

TRIGONOMETRY Class XI		
Q.1)	Solve the equation or find the general solutions $\cos x + \cos(2x) + \cos(3x) = 0$	
Sol.1)	<p>We have, $[\cos(3x) + \cos x] + \cos(2x) = 0$ $\Rightarrow 2\cos(2x) \cdot \cos(x) + \cos(2x) = 0$ $\Rightarrow \cos(2x)[2\cos x + 1] = 0$ $\Rightarrow \cos(2x) = 0$ $\Rightarrow \theta = (2n+1)\frac{\pi}{2}$ $\Rightarrow 2x = (2n+1)\frac{\pi}{2}$ $\Rightarrow x = (2n+1)\frac{\pi}{4}$ $\Rightarrow n \in \mathbb{Z}$</p> <p>$\Rightarrow 2\cos x + 1 = 0$ $\Rightarrow \cos x = \frac{-1}{2}$ $\Rightarrow \cos x = \cos\left(\pi - \frac{\pi}{3}\right)$ $\Rightarrow \cos x = \cos\left(\frac{2\pi}{3}\right)$ Company with $\cos \theta = \cos \alpha$ here, $\theta = x$ and $\alpha = \frac{2\pi}{3}$ $\theta = 2n\pi \pm \alpha$</p> <p>$\Rightarrow x = 2n\pi \pm \frac{2\pi}{3}$ $\therefore x = 2n + 1\left(\frac{\pi}{4}\right)$ and $x = 2n\pi \pm \frac{2\pi}{3}; n \in \mathbb{Z}$ ans.</p>	
Q.2)	Solve, $\sin(2x) + \cos(x) = 0$.	
Sol.2)	<p>We have, $\sin(2x) + \cos(x) = 0$ $\Rightarrow 2\sin x \cdot \cos x + \cos x = 0$ $\Rightarrow \cos x[2\sin x + 1] = 0$ $\Rightarrow \cos x = 0$ $\Rightarrow \cos \theta = 0$ $\Rightarrow \theta = (2n+1)\frac{\pi}{2}$ $\Rightarrow x = (2n+1)\frac{\pi}{2}$</p> <p>$\Rightarrow 2\sin x + 1 = 0$ $\Rightarrow \sin x = \frac{-1}{2}$ $\Rightarrow \sin x = \sin\left(\pi + \frac{\pi}{6}\right)$ $\Rightarrow \sin x = \sin\left(\frac{7\pi}{6}\right)$ Company with $\sin \theta = \sin \alpha$ here, $\theta = x$ and $\alpha = \frac{7\pi}{6}$ $\theta = n\pi + (-1)^n \alpha$</p> <p>$\Rightarrow x = (n\pi) + (-1)^n \frac{7\pi}{6}$ $\therefore x = 2n + 1\left(\frac{\pi}{2}\right)$ and $x = (n\pi) + (-1)^n \frac{7\pi}{6}; n \in \mathbb{Z}$ ans.</p>	
Q.3)	Solve the equation: $\sin x - 3\sin(2x) + \sin(3x) = \cos x - 3\cos(2x) + \cos(3x)$	
Sol.3)	<p>We have, $\sin x - 3\sin(2x) + \sin(3x) = \cos x - 3\cos(2x) + \cos(3x)$ $\Rightarrow (\sin(3x) + \sin x) - 3\sin(2x) = (\cos(3x) + \cos x) - 3\cos(2x)$ $\Rightarrow 2\sin(2x) \cdot \cos(x) - 3\sin(2x) = 2\cos(2x) \cos x - 3\cos(2x)$ $\Rightarrow \sin(2x)(2\cos x - 3) = \cos(2x)(2\cos x - 3)$ $\Rightarrow \sin(2x)(2\cos x - 3) - \cos(2x)(2\cos x - 3) = 0$ $\Rightarrow (2\cos x - 3)(\sin(2x) - \cos(2x)) = 0$ Either $2\cos x - 3 = 0$ OR $\sin(2x) - \cos(2x) = 0$ $\Rightarrow \cos x = \frac{3}{2}$ (not possible) $-1 \leq \cos \theta \leq 1$ $\Rightarrow \sin(2x) = \cos(2x)$ $\Rightarrow \tan(2x) = 1$ $\Rightarrow \tan(2x) = \tan\frac{\pi}{4}$</p> <p>Here, $\theta = 2x; \alpha = \frac{\pi}{4}$</p>	

	$\Rightarrow \theta = n\pi + \alpha$ $\Rightarrow 2x = n\pi + \frac{\pi}{4}$ $\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}, n \in \mathbb{Z}$ ans.	
Q.4)`	Solve, $2\cos^2 x + 3\sin x = 0$.	
