CBSE Class 11 Straight Lines Worksheet

Class 11^{th}

Q.1) The hypotenuse of a right angle triangle has its ends at the points (1,3) and $(-4,1)$. Find the equation of the lines (perpendicular sides) of the traingle.

Sol.1) Required lines are AC & BC

Clearly AC is parallel to $Y-axis$ and

Slope of $Y-axis = \frac{1}{0}$

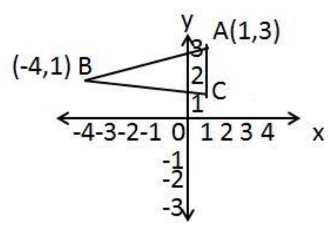
$\therefore$ slopw of $AC = \frac{1}{0}$

Now, equation of AC (using point-slope form, point $A(1,3)$ & slope= $\frac{1}{0}$)

$y-3 = \frac{1}{0}(x-1)$

$\Rightarrow 0 = x-1$

$\Rightarrow x = 1$



Now, BC is parallel to $X-axis$ &

Slope of $X-axis = 0$

$\therefore$ the slope of $BC = 0$

Equation of BC (using point-slope form, point $B(-4,1)$ & slope $= 0$)

$y-1 = 0(x+4)$

$\Rightarrow y-1 = 0$

$\Rightarrow y = 1$

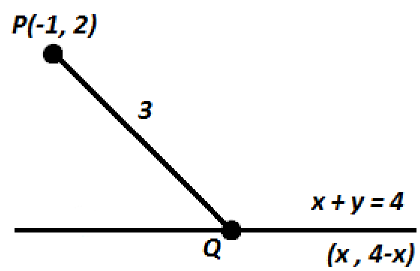
$\therefore$ required equations are $x = 1$ & $y = 1$ ans.

Q.2) Find the function in which a straight line must be drawn through the point $(-1,2)$, so that its point of intersection with the line $x+y=4$ may be at a distance of 3 $units$ from this point.

Sol.2) Let Q is $(x, 4-x)$

$PQ = 3$ (given)

$\sqrt{(x+1)^1 + (4-x-2)^2} = 3$



Squaring

$\Rightarrow (x+1)^2 + (2-x)^2 = 9$

$\Rightarrow x^2 + 1 + 2x + 4 + x^2 - 4x = 9$

$\Rightarrow 2x^2 - 2x - 4 = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x = 2$ & $x = -1$

$\therefore$ point Q is (2,2) or $(-1,5)$

Now equation of PQ (using two-point form, $P(-1,2), Q(2,2)$)

$y-2 = \frac{2-2}{2+1}(x+1)$

$y-2 = 0(x+1)$

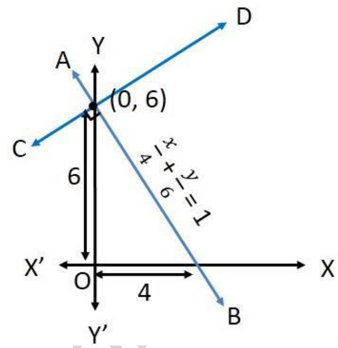
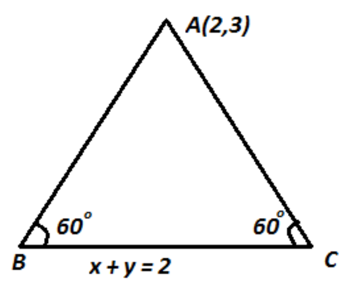
$y = 2$ is a line parallel to $X-axis$

Again equation of PQ (using $P(-1,2)$ & $Q(-1,5)$)

