CBSE Class 11 Straight Lines worksheet

Class 11th

Q.1) The perpendicular from the origin to the line $y = mx + c$ meets it at the point (-1, 2). Find the value of m and c.

Sol.1) Equation of $AB = y = mx + c$

Slope of $AB = m$

Slope of $CD = \frac{2-0}{-10} = -2$ .... $m = \frac{y_2 - y_1}{x_2 - x_1}$

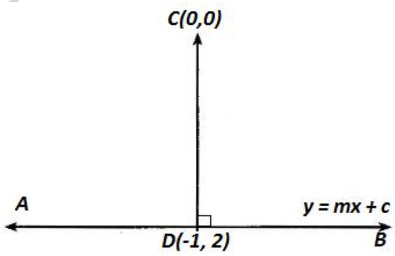
Since $CD \perp \mathrm{AB}$

$\therefore$ Slope of $AB = \frac{1}{2}$ .... $-ve\ respectively$

$m = \frac{1}{2}$ .... (since slope of AB is also m)

$\therefore$ equation of line AB becomes

$y = \frac{1}{2}x + c$



$\Rightarrow 2y = x + 2c$

$\Rightarrow x - 2y + 2c = 0$

This line passes through point $D(-1, 2)$

$\therefore\ -1 - 4 + 2c = 0$

$\Rightarrow c = \frac{5}{2}$

$\therefore m = 1$ and $c = \frac{5}{2}$ ans.

Q.2) Assuming that straight lines work as a plane mirror for a point in the line $x - 3y + 4 = 0$.

Sol.2) Let $P'(a, b)$ is the image of point $P(1, 2)$ equation of line $AB: x - 3y = -4$ (given) ...... (i)

Slope of $AB = -\frac{1}{-3} = \frac{1}{3}$ ....$\left\{m = \frac{-\,coefficient\ of\ x}{coefficient\ of\ y}\right\}$

Since $PQ \perp \mathrm{AB}$

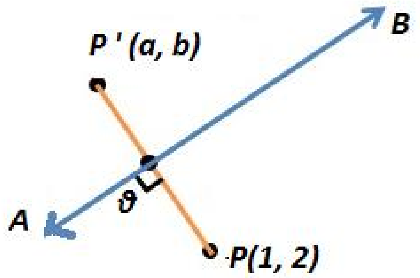
$\therefore$ Slope of $PQ = -3$ .... $-ve\ respectively$

Now equation of PQ (point slope form , point $P(1,2)$ slope $= -3$)

$y - 2 = -3(x - 1)$

$y - 2 = -3x + 3$

$\Rightarrow 3x + y = 5$ .... (ii)



Solving equation (i) & (ii)

We get $x = \frac{11}{10}$ And $y = \frac{17}{10}$

$\therefore\ \theta\left(\frac{11}{10}, \frac{17}{10}\right)$

Now θ is the mid-point of $P(1,2)$ & $P'(a,b)$

By mid-point formula,

$\frac{11}{10} = \frac{1+a}{2}$ And $\frac{17}{10} = \frac{2+b}{2}$

$\Rightarrow 22 = 10 + 10a$ and $34 = 20 + 10b$

$\Rightarrow a = \frac{6}{5}$ $\qquad \Rightarrow b = \frac{7}{5}$

$\therefore$ image is $P'\left(\frac{6}{5}, \frac{7}{5}\right)$ Ans.

Q.3) Show that the area of the triangle formed by the lines $y = m_1x + c_1$; $y = m_2x + c_2$ And $x = 0$ is $\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}$

Sol.3) Equation of $AC: x = 0$ .... (i)

Equation of $AB: y = m_1x + c_1$ .... (ii)

Equation of $BC: y = m_2x + c_2$ .... (iii)

Point A is the intersection point of side AB & AC

$\therefore$ solving (i) & (ii)

We get $x = 0$ and $y = c_1$

$\therefore A(0, c_2)$

Point B is the intersection point of AB & BC

Solving (ii) & (iii)

