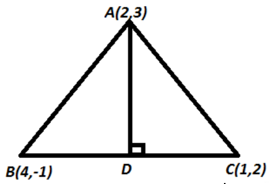
CBSE Class 11 Straight Lines worksheet

Class 11th

Q.1) If the ΔABC with vertices $A(2,3), B(4,-1)$ and $C(1,2)$. Find the equation and length of altitude from the vertex A.

Sol.1) Slope of $BC = \frac{2-(-1)}{1-4} = \frac{3}{-3} = -1$



Since $AD \perp \mathrm{BC}$

∴ slope of $AD = 1$ .... $(-ve\ reciprocal)$

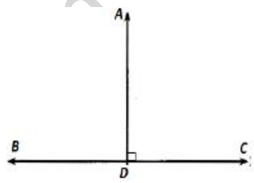
Equation of altitude AB (point slope form , point $AB(2,3)$, slope= 1)

$y - 3 = 1(x - 2)$

$\Rightarrow y - 3 = x - 2$

$\Rightarrow x - y + 1 = 0$ ans.

Now equation of BC (two point form, $B(4,-1)$ & $C(1,2)$)



$y + 1 = \frac{2-(-1)}{1-4}(x - 4)$

$\Rightarrow y + 1 = -1(x - 4)$

$\Rightarrow y + 1 = -x + 4$

$\Rightarrow x + y - 3 = 0$

Now, length of AD =perpendicular between the point $A(2,3)$ and the line BC

By distance formula,

$AD = \frac{|2+3-3|}{\sqrt{1+1}}$

$AD = \frac{2}{\sqrt{2}}$

$AD = \sqrt{2}$ ans.

Q.2) Find the perpendicular distance from the origin to the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\emptyset, \sin\emptyset)$.

Sol.2) Equation of line AB (two point form)

$y - \sin\theta = \left(\frac{\sin\emptyset - \sin\theta}{\cos\emptyset - \cos\theta}\right)(x - \cos\theta)$

$\Rightarrow y - \sin\theta = \left(\frac{2\cos\left(\frac{\emptyset+\theta}{2}\right).\sin\left(\frac{\emptyset-\theta}{2}\right)}{-2\sin\left(\frac{\emptyset+\theta}{2}\right).\sin\left(\frac{\emptyset-\theta}{2}\right)}\right).(x - \cos\theta)$

$\Rightarrow -y\sin\left(\frac{\emptyset+\theta}{2}\right) + \sin\theta.\sin\left(\frac{\emptyset+\theta}{2}\right) = x\cos\left(\frac{\emptyset+\theta}{2}\right) - \cos\theta.\cos\left(\frac{\emptyset+\theta}{2}\right)$

$\Rightarrow x\cos\left(\frac{\emptyset+\theta}{2}\right) + y\sin\left(\frac{\emptyset+\theta}{2}\right) - \cos\theta.\cos\left(\frac{\emptyset+\theta}{2}\right) - \sin\theta.\sin\left(\frac{\emptyset+\theta}{2}\right)$

$\Rightarrow x\cos\left(\frac{\emptyset+\theta}{2}\right) + y\sin\left(\frac{\emptyset+\theta}{2}\right) - \left\{\cos\theta.\cos\left(\frac{\emptyset+\theta}{2}\right) + \sin\theta.\sin\left(\frac{\emptyset+\theta}{2}\right)\right\}$

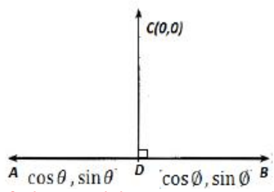
$\Rightarrow x\cos\left(\frac{\emptyset+\theta}{2}\right) + y\sin\left(\frac{\emptyset+\theta}{2}\right) - \left\{\cos\theta.\cos\left(\frac{\emptyset+\theta}{2}\right) + \sin\theta.\sin\left(\frac{\emptyset+\theta}{2}\right)\right\}$

$\Rightarrow x\cos\left(\frac{\emptyset+\theta}{2}\right) + y\sin\left(\frac{\emptyset+\theta}{2}\right) - \cos\left(\theta - \frac{\emptyset+\theta}{2}\right) = 0$ ....$\{\cos A\cos B + \sin A\sin B = \cos(A - B)\}$

