

<b>TRIGONOMETRY</b>	
Q.1)	<p>Find the value of <math>\cos(18^\circ)</math>, <math>\cos(36^\circ)</math>, <math>\sin(36^\circ)</math> <math>\sin(54^\circ)</math>?</p> <p>Sol.1) We know, <math>\cos \theta = \sqrt{1 - \sin^2 \theta}</math>, put <math>\theta = 18^\circ</math>  <math>\Rightarrow \cos(18^\circ) = \sqrt{1 - \sin^2(18^\circ)}</math>  <math>\Rightarrow \cos(18^\circ) = \sqrt{1 - \left(\frac{\sqrt{5}-1}{4}\right)^2} \dots \left\{ \sin(18^\circ) = \frac{\sqrt{5}-1}{4} \right\}</math>  <math>= \sqrt{1 - \left(\frac{5+1-2\sqrt{5}}{16}\right)}</math>  <math>= \sqrt{\frac{16-6+2\sqrt{5}}{16}}</math>  <math>= \cos(18^\circ) = \sqrt{\frac{10+2\sqrt{5}}{4}} \text{ ans.}</math></p> <p>Now, <math>\cos(36^\circ)</math>  <math>= \cos 2\theta = 1 - 2\sin^2 \theta</math>, put <math>\theta = 18^\circ</math>  <math>\Rightarrow \cos(36^\circ) = 1 - 2 \sin^2(18^\circ)</math>  <math>\Rightarrow \cos(36^\circ) = 1 - 2 \left(\frac{\sqrt{5}-1}{4}\right)^2 \dots \left\{ \sin(18^\circ) = \frac{\sqrt{5}-1}{4} \right\}</math>  <math>\Rightarrow 1 - 2 \left(\frac{5+1-2\sqrt{5}}{16}\right)</math>  <math>\Rightarrow \frac{16-12+4\sqrt{5}}{16} = \frac{4+4\sqrt{5}}{16}</math>  <math>\cos(36^\circ) = \frac{\sqrt{5}+1}{4} \text{ ans.}</math></p> <p>Now, <math>\sin(36^\circ)</math>  We have, <math>\sin \theta = \sqrt{1 - \cos^2 \theta}</math>, put <math>\theta = 36^\circ</math>  <math>\Rightarrow \sin(36^\circ) = \sqrt{1 - \cos^2(36^\circ)}</math>  <math>\Rightarrow \sin(36^\circ) = 1 - \left(\frac{\sqrt{5}+1}{4}\right)^2 \dots \left\{ \cos(36^\circ) = \frac{\sqrt{5}+1}{4} \right\}</math>  <math>\Rightarrow \sqrt{1 - \left(\frac{5+1+2\sqrt{5}}{16}\right)}</math>  <math>= \sqrt{\frac{16-6-2\sqrt{5}}{16}}</math>  <math>\sin(36^\circ) = \frac{\sqrt{10}-2\sqrt{5}}{4} \text{ ans.}</math></p> <p>Now, <math>\sin(54^\circ) = \sin(90^\circ - 36^\circ)</math>  <math>= \cos(36^\circ) = \frac{\sqrt{5}+1}{4} \text{ ans.}</math></p>
Q.2)	<p>Find the value of <math>\tan(9^\circ) - \tan(27^\circ) - \tan(63^\circ) + \tan(81^\circ)</math>?</p> <p>Sol.2) We have, <math>\tan(9^\circ) - \tan(27^\circ) - \tan(63^\circ) + \tan(81^\circ)</math>  <math>= \tan(9^\circ) - \tan(27^\circ) - \tan(90^\circ - 27^\circ) + \tan(90^\circ - 9^\circ)</math>  <math>= \tan(9^\circ) - \tan(27^\circ) - \cot(27^\circ) + \cot(9^\circ)</math>  <math>= (\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)</math>  <math>= \left(\frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\cos 9^\circ}{\sin 