$m_1x + c_1 = m_2x + c_2$

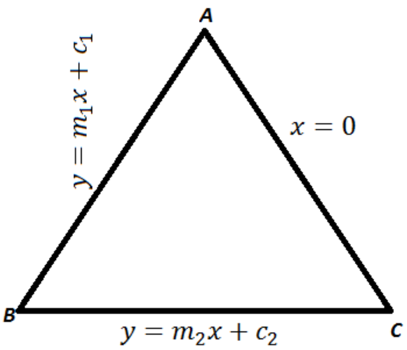
$\Rightarrow x(m_1 - m_2) = c_2 - c_1$

$\Rightarrow x = \left(\frac{c_2 - c_1}{m_1 - m_2}\right) + c_1$

$\Rightarrow y = \frac{m_1c_2 - m_1c_1 + m_1c_1 - m_2c_1}{m_1 - m_2}$

$\Rightarrow y = \frac{m_1c_1 - m_2c_1}{m_1 - m_2}$

$\therefore B\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right)$



Point C is the intersection point of side AC & BC

$\therefore$ solving (i) & (iii)

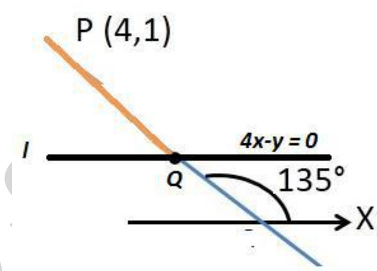
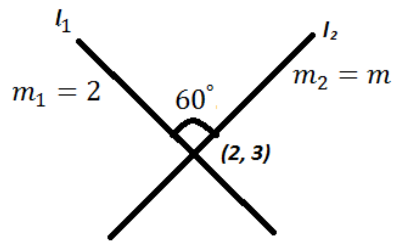
We get $x = 0$ and $y = c_2$

$\therefore C(0, c_2)$

$\therefore A(0_1c_1), B\left(\frac{c_2 - c_1}{m_1 - m_2}, \frac{m_1c_2 - m_2c_1}{m_1 - m_2}\right), C(0_1c_2)$

Now area of ΔABC is

$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

	$= -\frac{1}{2}\left\|0(y_2-y_3) + \frac{c_2-c_1}{m_1-m_2}(c_2-c_1) + 0(y_1-y_2)\right\|$ $= \frac{1}{2}\left\|\frac{(c_2-c_1)}{m_1-m_2}\right\|$ Area $= \frac{1}{2}\frac{(c_2-c_1)}{\|m_1-m_2\|}$ Ans.
Q.4)	Find the distance of the line $4x - y = 0$ from the point $P(4,1)$ measured along the line making an angle of 135° with $+ve\ X - axis$.
Sol.4)	Slope of line $l(4x-y) = 0 = \frac{-4}{-1} = 4$ Slope of $PQ = \tan(135)$ $= \tan(180-45)$ $= -\tan(45) = -1$ Equation of line PQ (point slope form, point $P(4,1), m = -1$) $y - 1 = -1(x-4)$ $y - 1 = -x + 4$ $\Rightarrow x + y = 5$ .... (i) Equation of given line (L): $4x - y = 0$ .... (ii) Solving (i) and (ii) $x = 1$ and $y = 4$ $\therefore$ coordinate of Q is $(1,4)$ Required distance $PQ = \sqrt{3^2+3^2}$ $= \sqrt{18} = 3\sqrt{2}\ units$ ans. 
Q.5)	Two lines passing through the point $(2,3)$ intersects each other at an angle of 60°. If the slope of one line is 2. Find the equation of other line.
Sol.5)	Slope of one line: $m_1 = 2$ Let slope of required line: $m_2 = m$ Angle between them is $\theta = 60^\circ$ We have, $\tan\theta = \left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $\Rightarrow \tan 60^\circ = \left\|\frac{2-m}{1+2m}\right\|$ $\Rightarrow \sqrt{3} = \left\|\frac{2-m}{1+2m}\right\|$ $\Rightarrow \pm\sqrt{3} = \left\|\frac{2-m}{1+2m}\right\|$ .... (if $\|x\| = y$ then $x = \pm y$) Case 1: $\sqrt{3} = \left\|\frac{2-m}{1+2m}\right\|$ $\Rightarrow \sqrt{3} + 2\sqrt{3}m = 2 - m$ $\Rightarrow 2\sqrt{3}m + m = 2 - \sqrt{3}$ $\Rightarrow m(2\sqrt{3}+1) = 2-\sqrt{3}$ $\Rightarrow m = \frac{2-\sqrt{3}}{2\sqrt{3}+1}$ Case 2: $-\sqrt{3} = \left\|\frac{2-m}{1+2m}\right\|$ $-\sqrt{3} - 2\sqrt{3}m = 2 - m$ $-2-\sqrt{3} = 2\sqrt{3}m - m$ $-2-\sqrt{3} = m(2\sqrt{3}-1)$ $\Rightarrow m = \frac{-2-\sqrt{3}}{2\sqrt{3}-1}$ Case 1: equation of required line (point slope form) $y - 3 = \frac{2-\sqrt{3}}{2\sqrt{3}+1}(x-2)$ $\Rightarrow (2\sqrt{3}+1)y - 6\sqrt{3} - 3 = (2-\sqrt{3})x - 4 + 2\sqrt{3}$ $\Rightarrow (2-\sqrt{3})x - (2\sqrt{3}+1)y + 8\sqrt{3} - 1 = 0$ ans. Case 2: equation of required line (point slope form) $y - 3 = \left(\frac{-2-\sqrt{3}}{2\sqrt{3}-1}\right)(x-2)$ $(2\sqrt{3}-1)y - 6\sqrt{3} + 3 = (-2-\sqrt{3})x + 4 + 2\sqrt{3}$ $\Rightarrow (2+\sqrt{3})x + (2\sqrt{3}-1)y = 8\sqrt{3}+1$ ans. 

