CHAPTER - 1

SETS

KEY POINTS

- A set is a well-defined collection of objects.
- There are two methods of representing a set :–
  - (a) Roster or Tabular form.
  - (b) Set-builder form or Rule method.
- Types of sets :–
  - (i) Empty set or Null set or void set
  - (ii) Finite set
  - (iii) Infinite set
  - (iv) Singleton set
- Subset :– A set A is said to be a subset of set B if $a \in A \Rightarrow a \in B$, $\forall\ a \in A$
- Equal sets :– Two sets A and B are equal if they have exactly the same elements i.e $A = B$ if $A \subset B$ and $B \subset A$
- Power set : The collection of all subsets of a set A is called power set of A, denoted by P(A) i.e. $P(A) = \{ B : B \subset A \}$
- If A is a set with $n(A) = m$ then $n[P(A)] = 2^m$.

Types of Intervals

Open Interval $(a, b) = \{ x \in R : a < x < b \}$

Closed Interval $[a, b] = \{ x \in R : a \leq x \leq b \}$

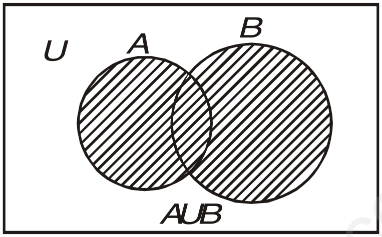
Semi open or Semi closed Interval,

$(a,b] = \{ x \in R : a < x \leq b\}$

$[a,b) = \{ x \in R : a \leq x < b\}$

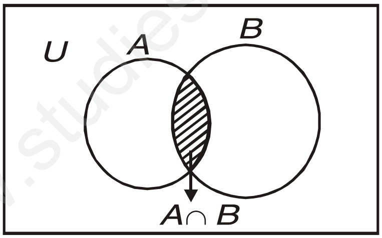
- Union of two sets A and B is,

  $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

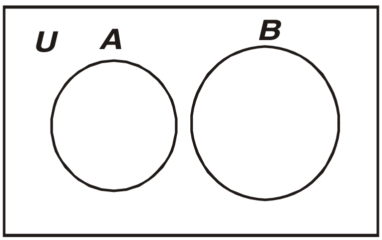


- Intersection of two sets A and B is,

  $A \cap B = \{ x : x \in A \text{ and } x \in B\}$

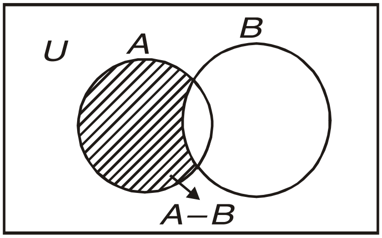


- Disjoint sets : Two sets A and B are said to be disjoint if $A \cap B = \phi$



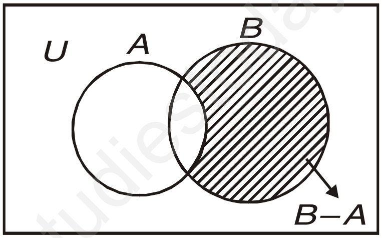
- Difference of sets A and B is,

  $A - B = \{ x : x \in A \text{ and } x \notin B\}$



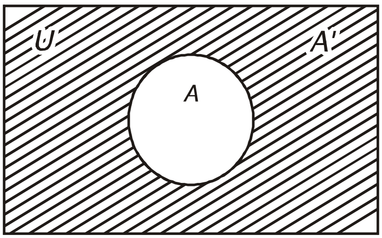
- Difference of sets B and A is,

  $B - A = \{ x : x \in B \text{ and } x \notin A \}$



- Complement of a set A, denoted by A' or A^c is

  $A' = A^c = U - A = \{ x : x \in U \text{ and } x \notin A\}$



- Properties of complement sets :

  1. Complement laws

     (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$ (iii) $(A')' = A$

2. De Morgan's Laws

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Note : This law can be extended to any number of sets.

3. $\phi' = \bigcup$ and $\bigcup' = \phi$

- $A - B = A \cap B'$
- Commutative Laws :–

  (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$
- Associative Laws :–

  (i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $(A \cap B) \cap C = A \cap (B \cap C)$
- Distributive Laws :–

  (i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  (ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Which of the following are sets? Justify your answer.

1. The collection of all the months of a year beginning with letter M
2. The collection of difficult topics in Mathematics.