9^\circ}\right) - \left(\frac{\sin 27^\circ}{\cos 27^\circ} + \frac{\cos 27^\circ}{\sin 27^\circ}\right)</math>  <math>= \left(\frac{\sin^2 9^\circ + \cos^2 9^\circ}{\sin 9^\circ \cos 9^\circ}\right) - \left(\frac{\sin^2 27^\circ + \cos^2 27^\circ}{\cos 27^\circ \sin 27^\circ}\right)</math>  <math>= \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\cos 27^\circ \sin 27^\circ}</math>  <math>= \frac{1}{2\sin(9^\circ)\cos(9^\circ)} - \frac{1}{2\cos(27^\circ)\sin(27^\circ)} \{ \text{multiply and divide by 2} \}</math>  <math>= \frac{2}{\sin(18^\circ)} - \frac{2}{\sin(54^\circ)}</math>  <math>= \frac{\sin(18^\circ)}{2} - \frac{\sin(90^\circ - 36^\circ)}{2}</math>  <math>= \frac{\sin(18^\circ)}{2} - \frac{\cos(36^\circ)}{2}</math></p>

	$= \frac{\frac{2}{\sqrt{5}-1}}{4} - \frac{\frac{2}{\sqrt{5}+1}}{4}$ $= \frac{8\sqrt{5}+8-8\sqrt{5}+8}{5-1} = \frac{16}{4} = 4 \text{ ans.}$	
Q.3)	Simplify $\frac{\cos x}{1+\sin x}$ .	
Sol.3)	<p>We have, <math>\frac{\cos x}{1+\sin x}</math></p> $= \frac{\sin\left(\frac{x}{2}-x\right)}{1+\cos\left(\frac{x}{2}-x\right)}$ $= \frac{2\sin\left(\frac{x-x}{2}\right)\cos\left(\frac{x-x}{2}\right)}{2\cos^2\left(\frac{x-x}{2}\right)} \left\{ \sin(\theta) = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \right\} \left\{ 1 + \cos\theta = 2\cos^2\frac{\theta}{2} \right\}$ $= \tan\left(\frac{x}{2}-\frac{x}{2}\right)$	
Q.4)	If $\tan x = \frac{b}{a}$ , find the value of $\sqrt{\frac{a-b}{a+b}} + \sqrt{\frac{a+b}{a-b}}$ .	
Sol.4)	<p>We have, <math>\sqrt{\frac{a-b}{a+b}} + \sqrt{\frac{a+b}{a-b}}</math></p> $= \sqrt{\frac{a-b}{a+b}} \times \sqrt{\frac{a-b}{a-b}} + \sqrt{\frac{a+b}{a-b}} \times \sqrt{\frac{a+b}{a+b}}$ $= \frac{a-b}{\sqrt{a^2-b^2}} + \frac{a+b}{\sqrt{a^2-b^2}}$ $= \frac{a-b+a+b}{\sqrt{a^2-b^2}}$ $= \frac{2a}{\sqrt{a^2-b^2}}$ <p>divide N &amp; D by a</p> $= \frac{\frac{2a}{\sqrt{a^2-b^2}}}{a} = \frac{2}{\sqrt{\frac{a^2-b^2}{a^2}}}$ $= \frac{2}{\sqrt{1-\frac{b^2}{a^2}}}$ <p>Put <math>\tan x = \frac{b}{a}</math>, given</p> $= \frac{2}{1-\tan^2 x} = \frac{2}{\sqrt{1-\frac{\sin^2 x}{\cos^2 x}}}$ $= \frac{2 \cos x}{\sqrt{\cos^2 x - \sin^2 x}}$ $= \frac{2 \cos x}{\sqrt{\cos(2x)}} \text{ ans. } \{\cos(2\theta) = \cos^2 \theta - \sin^2 \theta\}$	
Q.5)	Simplify $\frac{1-\cos x}{\sin x}$	
Sol.5)	<p>we have, <math>\frac{1-\cos x}{\sin x}</math></p> $= \frac{2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)}$ $= \tan\left(\frac{x}{2}\right)$	
Q.6)	Show that, $\sqrt{3} \operatorname{cosec}(20^\circ) - \sec(20^\circ) = 4$ ?	
