frac{A}{2}$ (vii) $a \cos\left(\frac{B-C}{2}\right) = (b+c) \sin\frac{A}{2}$

(viii) $\frac{a-b}{a+c} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$ (ix) $a(b \cos C - c \cos B) = b^2 - c^2$ (x) $\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}$ (xi) $\frac{b^2 - c^2}{a^2} = \frac{\sin(B-C)}{\sin(B+C)}$

(xii) $(b-c) \cot\frac{A}{2} + (c-a) \cot\frac{B}{2} + (a-b) \cot\frac{C}{2} = 0$ (xiii) $\frac{c}{a-b} = \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{\tan\frac{A}{2} - \tan\frac{B}{2}}$

(xiv) $\frac{(b^2 - c^2)}{a^2} \sin 2A + \frac{(c^2 - a^2)}{b^2} \sin 2B + \frac{(a^2 - b^2)}{c^2} \sin 2C = 0$ (xv) $\frac{c - b \cos A}{b - c \cos A} = \frac{\cos B}{\cos C}$

Q22. If $a \cos A = b \cos B$, then the triangle is either isosceles or right angled.

Q23. In ΔABC , $\frac{(b+c)}{12} = \frac{(c+a)}{13} = \frac{(a+c)}{15}$ then prove that $\frac{\cos A}{2} = \frac{\cos B}{7} = \frac{\cos C}{11}$

Q24. If in ΔABC , $\cos A + 2\cos B + \cos C = 2$, prove that sides of the Δ are in A.P.

----- Best of Luck -----

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