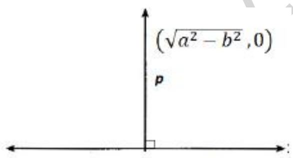
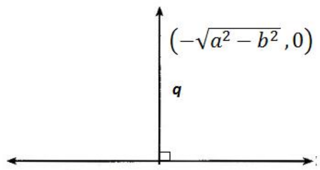
$\Rightarrow x\cos\left(\frac{\emptyset+\theta}{2}\right) + y\sin\left(\frac{\emptyset+\theta}{2}\right) - \cos\left(\frac{\theta-\emptyset}{2}\right) = 0$ is the equation of line AB

Now, the perpendicular distance of line AB from the point $(0,0)$ is given by distance

$= \frac{\left|0+0-\cos\left(\frac{\theta-\emptyset}{2}\right)\right|}{\sqrt{\cos^2\left(\frac{\theta-\emptyset}{2}\right)+\sin^2\left(\frac{\theta-\emptyset}{2}\right)}}$



$= \frac{\cos\left(\frac{\theta-\emptyset}{2}\right)}{1}$ .... $\{\cos^2\theta + \sin^2\theta = 1\}$

	$\therefore$ required distance $\cos\left(\frac{\theta-\emptyset}{2}\right)$ ans.
Q.3)	Prove that the product of the lengths of the perpendicular drawn from the points $\left(\sqrt{a^2-b^2}, 0\right)$ and $\left(-\sqrt{a^2-b^2}, 0\right)$ to the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is b^2.
Sol.3)	To prove: $pq = b^2$ Equation given line: $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ $\Rightarrow bx\cos\theta + ay\sin\theta - ab = 0$ Now, by distance formula $pq = \frac{\lvert b\sqrt{a^2-b^2}\cos\theta+0-ab\rvert}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}} \times \frac{\lvert -b\sqrt{a^2-b^2}\cos\theta+0-ab\rvert}{\sqrt{b^2\cos^2\theta+a^2\sin^2\theta}}$ $\Rightarrow x\cos\left(\frac{\emptyset+\theta}{2}\right) + y\sin\left(\frac{\emptyset+\theta}{2}\right) - \left\{\cos\theta.\cos\left(\frac{\emptyset+\theta}{2}\right) + \sin\theta.\sin\left(\frac{\emptyset+\theta}{2}\right)\right\}$ $\Rightarrow x\cos\left(\frac{\emptyset+\theta}{2}\right) + y\sin\left(\frac{\emptyset+\theta}{2}\right) - \left\{\cos\theta.\cos\left(\frac{\emptyset+\theta}{2}\right) + \sin\theta.\sin\left(\frac{\emptyset+\theta}{2}\right)\right\}$ $\Rightarrow x\cos\left(\frac{\emptyset+\theta}{2}\right) + y\sin\left(\frac{\emptyset+\theta}{2}\right) - \cos\left(\theta - \frac{\emptyset+\theta}{2}\right) = 0$ ....$\{\cos A\cos B + \sin A\sin B = \cos(A-B)\}$ $\Rightarrow \frac{\lvert b\sqrt{a^2-b^2}.\cos\theta-ab\rvert\times\lvert b\sqrt{a^2-b^2}.\cos\theta+ab\rvert}{b^2\cos^2\theta+a^2\sin^2\theta}$ $= \frac{\lvert b(a^2-b^2)\cos^2\theta-a^2b^2\rvert}{b^2\cos^2\theta+a^2\sin^2\theta}$ .... $\left\{\begin{matrix}\lvert -a-b\rvert = \lvert a+b\rvert \\ \lvert x\rvert\lvert y\rvert = \lvert xy\rvert\end{matrix}\right\}$ $= \frac{b^2\lvert(a^2-b^2)\cos^2\theta-a^2\rvert}{b^2\cos^2\theta+a^2\sin^2\theta}$ $= \frac{b^2\lvert a^2\cos^2\theta-b^2\cos^2\theta-a^2\rvert}{b^2\cos^2\theta+a^2\sin^2\theta}$ $= \frac{b^2\lvert a^2(\cos^2\theta-1)-b^2\cos^2\theta\rvert}{b^2\cos^2\theta+a^2\sin^2\theta}$ $= \frac{b^2\lvert -a^2\sin^2\theta-b^2\cos^2\theta\rvert}{b^2\cos^2\theta+a^2\sin^2\theta}$ $= \frac{b^2(a^2\sin^2\theta+b^2\cos^2\theta)}{b^2\cos^2\theta+a^2\sin^2\theta}$ .... $\{\lvert -a-b\rvert = (a+b)\}$ $\therefore pq = b^2$ (proved)  $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$  $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$
Q.4)	Find the equation of the line which is equidistant from the parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$.
Sol.4)	Equation of given lines $9x + 6y - 7 = 0$ Or $l_1: 3x + 2y - \frac{7}{3} = 0$ ....(i) And $l_3: 3x + 2y + 6 = 0$ .... (ii) Slope of l_1 or $l_3 = \frac{-3}{2}$ .... $\left\{m = \frac{-coefficient\ of\ x}{coefficient\ of\ y}\right\}$ Since l_2 is parallel to l_1 and l_3 $\therefore$ slope of $l_2 = \frac{-3}{2}$ Let equation of required line (l_2) is, $\Rightarrow y = mx + c$ $\Rightarrow y = -3x + c$ $\Rightarrow 2y = -3x + 2c$ $\Rightarrow 3x + 2y - 2c = 0$ (l_2) .... (iii) We are given that, Distance between l_1 and l_2= distance between l_2 and l_3 $\frac{\left\lvert\frac{-7}{3}+2c\right\rvert}{\sqrt{9+4}} = \frac{\lvert 6+2c\rvert}{\sqrt{9+4}}$ .... (formula distance $= \frac{\lvert c_1-c_2\rvert}{\sqrt{a^2+b^2}}$) $\Rightarrow \left\lvert\frac{-7}{3} + 2c\right\rvert = \lvert 6 + 2c\rvert$ $\Rightarrow \frac{-7}{3} + 2c = \pm(6 + 2c)$