Sol.6)	<p>L.H.S. <math>\sqrt{3} \operatorname{cosec}(20^\circ) - \sec(20^\circ)</math></p> $= \frac{\sqrt{3}}{\sin(20^\circ)} + \frac{1}{\cos(20^\circ)}$ $= \frac{\sqrt{3} \cos(20^\circ) - \sin(20^\circ)}{\sin(20^\circ) \cos(20^\circ)}$ $= \frac{2\left(\frac{\sqrt{3}}{2}\cos(20^\circ) - \frac{1}{2}\sin(20^\circ)\right)}{\frac{1}{2}(2\sin(20^\circ) \cos(20^\circ))}$	

	$  \begin{aligned}  &= \frac{2(\sin(60).\cos(20) - \cos(20).\sin(20))}{\frac{1}{2}\sin(40)} \quad \{\sin(2\theta) = 2 \sin \theta \cdot \cos \theta\} \\  &= \frac{4 \sin(60-20)}{\sin(40)} = \frac{4 \sin(40)}{\sin(40)} = 4 \text{ R.H.S. ans.}  \end{aligned}  $	
Q.7)	Simplify $\frac{1-\sin x}{\cos x}$ .	
Sol.7)	<p>We have, <math>\frac{1-\sin x}{\cos x}</math></p> $  \begin{aligned}  &= \frac{1-\cos(\frac{\pi}{2}-x)}{\sin(\frac{\pi}{2}-x)} \\  &= \frac{2 \sin^2(\frac{\pi}{4}-\frac{x}{2})}{2\sin(\frac{\pi}{4}-\frac{x}{2}).\cos(\frac{\pi}{4}-\frac{x}{2})} \\  &= \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)  \end{aligned}  $	
Q.8)	Show that, $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$ ?	
Sol.8)	<p>L.H.S. <math>\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1}</math></p> $  \begin{aligned}  &= \frac{\frac{\sin A}{\cos A} + \frac{1}{\cos A} - 1}{\frac{\sin A}{\cos A} - \frac{1}{\cos A} + 1} \\  &= \frac{\sin A - 1 + \cos A}{\sin A + 1 - \cos A} \\  &= \frac{\sin A - 1 + \cos A}{\sin A - (1 - \cos A)} \\  &= \frac{\sin A + 2 \sin^2 \frac{A}{2}}{\sin A - 2 \sin^2 \frac{A}{2}} \quad \left\{ 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2} \right\} \\  &= \frac{2\sin(\frac{A}{2})\cos(\frac{A}{2}) + 2\sin^2(\frac{A}{2})}{2\sin(\frac{A}{2})\cos(\frac{A}{2}) - 2\sin^2(\frac{A}{2})} \quad \left\{ \sin(\theta) = 2 \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} \right\} \\  &= \frac{2\sin(\frac{A}{2})[\cos(\frac{A}{2}) + \sin(\frac{A}{2})]}{2\sin(\frac{A}{2})[\cos(\frac{A}{2}) - \sin(\frac{A}{2})]}  \end{aligned}  $ <p>Rationalize as we have to make the angle to <math>\cos A</math> from <math>\frac{A}{2}</math></p> $  \begin{aligned}  &= \frac{[\cos(\frac{A}{2}) + \sin(\frac{A}{2})][\cos(\frac{A}{2}) + \sin(\frac{A}{2})]}{[\cos(\frac{A}{2}) - \sin(\frac{A}{2})][\cos(\frac{A}{2}) + \sin(\frac{A}{2})]} \\  &= \frac{\cos^2(\frac{A}{2}) + \sin^2(\frac{A}{2}) + 2\sin(\frac{A}{2})\cos(\frac{A}{2})}{\cos^2(\frac{A}{2}) - \sin^2(\frac{A}{2})} \\  &= \frac{1 + \sin A}{\cos A} \text{ R.H.S (proved)} \quad \left\{ \begin{array}{l} 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2}, \\ \cos(2\theta) = \cos^2 \theta - \sin^2 \theta \end{array} \right\}  \end{aligned}  $	
Q.9)	Show that $\sin(4A) = 4 \sin A \cdot \cos^3 A \cdot \sin^3 A$ ?	