Q.6) Show that the equation of the line passing through the origin & making an angle θ with the line $y = mx + c$ is $\frac{y}{x} = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$.

Sol.6) Equation of given line $y = mx + c$

Slope of given line $= m(m_1)$

Let slope of required line $M(m_2)$

Angle between them $= \theta$

We have, $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \tan\theta = \left|\frac{m - M}{1 + mM}\right|$

$\Rightarrow \tan\theta = \frac{m - M}{1 + mM}$

Case 1: $\tan\theta = \frac{m - M}{1 + mM}$

$\Rightarrow \tan\theta + mM\tan\theta = m - M$

$\Rightarrow M + mM\tan\theta = m - \tan\theta$

$\Rightarrow M(1 + m\tan\theta) = m - \tan\theta$

$\Rightarrow M = \frac{m - \tan\theta}{1 + m\tan\theta}$

Case 2: $-\tan\theta = \frac{m - M}{1 + mM}$

$-\tan\theta - mM\tan\theta = m - M$

$M - mM\tan\theta = m + \tan\theta$

$M(1 - m\tan\theta) = m + \tan\theta$

$\Rightarrow M = \frac{m \pm \tan\theta}{1 - m\tan\theta}$

$\Rightarrow \qquad M = \frac{m \pm \tan\theta}{1 \mp m\tan\theta}$

Now, equation of required line: (point slope form, point (0,0), slope $= M$)

$y - 0 = \frac{m \pm \tan\theta}{1 - m\tan\theta}(x - 0)$

$\Rightarrow \frac{y}{x} = \frac{m \pm \tan\theta}{1 - m\tan\theta}$ Ans.

Q.7) Point $R(h, k)$ divides a line segment between the axis in the ratio $1:2$. Find the equation of line.

Sol.7) Let $A(x, 0)$ and $B(0, y)$

R divides AB in ratio $1:2$

By section formula,

$h = \frac{2x + 0}{2 + 1}$ And $k = \frac{0 + y}{2 + 1}$

$\Rightarrow x = \frac{3h}{2}$ And $y = 3k$

$\therefore A(\frac{3h}{2}, 0)$ and $B(0, 3k)$

$\Rightarrow X$ intercept: $a = \frac{3h}{2}$ And Y intercept: $b = 3k$

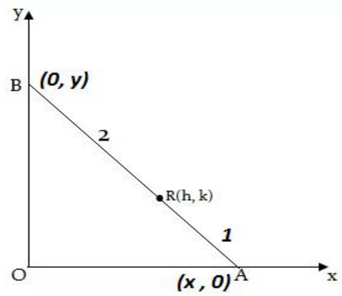
By intercept form,

$\frac{x}{a} + \frac{y}{b} = 1$

$\Rightarrow \frac{x}{\frac{3h}{2}} = \frac{y}{3k} = 1$

$\Rightarrow \frac{2x}{3h} + \frac{y}{3k} = 1$

$\Rightarrow 2x + y = 3k$ ans.



Q.8) Find the point on $Y - axis$ whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units.