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$\Rightarrow \frac{-7}{3} + 2c = 6 + 2c$

$\Rightarrow$ there is no value of c

$\frac{-7}{3} + 2c = -6 - 2c$

$4c = -6 + \frac{7}{3}$

$4c = \frac{-11}{3} \Rightarrow c = \frac{-11}{12}$

$\therefore$ equation of required line l_2 from equation (iii)

$3x + 2y - 2\left(\frac{-11}{12}\right) = 0$

$\Rightarrow 3x + 2y + \frac{11}{6} = 0$

$\Rightarrow 18x + 12y + 11 = 0$ ans.

Q.5) A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y = 4$ is bisected at the point $(1,5)$ obtain its equation.

Sol.5) Point $P(x_1, y_1)$ lies on line l_2

$\therefore 5x_1 - y_1 = -4$ .... (i)

Point $Q(x_2, y_2)$ lies on line l_3

$\therefore 3x_2 + 4y_2 = 4$ .... (ii)

Now $R(1,5)$ is the mid-point of $P(x_1, y_1)$ & $Q(x_2, y_2)$

$\therefore 1 = \frac{x_1 + x_2}{2}$ and $5 = \frac{y_1 + y_2}{2}$

$\Rightarrow x_1 + x_2 = 2$ and $y_1 + y_2 = 10$

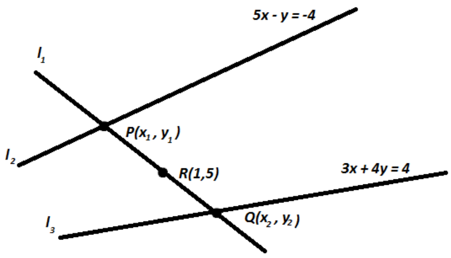
$\Rightarrow x_2 = 2 - x_1$ and $y_2 = 10 - y_1$

Put value of x_2 and y_2 in equation (ii)