Sol.9)	HINT: $\sin(4A) = 2 \sin(2A) \cdot \cos(2A)$	
Q.10)	Show that, $\cos^2(A - B) + \cos^2 B - 2 \cos(A - B) \cdot \cos A \cos B = \sin^2 A$ ?	
Sol.10)	<p>L.H.S. <math>\cos^2(A - B) + \cos^2 B - 2 \cos(A - B) \cdot \cos A \cos B</math></p> $  \begin{aligned}  &= \cos(A - B)[\cos(A - B) - 2 \cos A \cos B] + \cos^2 B \\  &= \cos(A - B)[\cos A \cos B + \sin A \sin B - 2 \cos A \cos B] + \cos^2 B \\  &= \cos(A - B)[- \cos A \cos B + \sin A \sin B] + \cos^2 B \dots\dots \\  &\quad (- \text{ common for making a formula}) \\  &= -\cos(A - B)[\cos A \cos B - \sin A \sin B] + \cos^2 B \\  &= -\cos(A - B) \cdot \cos(A + B) + \cos^2 B \\  &= -[\cos^2 A - \sin^2 B] + \cos^2 B \dots\dots \quad [\cos(A + B) \cdot \cos(A - B) = \cos^2 A - \sin^2 B] \\  &= -\cos^2 A + (\sin^2 B + \cos^2 B) \\  &= -\cos^2 A + 1 = 1 - \cos^2 A \dots\dots \{\cos^2 \theta + \sin^2 \theta = 1\} \\  &= \sin^2 A = \text{R.H.S. (proved)}  \end{aligned}  $	
Q.11)	Show that, $\cos A \cdot \cos(2A) \cdot \cos(2^2 A) \cdot \cos(2^3 A) \dots \cos(2^{n-1} A) = \frac{\sin(2^n A)}{2^n \sin A}$ ?	



Sol.11)	<p>Taking, L.H.S. <math>\cos A \cdot \cos(2A) \cdot \cos(2^2 A) \cdot \cos(2^3 A) \cdot \dots \cdot \cos(2^{n-1} A)</math></p> <p>Multiply &amp; divide by <math>2\sin A</math></p> $= \frac{1}{2\sin A} [(2\sin A \cdot \cos A) \cdot \cos(2A) \cdot \cos(2^2 A) \cdot \cos(2^3 A) \cdot \dots \cdot \cos(2^{n-1} A)]$ $= \frac{1}{2\sin A} [\sin(2A) \cdot \cos(2A) \cdot \cos(2^2 A) \cdot \cos(2^3 A) \cdot \dots \cdot \cos(2^{n-1} A)]$ <p>Multiply &amp; divide by 2</p> $= \frac{1}{2^2 \sin A} [2\sin(2A) \cdot \cos(2A) \cdot \cos(2^2 A) \cdot \cos(2^3 A) \cdot \dots \cdot \cos(2^{n-1} A)]$ $= \frac{1}{2^2 \sin A} [(\sin(4A) \cdot \cos(4A)) \cdot \cos(2^3 A) \cdot \dots \cdot \cos(2^{n-1} A)]$ $= \frac{1}{2^3 \sin A} [(2\sin(4A) \cdot \cos(4A)) \cdot \cos(2^3 A) \cdot \dots \cdot \cos(2^{n-1} A)]$ $= \frac{1}{2^3 \sin A} [\sin(2^3 A) \cdot \cos(2^3 A) \cdot \dots \cdot \cos(2^{n-1} A)]$ <p>Once the process gives on.....</p> $= \frac{1}{2^{n-1} \sin A} [\sin(2^{n-1} A) \cdot \cos(2^{n-1} A)]$ <p>Multiply &amp; divide by 2</p> $= \frac{1}{2^n \sin A} [2\sin(2^{n-1} A) \cdot \cos(2^{n-1} A)]$ $= \frac{1}{2^n \sin A} [\sin(2 \cdot 2^{n-1} A)] \dots \{ \sin(2\theta) = 2 \sin \theta \cdot \cos \theta \}$ $= \frac{1}{2^n \sin A} \cdot \sin(2^n A) \cdot \text{R.H.S. (proved)}$
Q.12)	Show that, $\cos\left(\frac{\pi}{7}\right) \cdot \cos\left(\frac{2\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right) = -\frac{1}{8}$ ?
