CHAPTER - 10

STRAIGHT LINES

- Slope or gradient of a line is defined as $m = \tan\theta$, $(\theta \neq 90°)$, where θ is angle which the line makes with positive direction of x-axis measured in anticlockwise direction, $0 \leq \theta < 180°$
- Slope of x-axis is zero and slope of y-axis is not defined.
- Slope of a line through given points (x_1, y_1) and (x_2, y_2) is given by $\dfrac{y_2 - y_1}{x_2 - x_1}$
- Two lines are parallel to each other if and only if their slopes are equal.
- Two lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.
- Acute angle α between two lines, whose slopes are m_1 and m_2 is given by $\tan\alpha = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$, $1 + m_1 m_2 \neq 0$
- $x = a$ is a line parallel to y-axis at a distance of a units from y-axis. $x = a$ lies on right or left of y-axis according as a is positive or negative.
- $y = b$ is a line parallel to x-axis at a distance of 'b' units from x-axis. $y=b$ lies above or below x-axis, according as b is positive or negative.
- Equation of a line passing through given point (x_1, y_1) and having slope m is given by

  $$y - y_1 = m(x - x_1)$$

- Equation of a line passing through given points (x_1, y_1) and (x_2, y_2) is given by $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- Equation of a line having slope m and y-intercept c is given by

  $$y = mx + c$$

- Equation of line having intercepts a and b on x-axis and y-axis respectively is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$

- Equation of line in normal form is given by x $\cos\alpha$ + y $\sin\alpha$ = p,

  p = Length of perpendicular segment from origin to the line

  α = Angle which the perpendicular segment makes with positive direction of x-axis

- Equation of line in general form is given by Ax + By + C = 0, A, B and C are real numbers and at least one of A or B is non zero.

- Distance of a point (x_1, y_1) from line Ax + By + C = 0 is given by

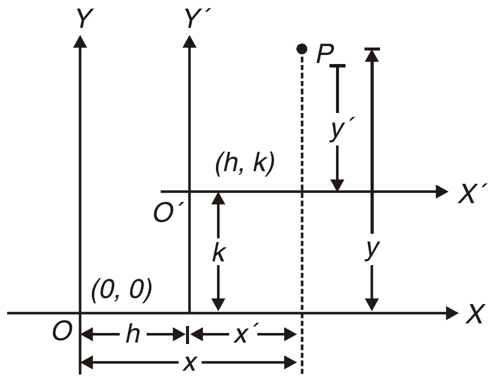
$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

- Distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$

- Shifting of origin to a new point without changing the direction of the axes is known as translation of axes.

  Let OX, OY be the original axes and O' be the new origin. Let coordinates of O' referred to original axes be (h, k). Let P(x, y) be point in plane



Let O'X' and O'Y' be drawn parallel to and in same direction as OX and OY respectively. Let coordinates of P referred to new axes O'X' and O'Y' be (x', y') then $x = x' + h$, $y = y' + k$

or $\qquad x' = x - h, \; y' = y - k$

Thus

(i) The point whose coordinates were (x, y) has now coordinates $(x - h, y - k)$ when origin is shifted to (h, k).

(ii) Coordinates of old origin referred to new axes are $(-h, -k)$.

- Equation of family of lines parallel to $Ax + By + C = 0$ is given by $Ax + By + k = 0$, for different real values of k
- Equation of family of lines perpendicular to $Ax + By + C = 0$ is given by $Bx - Ay + k = 0$, for different real values of k.
- Equation of family of lines through the intersection of lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ is given by $(A_1x + B_1y + C_1) + k(A_2x + B_2y + C_2) = 0$, for different real values of k.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Three consecutive vertices of a parallelogram are $(-2, -1)$, $(1, 0)$ and $(4, 3)$, find the fourth vertex.
2. For what value of k are the points $(8, 1)$, $(k, -4)$ and $(2, -5)$ collinear?
3. The mid point of the segment joining (a, b) and $(-3, 4b)$ is $(2, 3a + 4)$. Find a and b.
4. Coordinates of centroid of ΔABC are $(1, -1)$. Vertices of ΔABC are $A(-5, 3)$, $B(p, -1)$ and $C(6, q)$. Find p and q.
5. In what ratio y-axis divides the line segment joining the points $(3,4)$ and $(-2, 1)$?
6. What are the possible slopes of a line which makes equal angle with both axes?
7. Determine x so that slope of line through points $(2, 7)$ and $(x, 5)$ is 2.
8. Show that the points $(a, 0)$, $(0, b)$ and $(3a - 2b)$ are collinear.

9. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through (2, 5).

10. Find k so that the line $2x + ky - 9 = 0$ may be perpendicular to $2x + 3y - 1 = 0$

11. Find the acute angle between lines $x + y = 0$ and $y = 0$

12. Find the angle which $\sqrt{3}x + y + 5 = 0$ makes with positive direction of x-axis.

13. If origin is shifted to (2, 3), then what will be the new coordinates of (–1, 2)?

14. On shifting the origin to (p, q), the coordinates of point (2, –1) changes to (5, 2). Find p and q.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

15. If the image of the point (3, 8) in the line $px + 3y - 7 = 0$ is the point (–1, –4), then find the value of p.

16. Find the distance of the point (3,2) from the straight line whose slope is 5 and is passing through the point of intersection of lines $x + 2y = 5$ and $x - 3y + 5 = 0$

17. The line $2x - 3y = 4$ is the perpendicular bisector of the line segment AB. If coordinates of A are (–3, 1) find coordinates of B.

18. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line $y = 2x + c$. Find c and remaining two vertices.

19. If two sides of a square are along $5x - 12y + 26 = 0$ and $5x - 12y - 65 = 0$ then find its area.

20. Find the equation of a line with slope –1 and whose perpendicular distance from the origin is equal to 5.

21. If a vertex of a square is at (1, –1) and one of its side lie along the line $3x - 4y - 17 = 0$ then find the area of the square.

22. Find the coordinates of the orthocentre of a triangle whose vertices are (–1, 3) (2, –1) and (0, 0). [Orthocentre is the point of concurrency of three altitudes].

23. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $2x - 5y + 7 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.

24. If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

25. Find points on the line $x + y + 3 = 0$ that are at a distance of $\sqrt{5}$ units from the line $x + 2y + 2 = 0$

26. Find the equation of a straight line which makes acute angle with positive direction of x–axis, passes through point(–5, 0) and is at a perpendicular distance of 3 units from origin.

27. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1,1). Find the equation of other three sides.

28. If (1,2) and (3, 8) are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.

29. Find the equations of the straight lines which cut off intercepts on x–axis twice that on y–axis and are at a unit distance from origin.

30. Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals is $11x + 7y = 4$, find the equation of the other diagonal.

ANSWERS

1. (1, 2)	2. $k = 3$
3. $a = 7, b = 10$	4. $p = 2, q = -5$
5. 3 : 2 (internally)	6. ± 1
7. 1	9. $x + y = 7$

10. $\frac{-4}{3}$

11. $\frac{\pi}{4}$

12. $\frac{2\pi}{3}$

13. (–3, –1)

14. p = –3, q = –3

15. 1

16. $\frac{10}{\sqrt{26}}$

17. (1, –5)

18. c = –4, (2,0), (4, 4)

19. 49 square units

20. $x + y + 5\sqrt{2} = 0$, $x + y - 5\sqrt{2} = 0$

21. 4 square units

22. (–4, –3)

23. $x + 2y = 1$

24. $x + y - 5 = 0$

25. (1, –4), (–9 , 6)

26. $3x - 4y + 15 = 0$

27. $4x + 7y - 11 = 0$, $7x - 4y + 25 = 0$

$7x - 4y - 3 = 0$

28. $x - 2y + 3 = 0$, $2x + y - 14 = 0$,

$x - 2y + 13 = 0$, $2x + y - 4 = 0$

$3x - y - 1 = 0$, $x + 3y - 17 = 0$

29. $x + 2y + \sqrt{5} = 0$, $x + 2y - \sqrt{5} = 0$

30. $x = y$