$y-2 = \frac{5-2}{-1+1}(x+1)$

$y-2 = \frac{3}{0}(x+1)$

$\Rightarrow 0 = 3(x+1)$

	$\Rightarrow x = -1$ is a line parallel to $Y - axis$ $\therefore$ Direction of line is parallel to $X - axis$ or parallel to $Y - axis$.
Q.3)	Find the equation of line drawn perpendicular to the line $\frac{x}{4} + \frac{y}{6} = 1$ where it (given line) meets the $Y - axis$.
Sol.3)	Equation of given line  $\frac{x}{4} + \frac{y}{6} = 1$ $\Rightarrow 3x + 2y = 12$ Slope of given line $= \frac{-1}{2}$ $(m = \frac{-coefficient\ of\ x}{coefficient\ of\ y})$ Since required line is perpendicular to the given line $\therefore$ slope of required line $= \frac{2}{3}$ .... (-ve reciprocal) Given line meets the $\mathrm{Y} - \mathrm{axis}$ $\therefore x = 0$ $\Rightarrow y = 6$ (0,6) is also the point on the required line $\therefore$ equation of required line (using point-slope form, point $(0,6)$, $\mathrm{m} = \frac{2}{3}$) $y - 6 = \frac{2}{3}(x - 0)$ $\Rightarrow 3y - 18 = 2x$ $\Rightarrow 2x - 3y = -18$ ans.
Q.4)	A vertex of an equilateral triangle is $(2,3)$ & the opposite side is $x + y = 2$. Find the equation of the other side.
Sol.4)	Equation of $BC = x + y = 2$ (given)  Slope of $BC = \frac{-1}{1} = -1$ Let slope of $AB = m$ Angle between AB & $BC = 60^{\circ}$ Now angle between two lines $\tan\theta = \left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$ $\Rightarrow \tan 60 = \left\|\frac{m+1}{1+m}\right\|$ $\Rightarrow \sqrt{3} = \left\|\frac{m+1}{1+m}\right\|$ $\pm\sqrt{3} = \frac{m+1}{1-m}$ Case 1: $\sqrt{3} = \frac{m+1}{1-m}$ $\Rightarrow \sqrt{3} - \sqrt{3}m = m + 1$ $\Rightarrow \sqrt{3} - 1 = \sqrt{3}m + m$ $\Rightarrow \sqrt{3} - 1 = m(\sqrt{3} + 1)$ $\Rightarrow m = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ Case 2: $-\sqrt{3} = \frac{m+1}{1-m}$ $-\sqrt{3} + \sqrt{3}m = m + 1$ $\sqrt{3}m - m = 1 + \sqrt{3}$ $m(\sqrt{3} - 1) = 1 + \sqrt{3}$ $\Rightarrow m = \frac{1+\sqrt{3}}{\sqrt{3}-1}$ Equation of AB (using point slope form, point $(2,3)$ & $m = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ $y - 3 = \frac{\sqrt{3}-1}{\sqrt{3}+1}(x - 2)$ $(\sqrt{3} + 1)y - 3\sqrt{3} - 3 = (\sqrt{3} - 1)x - 2\sqrt{3} + 2$ $(\sqrt{3} - 1)x - (\sqrt{3} + 1)y + 5 + \sqrt{3} = 0$

Equation of AC (point $A(2,3)$ & $m = \frac{1+\sqrt{3}}{\sqrt{3}-1}$

$y - 3 = \frac{1+\sqrt{3}}{\sqrt{3}-1}(x-2)$

$\Rightarrow (\sqrt{3}-1)y - 3\sqrt{3} + 3 = (\sqrt{3}+1)x - 2 - 2\sqrt{3}$

$\Rightarrow (\sqrt{3}+1)x - (\sqrt{3}-1)y + \sqrt{3} - 5 = 0$

$\therefore$ equation of other two sides are

$(\sqrt{3}-1)x - (\sqrt{3}+1)y + 5 + \sqrt{3} = 0$

And $(\sqrt{3}+1)x - (\sqrt{3}-1)y + \sqrt{3} - 5 = 0$ ans.

Q.5) The opposite angular points of a square are $(3,4)$ and $(1,-1)$. Find the coordinates of the other two sides.

Sol.5) Slope of $AC = \frac{-1-4}{1-3}$

$= \frac{-5}{-2} = \frac{5}{2}$

Let slope of $AB = m$

$\theta = 45^\circ$

$\tan 45 = \left|\frac{m-\frac{5}{2}}{1+\frac{5}{2}m}\right|$

$1 = \left|\frac{2m-5}{2+m}\right|$

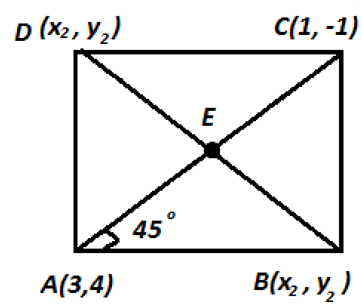
$\pm 1 = \frac{2m-5}{2+5m}$

$1 = \frac{2m-5}{2+5m}$ $\quad$ $-1 = \frac{2m-5}{2+5m}$

$2 + 5m = 2m - 5$ $\quad$ $-2 - 5m = 2m - 5$

$3m = -7$ $\quad$ $3 = 7m$

$m = -\frac{7}{3}$ $\quad$ $m = \frac{3}{7}$



Let slope of $AB = -\frac{7}{3}$ and slope of $BC = \frac{3}{7}$

Find equation of AB & BC by using point slope forms

$AB: 7x + 3y - 4 = 0$ and $BC: 3x - 7y + 19 = 0$

Solve equation of AB and BC simultaneously to get point $B(-\frac{1}{2}, \frac{5}{2})$