Sol.12)	<p>L.H.S. <math>\cos\left(\frac{\pi}{7}\right) \cdot \cos\left(\frac{2\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right)</math></p> <p>Multiply &amp; divide by <math>2\sin\left(\frac{\pi}{7}\right)</math></p> $= \frac{1}{2\sin\left(\frac{\pi}{7}\right)} \left[ \left( 2\sin\left(\frac{\pi}{7}\right) \cdot \cos\left(\frac{\pi}{7}\right) \right) \cdot \cos\left(\frac{2\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right) \right]$ $= \frac{1}{2\sin\left(\frac{\pi}{7}\right)} \left[ \sin\left(\frac{2\pi}{7}\right) \cdot \cos\left(\frac{2\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right) \right]$ $= \frac{1}{2^2 \sin\left(\frac{\pi}{7}\right)} \left[ \left( 2\sin\frac{2\pi}{7} \cdot \cos\frac{2\pi}{7} \right) \cdot \cos\left(\frac{4\pi}{7}\right) \right]$ $= \frac{1}{2^2 \sin\left(\frac{\pi}{7}\right)} \left[ \sin\left(\frac{4\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right) \right]$ $= \frac{1}{2^3 \sin\left(\frac{\pi}{7}\right)} \left[ 2\sin\left(\frac{4\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right) \right]$ $= \frac{1}{2^3 \sin\left(\frac{\pi}{7}\right)} \cdot \sin\left(\frac{8\pi}{7}\right)$ $= \frac{1}{8\sin\left(\frac{\pi}{7}\right)} \cdot \sin\left(\pi + \frac{\pi}{7}\right)$ $= \frac{1}{8\sin\left(\frac{\pi}{7}\right)} \cdot (-\sin\left(\frac{\pi}{7}\right))$ $= -\frac{1}{8} \text{ R.H.S. (proved)}$
Q.13)	Show that, $\cos\left(\frac{2\pi}{15}\right) \cdot \cos\left(\frac{4\pi}{15}\right) \cdot \cos\left(\frac{8\pi}{15}\right) \cdot \cos\left(\frac{14\pi}{15}\right) = \frac{1}{16}$ ?
Sol.13)	<p>L.H.S. <math>\cos\left(\frac{2\pi}{15}\right) \cdot \cos\left(\frac{4\pi}{15}\right) \cdot \cos\left(\frac{8\pi}{15}\right) \cdot \cos\left(\frac{14\pi}{15}\right)</math></p> $= \cos\left(\frac{2\pi}{15}\right) \cdot \cos\left(\frac{4\pi}{15}\right) \cdot \cos\left(\frac{8\pi}{15}\right) \cdot \cos\left(\pi - \frac{\pi}{15}\right)$ $= \cos\left(\frac{2\pi}{15}\right) \cdot \cos\left(\frac{4\pi}{15}\right) \cdot \cos\left(\frac{8\pi}{15}\right) \cdot (-\cos\frac{\pi}{15})$ $= -\cos\left(\frac{\pi}{15}\right) \cdot \cos\left(\frac{2\pi}{15}\right) \cdot \cos\left(\frac{4\pi}{15}\right) \cdot \cos\left(\frac{8\pi}{15}\right)$ <p>Multiply &amp; divide by <math>2\sin\left(\frac{\pi}{15}\right)</math></p> $= \frac{-1}{2\sin\left(\frac{\pi}{15}\right)} \left[ 2\sin\left(\frac{\pi}{15}\right) \cdot \cos\left(\frac{\pi}{15}\right) \cdot \cos\left(\frac{2\pi}{15}\right) \cdot \cos\left(\frac{4\pi}{15}\right) \cdot \cos\left(\frac{8\pi}{15}\right) \right]$