$\therefore 3(2 - x_1) + 4(10 - y_1) = 4$

$\Rightarrow 6 - 3x_1 + 40 - 4y_1 = 4$

$\Rightarrow 3x_1 + 4y_1 = 42$ ....(iii)



Solving (i) & (iii), we get

$x = \frac{26}{23}$ and $y = \frac{222}{23}$

$\therefore P\left(\frac{26}{23}, \frac{222}{23}\right)$

Now, equation of required line l_1......... (using two point form) points (1,5) and $\left(\frac{26}{23}, \frac{222}{23}\right)$

$\Rightarrow y - 5 = \left(\frac{\frac{222}{23} - 5}{\frac{26}{23} - 1}\right)(x - 1)$

$\Rightarrow y - 5 = \frac{107}{3}(x - 1)$

$\Rightarrow 107x - 34 = 92$ ans.

Q.6) If the lines $2x + y = 3, 5x + ky - 3 = 0$ and $3x - y - 2 = 0$ are concurrent (intersect at one point). Find the value of k.

Sol.6) We have,

$2x + y = 3$ .... (i)

$5x + ky = 3$ .... (ii)

$3x - y = 2$ .... (iii)

Solving (i) & (iii), we get

$x = 1$ & $y = 1$

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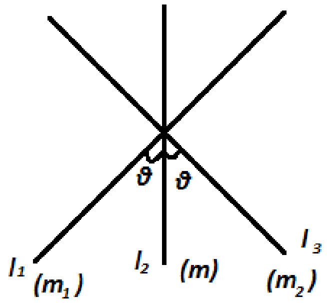
	Put these values in equation (ii) $\Rightarrow 5 + k - 3 = 0$ $\Rightarrow k = -2$ ans.
Q.7)	If the lines $y = 3x + 1$ nd $2y = x + 3$ are equally included to the line $y = mx + 4$. Find the value of m.
Sol.7)	Equation of given lines: $l_1: y = 3x + 1$ $m_1 = 3$ ………… (compared with $y = mx + c$) $l_3: 2y = x + 3$ $\Rightarrow y = \frac{1}{2}x + \frac{3}{2}$ $\therefore m_3 = \frac{1}{2}$ Let slope of required line $l_2 = m$  Angle between line l_1 and l_2 $\tan\theta = \left\lvert\frac{3-m}{1+3m}\right\rvert$ ………. (i) Angle between line l_2 and l_3: $\tan\theta = \left\lvert\frac{\frac{1}{2}-m}{1+\frac{1}{2}m}\right\rvert$ $\tan\theta = \left\lvert\frac{1-2m}{2+m}\right\rvert$ ………. (ii) From equation (i) & (ii) $\Rightarrow \left\lvert\frac{3-m}{1+3m}\right\rvert = \left\lvert\frac{1-2m}{2+m}\right\rvert$ $\Rightarrow \frac{3-m}{1+3m} = \pm\left(\frac{1-2m}{2+m}\right)$ …….. (if |x| = |y| then x = ±y) Case 1: $\frac{3-m}{1+3m} = \frac{1-2m}{2+m}$ $\Rightarrow 6 - 2m + 3m - m^2 = 1 - 2m + 3m - 6m^2$ $\Rightarrow 5m^2 = -5$ $\Rightarrow m^2 = -1 \Rightarrow m = \pm 1$

(rejected, no real value of m)

Case 2: $\frac{3-m}{1+3m} = -\left(\frac{1-2m}{2+m}\right)$

$\Rightarrow 6 - 2m + 3m - m^2 = -1 + 2m - 3m + 6m^2$

$\Rightarrow 7m^2 - 2m - 7 = 0$

$\Rightarrow m = \frac{2 \pm \sqrt{4+196}}{14}$ .... (by quadratic formula)

$\Rightarrow m = \frac{2 \pm \sqrt{200}}{14}$

$\Rightarrow m = \frac{2 \pm 10\sqrt{2}}{14}$

$\therefore m = \frac{1 \pm 5\sqrt{2}}{7}$ ans.

Q.8) Find the values of α and p if the equation $x\cos\alpha + y\sin\alpha = p$ is the normal form of $\sqrt{3}x + y + 2 = 0$.