Sol.4)	We have, $2\cos^2 x + 3\sin x = 0$ $\Rightarrow 2(1 - \sin^2 x) + 3\sin x = 0$ $\Rightarrow 2 - 2\sin^2 x + 3\sin x = 0$ $\Rightarrow 2\sin^2 x - 3\sin x - 2 = 0$ $\Rightarrow 2\sin^2 x - 4\sin x + \sin x - 2 = 0$ $\Rightarrow 2\sin x(\sin x - 2) + 1(\sin x - 2) = 0$ $\Rightarrow (2\sin x + 1)(\sin x - 2) = 0$ Either $2\sin x + 1 = 0$ OR $\sin x - 2 = 0$ $\Rightarrow \sin x = -\frac{1}{2}$ $\Rightarrow \sin x = \sin\left(\pi + \frac{\pi}{6}\right)$ $\Rightarrow \sin x = \sin\left(\frac{7\pi}{6}\right)$ Here, $\theta = x ; \alpha = \frac{7\pi}{6}$ $\Rightarrow \theta = n\pi + (-1)^n\alpha$ $\Rightarrow x = (n\pi) + (-1)^n\frac{7\pi}{6}; n \in \mathbb{Z}$ ans.	$\Rightarrow \sin x = 2$ (not possible) $-1 \leq \sin \theta \leq 1$
Q.5)	Solve, $\cot^2 \theta + \frac{3}{\sin \theta} + 3 = 0$	
Sol.5)	We have, $\cot^2 \theta + \frac{3}{\sin \theta} + 3 = 0$ $\Rightarrow (\operatorname{cosec}^2 \theta - 1) + 3 \operatorname{cosec} \theta + 3 = 0$ $\Rightarrow \operatorname{cosec}^2 \theta + 3 \operatorname{cosec} \theta + 2 = 0$ $\Rightarrow \operatorname{cosec}^2 \theta + 2 \operatorname{cosec} \theta + \operatorname{cosec} \theta + 2 = 0$ $\Rightarrow \operatorname{cosec} \theta(\operatorname{cosec} \theta + 2) + 1(\operatorname{cosec} \theta + 2) = 0$ $\Rightarrow (\operatorname{cosec} \theta + 1)(\operatorname{cosec} \theta + 2) = 0$ $\Rightarrow \operatorname{cosec} \theta = -1$ $\Rightarrow \sin \theta = -1$ $\Rightarrow \operatorname{cosec} \theta = -2$ $\Rightarrow \sin \theta = \frac{-1}{2}$ $\Rightarrow \theta = n\pi + (-1)^n\frac{3\pi}{2}$ OR $\theta = n\pi + (-1)^n\frac{7\pi}{6}, n \in \mathbb{Z}$ ans.	
Q.6)	Solve the equation: $\tan(2x) = -\cot\left(x + \frac{\pi}{3}\right)$	
Sol. 6)	We have, : $\tan(2x) = -\cot\left(x + \frac{\pi}{3}\right)$ $\Rightarrow \tan(2x) = \tan\left(\frac{\pi}{2} + \left(x + \frac{\pi}{3}\right)\right)$ $\Rightarrow \tan(2x) = \tan\left(\frac{5\pi}{6} + x\right)$ Here $\theta = 2x ; \alpha = \frac{5\pi}{6} + x$ $\theta = n\pi + \alpha$ $\Rightarrow 2x = n\pi + \frac{5\pi}{6} + x$ $\Rightarrow 2x - x = n\pi + \frac{5\pi}{6}$ $\Rightarrow x = n\pi + \frac{5\pi}{6}, n \in \mathbb{Z}$ ans.	