	$= \frac{-1}{2\sin\left(\frac{\pi}{15}\right)} \left[ 2\sin\left(\frac{2\pi}{15}\right) \cdot \cos\left(\frac{2\pi}{15}\right) \cdot \cos\left(\frac{4\pi}{15}\right) \cdot \cos\left(\frac{8\pi}{15}\right) \right]$ <p>Proceed as Q.90</p> $= \frac{-1}{2^4 \sin\left(\frac{\pi}{15}\right)} \cdot \sin\left(\frac{16\pi}{15}\right)$ $= \frac{-1}{2^4 \sin\left(\frac{\pi}{15}\right)} \cdot \sin\left(\pi + \frac{\pi}{7}\right)$ $= \frac{-1}{16\sin\left(\frac{\pi}{15}\right)} \cdot \left(-\sin\left(\frac{\pi}{15}\right)\right)$ $= \frac{1}{16} \text{ R.H.S. (proved) ans.}$	
Q.14)	Show that, $\cot A + \cot(60 + A) - \cot(60 - A) = 3\cot(3A)$	
Sol.14)	<p>L.H.S. <math>\cot A + \cot(60 + A) - \cot(60 - A)</math></p> $= \frac{1}{\tan A} + \frac{1}{\tan(60+A)} - \frac{1}{\tan(60-A)}$ $= \frac{1}{\tan A} + \frac{1-\sqrt{3}\tan A}{\sqrt{3}+\tan A} - \frac{1+\sqrt{3}\tan A}{\sqrt{3}-\tan A} \quad [\tan(A+B) \cdot \tan(A-B) \text{ formula}]$ $= \frac{1}{\tan A} + \left[ \frac{(1-\sqrt{3}\tan A)(\sqrt{3}-\tan A) - (1+\sqrt{3}\tan A)(\sqrt{3}+\tan A)}{(\sqrt{3}+\tan A)(\sqrt{3}-\tan A)} \right]$ $= \frac{1}{\tan A} + \left[ \frac{\sqrt{3}-\tan A - 3\tan A + \sqrt{3}\tan^2 A - \sqrt{3}-\tan A - 3\tan A - \sqrt{3}\tan^2 A}{3-\tan^2 A} \right]$ $= \frac{1}{\tan A} + \frac{-8\tan A}{3-\tan^2 A}$ $= \frac{3-\tan^2 A - 8\tan^2 A}{\tan A(3-\tan^2 A)}$ $= \frac{3-9\tan^2 A}{3\tan A - \tan^3 A}$ $= \frac{3(1-3\tan^2 A)}{3\tan A - \tan^3 A}$ $= \frac{3}{\tan(3A)} \quad \left\{ \tan(3\theta) = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} \right\}$ $= 3\cot(3A) = \text{R.H.S. (proved) ans.}$	
Q.15)	Show that, $2\sin^2 \beta + 4 \cos(\alpha + \beta) \cdot \sin \alpha \sin \beta + \cos(2\alpha + 2\beta) = \cos(2\alpha)$ ?	
Sol.15)	<p>L.H.S. <math>2\sin^2 \beta + 4 \cos(\alpha + \beta) \cdot \sin \alpha \sin \beta + \cos(2\alpha + 2\beta)</math></p> $= 2\sin^2 \beta + 4(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \cdot \sin \alpha \sin \beta + (\cos 2\alpha + 2\beta - \sin(2\alpha) \cdot \sin(2\beta))$ $= 2\sin^2 \beta + 4 \cos \alpha \cos \beta \cdot \sin \alpha \sin \beta - 4 \sin^2 \alpha \cdot \sin^2 \beta + \cos(2\alpha) \cos(2\beta) \cdot \sin(2\alpha) \cdot \sin(2\beta)$ $= 2\sin^2 \beta + (2 \sin \alpha \cos \alpha) (2 \sin \beta \cos \beta) - 4 \sin^2 \alpha \cdot \sin^2 \beta +$ $\cos(2\alpha) \cos(2\beta) - \sin(2\alpha) \cdot \sin(2\beta)$ $= 2\sin^2 \beta + \sin(2\alpha) \cdot \sin(2\beta) - 4 \sin^2 \alpha \cdot \sin^2 \beta + \cos(2\alpha) \cos(2\beta) - \sin(2\alpha) \cdot \sin(2\beta)$ $= (1 - \cos 2\beta) - (1 - \cos(2\alpha))(1 - \cos(2\beta)) + \cos(2\alpha) \cos(2\beta) \quad \left\{ \sin^2 \theta = \frac{1 - \cos(2\theta)}{2} \right\}$ $= 1 - \cos(2\beta) - (1 - \cos 2\beta - \cos(2\alpha) + \cos(2\alpha) \cos(2\beta)) + \cos(2\alpha) \cos(2\beta)$ $= 1 - \cos(2\beta) - 1 + \cos(2\beta) + \cos(2\alpha) - \cos(2\alpha) \cos(2\beta) + \cos(2\alpha) \cos(2\beta)$ $= \cos(2\alpha) = \text{R.H.S. (proved) ans.}$	
Q.16)	If $x \cos \theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right)$ , then find the value of $xy + yz + zx$ ?	