E is the mid point of $A(3,4)$ & $C(1,-1)$

$\therefore$ coordinate of E is $(2, \frac{3}{2})$

E is also the mid point of $B(\frac{1}{2}, \frac{5}{2})$ and $D(x_2, y_2)$.

By mid point formula, coordinate of D is $(\frac{9}{2}, \frac{1}{2})$

$\therefore$ the other two vertices are $(-\frac{1}{2}, \frac{5}{2})$ and $(\frac{9}{2}, \frac{1}{2})$ ans.

Q.6) A point moves such that its distance from the point $(4,0)$ is half that of its distance from the line $x = 16$. Find the locus of the point.

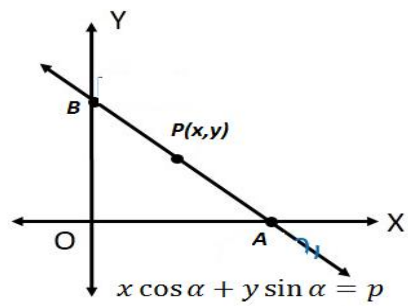
Sol.6) Let (x, y) be the coordinate of moving point

Distance between (x,y) & $(4,0) = \sqrt{(x-4)^2 + y^2}$

Distance between (x,y) and line $x - 16 = 0$

$\frac{|x-16|}{\sqrt{1+0}}$

Given that, $\sqrt{(x-4)^2 + y^2} = \frac{1}{2}\frac{|x-16|}{\sqrt{1+0}}$

	Squaring both sides $x^2 + 16 - 8x + y^2 = \frac{1}{4}(x^2 + 256 - 32x)$ $\Rightarrow 4x^2 + 64 - 32x + 4y^2 = x^2 + 256 - 32x$ $\Rightarrow 3x^2 + 4y^2 = 192$ is the required equation.
Q.7)	Show that the locus of the mid point of the distance between the axis of the variable line $x\cos\alpha + y\sin\alpha = p$ is $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ where p is constant.
Sol.7)	Line $x\cos\alpha + y\sin\alpha = p$ For point A, Put $y = 0$ $x = \frac{p}{\cos\alpha}$ $\quad\therefore A(\frac{p}{\cos\alpha}, 0)$ For point B put $x = 0$ $y\sin\alpha = p \Rightarrow y = \frac{p}{\sin\alpha}$ $\quad\therefore B(0, \frac{p}{\sin\alpha})$ Now, $\mathrm{P(x,y)}$ is the mid-point of A & B $\mathrm{x} = \frac{\frac{p}{\cos\alpha}}{2} + 0$ and $\mathrm{y} = 0 + \frac{\frac{p}{\sin\alpha}}{2}$ $x = \frac{p}{2\cos\alpha}$ and $y = \frac{p}{2\sin\alpha}$ $\cos\alpha = \frac{p}{2x}$ and $\sin\alpha = \frac{p}{2y}$ Squaring & adding these equations $\cos^2\alpha = \sin^2\alpha = \frac{p^2}{4x^2} + \frac{p^2}{4p^2}$ $1 = \frac{p^2}{4x^2} + \frac{p^2}{4y^2}$ $\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ (proved) 
Q.8)	If the equation of the base of an equilateral triangle is $x + y = 2$ & the vertex is $(2,1)$. Find the area of the traingle.
Sol.8)	AD is the perpendicular distance between the point A and line BC $(x + y - 20)$. $\therefore AD = \frac{\|2-1-2\|}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$ In ΔABD $\sin 60 = \frac{AD}{a}$ ……… (a is a side of triangle) $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}a}$ $\frac{1}{a} = \frac{\sqrt{2}\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow a = \frac{\sqrt{2}}{\sqrt{3}}$ Area of equilateral triangle $= \frac{\sqrt{3}}{4}a^2$ $= \frac{\sqrt{3}}{4}\left(\frac{2}{3}\right) = \frac{1}{2\sqrt{3}}$ *square units*