Sol.8) We have, $\sqrt{3}x + y + 2 = 0$

$\Rightarrow \sqrt{3}x + y = -2$

Multiply by -1

$\Rightarrow -\sqrt{3}x - y = 2$ .... (make RHS +ve)

Here $a = \sqrt{3}$ and $b = -1$

Divide both sides by $\sqrt{a^2 + b^2} = \sqrt{3+1} = 2$

$\Rightarrow -\frac{\sqrt{3}}{2}x + \frac{-1}{2}y = 1$

$\Rightarrow x\cos\left(\pi + \frac{\pi}{6}\right) + y\sin\left(\pi + \frac{\pi}{6}\right) = 1$

$\Rightarrow x\cos\left(\frac{7\pi}{6}\right) + y\sin\left(\frac{7\pi}{6}\right) = 1$

Compare with $x\cos\alpha + y\sin\alpha = p$

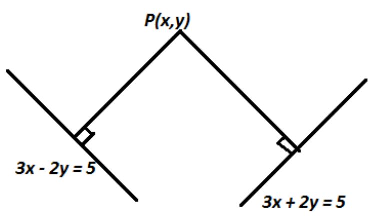
$\alpha = \frac{7\pi}{6}$ & $p = 1$ ans.

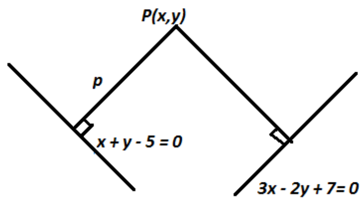
Q.9) Show that the path of a moving point such that its distance from two lines $3x - 2y = 5$ and $3x + 2y = 5$ are equal is a straight line.

Sol.9) Given lines are $3x - 2y - 5 = 0$ .... (i)

and $3x - 2y - 5 = 0$ .... (ii)

let $P(x, y)$ be the moving point, whose distance from the line (i) & (ii) are equal



	∴ By distance formula, $\frac{\lvert 3x-2y-5\rvert}{\sqrt{9+4}} = \frac{\lvert 3x+2y-5\rvert}{\sqrt{9+4}}$ $\Rightarrow \lvert 3x-2y-5\rvert = \lvert 3x+2y-5\rvert$ $\Rightarrow 3x-2y-5 = \pm(3x+2y-5)$ ..... (if $\lvert x\rvert = \lvert y\rvert$ then $x = y$) $\Rightarrow 3x-2y-5 = 3x+2y-5$ $\quad 3x-2y-5 = -3x-2y+5$ $\Rightarrow -4y = 0$ $\quad 6x = 10$ $\Rightarrow y = 0$ and $x = \frac{5}{3}$ Clearly lines represent the equation of line ∴ point P must moves on a straight line.
Q.10)	If sum of the perpendicular distances of available point $P(x, y)$ from the lines $x + y - 5 = 0$ and $3x - 2y + 7 = 0$ is always 10. Show that P must moves on a line.
Sol.10)	Given: $p + q = 10$ $\frac{\lvert x+y-5\rvert}{\sqrt{1+1}} = \frac{\lvert 3x-2y+7\rvert}{\sqrt{9+4}} = 10$  $\Rightarrow \sqrt{13}\,\lvert x+y-5\rvert + \sqrt{2}\,\lvert 3x-2y+7\rvert = \sqrt{2}\sqrt{13}10$ $\Rightarrow \sqrt{13}\,\lvert x+y-5\rvert + \sqrt{2}\,\lvert 3x-2y+7\rvert = 10\sqrt{26}$ There are four cases: $(+,+), (+,-), (-,+), (-,-)$ Consider 1st case: $\sqrt{13}\,(x+y-5) + \sqrt{2}(3x-2y+7) = 10\sqrt{26}$ $\Rightarrow x(\sqrt{13}+3\sqrt{2}) + \sqrt{13} - 2\sqrt{2}) - 5\sqrt{13} + 7\sqrt{2} - 10\sqrt{26} = 0$ Clearly the equation represents the equation of a straight line. ∴ point P must moves on a straight line. Similarly, remaining 3 cases can be done.