Sol.16)	<p>We can write, <math>xy + yz + zx = xyz \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right)</math></p> <p>Let <math>x \cos \theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right) = k</math> (say)</p> <p>Then, <math>\frac{1}{x} = \frac{\cos \theta}{k}</math>, <math>\frac{1}{y} = \frac{\cos\left(\theta + \frac{2\pi}{3}\right)}{k}</math>, <math>\frac{1}{z} = \frac{\cos\left(\theta + \frac{4\pi}{3}\right)}{k}</math></p> <p>Now, <math>\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{k} \left[ \cos \theta + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{4\pi}{3}\right) \right]</math></p> $= \frac{1}{k} \left[ \cos \theta + \cos \theta \cdot \cos \frac{2\pi}{3} - \sin \theta \cdot \sin \frac{2\pi}{3} + \cos \theta \cdot \cos \frac{4\pi}{3} - \sin \theta \cdot \sin \left( \frac{4\pi}{3} \right) \right]$ $= \frac{1}{k} \left[ \cos \theta + \cos \theta \cdot \cos \left( \pi - \frac{\pi}{3} \right) - \sin \theta \cdot \sin \left( \pi - \frac{\pi}{3} \right) + \cos \theta \cdot \cos \left( \pi + \frac{\pi}{3} \right) - \sin \theta \cdot \sin \left( \pi - \frac{\pi}{3} \right) \right]$ $= \frac{1}{k} \left[ \cos \theta + \cos \theta \cdot \cos \left( 1 - \frac{1}{2} \right) - \sin \theta \left( \frac{\sqrt{3}}{2} \right) + \cos \theta \left( \frac{-1}{2} \right) - \sin \theta \left( -\frac{\sqrt{3}}{2} \right) \right]$	

	$  \begin{aligned}  &= \frac{1}{k} \left[ \cos \theta - \frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta - \frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta \right] \\  &= \frac{1}{k} [\cos \theta - \cos \theta] = 0 \\  &\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0 \\  \text{Since, } xy + yz + zx &= xyz \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) \\  &= xyz \times (0) = 0 = \text{R.H.S. ans.}  \end{aligned}  $	
Q.17)	Find the value of expansion $3 \left[ \sin^4 \left( \frac{3\pi}{2} - \alpha \right) + \sin^4(3\pi + \alpha) \right] - 2 \left[ \sin^6 \left( \frac{\pi}{2} + \alpha \right) + \sin^6(5\pi + \alpha) \right]$ ?	
Sol.17)	$  \begin{aligned}  &3 \left[ \sin^4 \left( \frac{3\pi}{2} - \alpha \right) + \sin^4(3\pi + \alpha) \right] - 2 \left[ \sin^6 \left( \frac{\pi}{2} + \alpha \right) + \sin^6(5\pi + \alpha) \right] \\  &= 3[(\cos^2 \alpha + \sin^2 \alpha)^2 - 2 \cos^2 \alpha \sin^2 \alpha] - 2[(\cos^2 \alpha + \sin^2 \alpha).(\cos^4 \alpha + \sin^4 \alpha) - \cos^2 \alpha \sin^2 \alpha] \\  &= 3[1 - 2 \cos^2 \alpha \sin^2 \alpha] - 2[(1)(\cos^2 \alpha + \sin^2 \alpha)^2 - 2 \cos^2 \alpha \sin^2 \alpha - \cos^2 \alpha \sin^2 \alpha] \\  &= 3 - 6 \cos^2 \alpha \sin^2 \alpha - 2[1 - 3 \cos^2 \alpha \sin^2 \alpha] \\  &= 3 - 6 \cos^2 \alpha \sin^2 \alpha - 2 + 6 \cos^2 \alpha \sin^2 \alpha \\  &= 3 - 2 = 1 \text{ ans.}  \end{aligned}  $	
Q.18)	If $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$ , then show that $\cos(2\alpha) \cos(2\beta) = -2 \cos(\alpha + \beta)$ ?	