   Let $A = \{1,3,5,7,9\}$. Insert the appropriate symbol $\in$ or $\notin$ in blank spaces :– (Question- 3,4)
3. 2 —— A
4. 5 —— A
5. Write the set $A = \{x : x$ is an integer, $-1 \le x < 4\}$ in roster form
6. List all the elements of the set,

$$A = \left\{x : x \in Z, -\frac{1}{2} < x < \frac{11}{2}\right\}$$

7. Write the set B = {3,9,27,81} in set-builder form.

Which of the following are empty sets? Justify. (Question- 8,9)

8. $A = \{ x : x \in N \text{ and } 3 < x < 4\}$

9. $B = \{ x : x \in N \text{ and } x^2 = x\}$

Which of the following sets are finite or Infinite? Justify. (Question-10,11)

10. The set of all the points on the circumference of a circle.

11. $B = \{ x : x \in N$ and x is an even prime number$\}$

12. Are sets $A = \{ -2,2\}$, $B = \{ x : x \in Z, x^2 - 4 = 0\}$ equal? Why?

13. Write $(-5,9]$ in set-builder form

14. Write $\{ x : -3 \leq x < 7\}$ as interval.

15. If $A = \{ 1,3,5\}$, how many elements has P(A)?

16. Write all the possible subsets of $A = \{5,6\}$.

If $A = \{2,3,4,5\}$, $B = \{ 3,5,6,7\}$ find (Question- 17,18)

17. $A \cup B$

18. $A \cap B$

19. If $A = \{1,2,3,6\}$, $B = \{1, 2, 4, 8\}$ find $B - A$

20. If $A = \{p, q\}$, $B = \{p, q, r\}$, is B a superset of A? Why?

21. Are sets $A = \{1,2,3,4\}$, $B = \{ x : x \in N \text{ and } 5 \leq x \leq 7\}$ disjoint? Why?

22. If X and Y are two sets such that n(X) = 19, n(Y) = 37 and $n(X \cap Y) = 12$, find $n(X \cup Y)$.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

23. If $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,3,5,7,9\}$, $B = \{ 1,2,4,6\}$, verify

(i) $(A \cup B)' = A' \cap B'$

(ii) $B - A = B \cap A' = B - (A \cap B)$

24. Let A, B be any two sets. Using properties of sets prove that,

    (i) $(A - B) \cup B = A \cup B$

    (ii) $(A \cup B) - A = B - A$

    [**Hint :** $A - B = A \cap B'$ and use distributive law.]

25. In a group of 800 people, 500 can speak Hindi and 320 can speak English. Find

    (i) How many can speak both Hindi and English?

    (ii) How many can speak Hindi only?

26. A survey shows that 84% of the Indians like grapes, whereas 45% like pineapple. What percentage of Indians like both grapes and pineapple?

27. In a survey of 450 people, it was found that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

28. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?

29. If $A = [-3, 5)$, $B = (0, 6]$ then find (i) $A - B$, (ii) $A \cup B$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

30. In a survey it is found that 21 people like product A, 26 people like product B and 29 like product C. If 14 people like product A and B, 15 people like product B and C, 12 people like product C and A, and 8 people like all the three products. Find

    (i) How many people are surveyed in all?

    (ii) How many like product C only?

31. A college awarded 38 medals in football, 15 in basket ball and 20 in cricket. If these medals went to a total of 50 men and only five men got medals in all the three sports, how many received medals in exactly two of the three sports?

ANSWERS

1. Set
2. Not a set
3. $\notin$
4. $\in$
5. $A = \{-1, 0, 1, 2, 3\}$
6. $A = \{0,1,2,3,4,5\}$
7. $B = \{x : x = 3^n, n \in N \text{ and } 1 \leq n \leq 4\}$
8. Empty set
9. Non-empty set
10. Infinite set
11. Finite set
12. Yes
13. $\{x : x \in R, -5 < x \leq 9\}$
14. $[-3,7)$
15. $2^3 = 8$
16. ϕ, $\{5\}$, $\{6\}$, $\{5,6\}$
17. $A \cup B = \{2,3,4,5,6,7\}$
18. $A \cap B = \{3, 5\}$
19. $B - A = \{4,8\}$
20. Yes, because A is a subset of B
21. Yes, because $A \cap B = \phi$
22. $n(X \cup Y) = 44$
25. (i) 20 people can speak both Hindi and English

    (ii) 480 people can speak Hindi only
26. 29% of the Indians like both grapes and pineapple.
27. **Hint :** $\cup$ – set of people surveyed

    A – set of people who play cricket

    B – set of people who play tennis

    Number of people who play neither cricket nor tennis

    $$= n[(A \cup B)'] = n(U) - n(A \cup B)$$

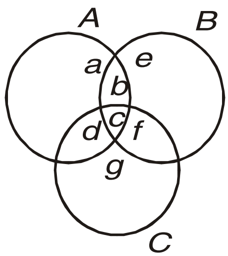
    $$= 450 - 200$$

    $$= 250$$
28. There are 280 students in the group.

29. (i) [–3, 0]; (ii) [–3, 6]

30. **Hint :** Let A, B, C denote respectively the set of people who like product A, B, C.

a, b, c, d, e, f, g – Number of elements in bounded region

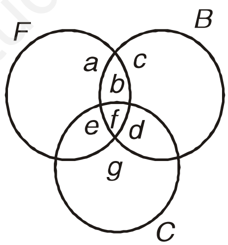


(i) Total number of Surveyed people = a + b + c + d + e + f + g = 43

(ii) Number of people who like product C only = g = 10

31. 13 people got medals in exactly two of the three sports.

Hint :



f = 5

a + b + f + e = 38

b + c + d + f = 15

e + d + f + g = 20

a + b + c + d + e + f + g = 50

we have to find *b* + *d* + *e*