Q.7)	Solve, $\sin(3x) + \cos(2x) = 0$	
Sol. 7)	We have, $\sin(3x) + \cos(2x) = 0$	

	$\Rightarrow \cos(2x) = -\sin(3x)$ $\Rightarrow \cos(2x) = \cos\left(\frac{\pi}{2} + 3x\right)$ Comparing with $\cos \theta = \cos \alpha$ $\Rightarrow \theta = 2x ; \alpha = \frac{\pi}{2} + 3x$ $\Rightarrow \theta = 2n\pi \pm \alpha$ $\Rightarrow 2x = 2n\pi \pm \left(\frac{\pi}{2} + 3x\right)$ Case 1 $2x = 2n\pi + \frac{\pi}{2} + 3x$ $\Rightarrow -x = 2n\pi + \frac{\pi}{2}$ $\Rightarrow x = -\left(2n\pi + \frac{\pi}{2}\right); n \in \mathbb{Z} \text{ ans.}$ Case 2 $2x = 2n\pi - \frac{\pi}{2} - 3x$ $\Rightarrow 5x = 2n\pi - \frac{\pi}{2}$ $\Rightarrow x = \frac{2n\pi}{5} - \frac{\pi}{10}; n \in \mathbb{Z} \text{ ans.}$	
Q.8)	Solve, $\sqrt{3} \cos x + \sin x = \sqrt{2}$	
Sol.8)	We have, $\sqrt{3} \cos x + \sin x = \sqrt{2}$ Here, $a = \sqrt{3}$ and $b = 1$ Divide both sides by $\sqrt{3 + 1} = 2$ $\Rightarrow \frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x = \frac{\sqrt{3}}{2}$ $\Rightarrow \cos\left(\frac{\pi}{6}\right) \cos x + \sin\left(\frac{\pi}{6}\right) \sin x = \cos\left(\frac{\pi}{4}\right)$ $\Rightarrow \cos\left(\pi - \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right) \dots \{\cos A \cos B + \sin A \sin B = \cos(A - B)\}$ Form $\cos \theta = \cos \alpha$ $\theta = \pi - \frac{\pi}{6}; \alpha = \frac{\pi}{4}$ $\theta = 2n\pi \pm \alpha$ $x - \frac{\pi}{6} = 2n\pi + \frac{\pi}{4}$ $x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{6}$ Case 1 $x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{6}$ $x = 2n\pi + \frac{5\pi}{12}; n \in \mathbb{Z} \text{ ans.}$ Case 2 $x = 2n\pi - \frac{\pi}{4} + \frac{\pi}{6}$ $x = 2n\pi - \frac{5\pi}{12}; n \in \mathbb{Z} \text{ ans.}$	
Q.9)	Solve the equation, $\cot \theta + \operatorname{cosec} \theta = \sqrt{3}$	
Sol.9)	We have, $\cot \theta + \operatorname{cosec} \theta = \sqrt{3}$ $\Rightarrow \frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta} = \sqrt{3}$ $\Rightarrow \cos \theta + 1 = \sqrt{3} \sin \theta$ $\Rightarrow \cos \theta - \sqrt{3} \sin \theta = -1$ Here, $a = 1$ and $b = -\sqrt{3}$ Divide both sides by $\sqrt{a^2 + b^2} = \sqrt{1 + 3} = 2$ $\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta = \frac{-1}{2}$ $\Rightarrow \cos\left(\frac{\pi}{3}\right) \cos \theta - \sin\left(\frac{\pi}{3}\right) \sin \theta = \cos\left(\pi - \frac{\pi}{3}\right)$ $\Rightarrow \cos\left(\theta + \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$ Here, $\alpha = \frac{2\pi}{3}$	

	$\Rightarrow \theta + \frac{\pi}{3} = 2n\pi \pm \frac{2\pi}{3}$ $\Rightarrow \theta = 2n\pi \pm \frac{2\pi}{3} - \frac{\pi}{3}$ <p>Case 1</p> $\Rightarrow \theta = 2n\pi + \frac{2\pi}{3} - \frac{\pi}{3}$ $\Rightarrow \theta = 2n\pi + \frac{\pi}{3}$	<p>Case 2</p> $\Rightarrow \theta = 2n\pi - \frac{2\pi}{3} - \frac{\pi}{3}$ $\Rightarrow \theta = 2n\pi - \pi$ <p>$n \in \mathbb{Z}$ ans.</p>	
Q.10)	Solve, $\tan \theta + \tan(2\theta) + \tan(3\theta) = \tan \theta \tan(2\theta) \tan(3\theta)$		
Sol.10)	<p>We have, $\tan \theta + \tan(2\theta) + \tan(3\theta) = \tan \theta \tan(2\theta) \tan(3\theta)$</p> $\Rightarrow \tan \theta + \tan(2\theta) = -\tan(3\theta) + \tan \theta \tan(2\theta) \tan(3\theta)$ $\Rightarrow \tan \theta + \tan(2\theta) = -\tan(3\theta)[1 - \tan \theta \cdot \tan(2\theta)]$ $\Rightarrow \frac{\tan \theta + \tan(2\theta)}{1 - \tan \theta \cdot \tan(2\theta)} = -\tan(3\theta)$ $\Rightarrow \tan(\theta + 2\theta) = -\tan(3\theta)$ $\Rightarrow \tan(3\theta) + \tan(3\theta) = 0$ $\Rightarrow 2\tan(3\theta) = 0$ $\Rightarrow \tan(3\theta) = 0$ $\Rightarrow 3\theta = n\pi$ $\Rightarrow \theta = \frac{n\pi}{3}; n \in \mathbb{Z}$ ans.		