Sol.8) Equation of given line $\frac{x}{3} + \frac{y}{4} = 1$

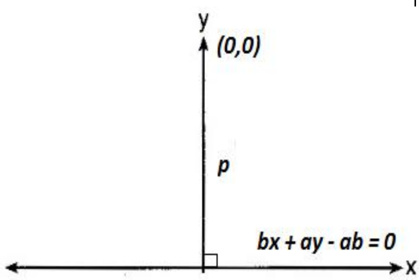
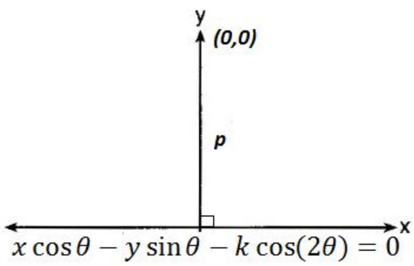
$\Rightarrow 4x + 3y = 12$

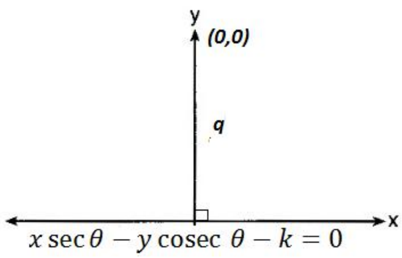
$\Rightarrow 4x + 3y - 12 = 0$

Let point on the $Y - axis$ is $(0, y)$ distance $= 4$ (given)

Distance between point and line is $= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Here, distance $= 4$, point $(0, y)$ & line: $4x + 3y - 12 = 0$

	$\Rightarrow 4 = \frac{\lvert 0+3y-12\rvert}{\sqrt{16+9}}$ $\Rightarrow 20 = \lvert 3y-12\rvert$ $\Rightarrow \pm 20 = 3y - 12$ $\Rightarrow 20 = 3y - 12$ and $-20 = 3y - 12$ $\Rightarrow 3y = 32$ and $3y = -8$ $\Rightarrow y = \frac{32}{3}$ And $y = \frac{-8}{3}$ $\therefore$ the required points are $(0, \frac{32}{3})$ and $(0, \frac{-8}{3})$ ans.
Q.9)	If p is the length of perpendicular from the origin to the line whose intercepts on the axis are a & b. Show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.
Sol.9)	Let equation of line is $\frac{x}{a} + \frac{y}{b} = 1$ $\Rightarrow bx + ay - ab = 0$ Point $(0,0)$; distance= p and line: $bx + ay - ab = 0$ By distance formula, $p = \frac{\lvert 0+0-ab\rvert}{\sqrt{b^2+a^2}}$ $\Rightarrow p = \frac{ab}{\sqrt{b^2+a^2}}$  Squaring $p^2 = \frac{a^2b^2}{b^2+a^2}$ $\Rightarrow \frac{1}{p^2} = \frac{b^2+a^2}{a^2b^2}$ $\Rightarrow \frac{1}{p^2} = \frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2}$ $\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (proved)
Q.10)	if p and q are the length of perpendicular from the origin to the lines $x\sec\theta - y\sin\theta = k\cos(2\theta)$ and $x\sec\theta + y\operatorname{cosec}\theta = k$ respectively. Prove that $p^2 + 4q^2 = k^2$.
Sol.10)	By distance formula, $p = \frac{\lvert 0-0-k\cos(2\theta)\rvert}{\sqrt{\cos^2\theta+\sin^2\theta}}$ $\Rightarrow p = \frac{k\cos(2\theta)}{1}$ $\Rightarrow p = k\cos(2\theta)$ Now, $q = \frac{\lvert 0+0-k\rvert}{\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta}}$ $\Rightarrow q = \frac{k}{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}$ $\Rightarrow q = \frac{k}{\sqrt{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta.\cos^2\theta}}}$ $\Rightarrow q = \frac{k}{\frac{1}{\sin\theta.\cos\theta}}$ 



	$\Rightarrow q = \frac{k}{\frac{1}{\sin\theta.\cos\theta}}$ $\Rightarrow q = k\sin\theta\cos\theta$ Taking L.H.S. $p^2 + 4q^2$ $= k^2\cos^2(2\theta) + 4k^2\sin^2\theta\,.\cos^2\theta$ $= k^2[\cos^2(2\theta) + 4\sin^2\theta\,.\cos^2\theta]$ $= k^2[\cos^2(2\theta) + (2\sin\theta\cos\theta)^2]$ $= k^2[\cos^2(2\theta) + \sin^2(2\theta)]$ ……….. $\{\therefore 2\sin\theta\cos\theta = \sin(2\theta)\}$ $= k^2(1)$ ……….. $\{\sin^2\theta + \cos^2\theta = 1\}$ $= k^2$ (proved)