Sol.18)	We have, $\cos \alpha + \cos \beta = \sin \alpha + \sin \beta = 0$ Then also we have, $(\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 = 0$ $\Rightarrow \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta - \sin^2 \alpha + \sin^2 \beta - 2 \sin \alpha \sin \beta = 0$ $\Rightarrow (\cos^2 \alpha - \sin^2 \alpha) + (\cos^2 \alpha - \sin^2 \alpha) + 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) = 0$ $\Rightarrow \cos(2\alpha) + \cos(2\beta) + 2 \cos(\alpha + \beta) = 0 \dots \{ \cos(2\theta) = \sin^2 \theta \cdot \cos^2 \theta \}$ $\Rightarrow \cos(2\alpha) + \cos(2\beta) = -2 \cos(\alpha + \beta) \text{ (proved)}$	
Q.19)	If $\tan \frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}} \tan \left( \frac{\phi}{2} \right)$ , show that $\cos \phi = \frac{\cos \theta - e}{1 - e \cos \theta}$ ?	
Sol.19)	We have, $\tan \left( \frac{\theta}{2} \right) = \sqrt{\frac{1-e}{1+e}} \tan \left( \frac{\phi}{2} \right)$ Squaring $\tan^2 \left( \frac{\theta}{2} \right) = \frac{(1-e)}{(1+e)} \tan^2 \left( \frac{\phi}{2} \right)$ $\Rightarrow \tan^2 \left( \frac{\theta}{2} \right) = \left( \frac{1-e}{1+e} \right) \tan^2 \left( \frac{\phi}{2} \right)$ Taking, L.H.S. $\cos \phi$ $= \frac{1-\tan^2 \left( \frac{\theta}{2} \right)}{1+\tan^2 \left( \frac{\theta}{2} \right)} \dots \{ \cos(2\theta) = \frac{1-\tan^2 \theta}{1+\tan^2 \theta} \}$ $= \frac{1-\left( \frac{1-e}{1+e} \right) \tan^2 \left( \frac{\phi}{2} \right)}{1+\left( \frac{1-e}{1+e} \right) \tan^2 \left( \frac{\phi}{2} \right)} \dots \{ \text{putting value } \tan^2 \left( \frac{\phi}{2} \right) \}$ $= \frac{(1-e)-(1+e)\tan^2 \left( \frac{\theta}{2} \right)}{(1-e)+(1+e)\tan^2 \left( \frac{\theta}{2} \right)}$ $= \frac{1-e-\tan^2 \frac{\theta}{2}-e\tan^2 \frac{\theta}{2}}{1-e+\tan^2 \frac{\theta}{2}+e\tan^2 \frac{\theta}{2}}$ $= \frac{\left( 1-\tan^2 \frac{\theta}{2} \right)-e\left( 1+\tan^2 \frac{\theta}{2} \right)}{\left( 1+\tan^2 \frac{\theta}{2} \right)-e\left( 1-\tan^2 \frac{\theta}{2} \right)}$ Divide N & D by $\left( 1 + \tan^2 \frac{\theta}{2} \right)$ $= \frac{\frac{1-\tan^2 \frac{\theta}{2}}{1+\tan^2 \frac{\theta}{2}}-e}{1-e\frac{\left( 1-\tan^2 \frac{\theta}{2} \right)}{\left( 1+\tan^2 \frac{\theta}{2} \right)}}$	



	$= \frac{\cos \theta - e}{1 - e \cos \theta} = \text{R.H.S.} \dots \left\{ \frac{1 - \tan^2 \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} = \cos(2\theta) \right\} \text{ans.}$	
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