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| Q.11)   | <p>Show that, <math>(A \cup B \cup C) \cap (A \cap B^1 \cap C^1) \cap C^1 = B \cap C^1</math></p>  |  |
| Sol.11) | <p>L.H.S. <math>(A \cup B \cup C) \cap (A \cap B^1 \cap C^1) \cap C^1</math><br/> <math>= (A \cup B \cup C) \cap (A \cap (B^1 \cap C^1)) \cap C^1</math> ..... {De – morgan's law}<br/> <math>= (A \cup B \cup C) \cap (A^1 \cup (B \cap C)) \cap C^1</math> ..... {De – morgan's law}<br/> <math>\Rightarrow ((B \cup C) \cup A) \cap ((B \cup C) \cup A^1) \cap C^1</math><br/> <math>= (B \cup C) \cup (A \cap A^1) \cap C</math> ..... {Distributive property}<br/> <math>= ((B \cup C) \cup \phi) \cap C^1</math> ..... {<math>A \cap A^1 = \phi</math>}<br/> <math>= (B \cup C) \cap C^1</math> ..... {<math>A \cup \phi = A</math>}<br/> <math>= (B \cap C^1) \cup (C \cap C^1)</math> ..... {Distributive law}<br/> <math>= (B \cap C^1) \cup \phi</math><br/> <math>= B \cap C^1</math> R.H.S. ans.</p> |  |
| Q.12)   | <p>If A and B are two sets containing 6 elements respectively, what can be the minimum number of elements in <math>A \cup B</math>. Find also maximum number of elements in <math>A \cup B</math> and <math>A \cap B</math>.</p>   |  |
| Sol.12) | <p><math>n(A) = 3 ; n(B) = 6</math><br/>         Minimum no. of elements in <math>A \cup B = 6</math><br/>         Maximum no. of elements in <math>A \cup B = 9</math><br/>         Maximum no. of elements in <math>A \cap B = 3</math></p>  |  |
| Q.13)   | <p>From 50 students taking examination in Maths, physics and chemistry, each of the students has passed in at least one of the subject, 37 passed maths, 24 passed physics and 43 passed chemistry. At most 19 passed maths &amp; physics, at most 29 passed maths &amp; chemistry and at most 20 passed physics &amp; chemistry.<br/>         What is the largest possible number that could have passed all the three subjects?</p>  |  |
| Sol.13) | <p>Given, <math>n(M \cup P \cup C) = 50</math><br/> <math>n(M) = 37, n(C) = 43; n(P) = 24</math><br/>         Since, at most if given<br/> <math>\therefore n(M \cap P) \leq 19</math><br/> <math>n(M \cap C) \leq 29</math><br/> <math>n(P \cap C) \leq 20</math><br/> <math>n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)</math><br/> <math>\Rightarrow 37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C) \leq 50</math><br/> <math>\Rightarrow n(M \cap P \cap C) \leq 50 - 36</math><br/> <math>\Rightarrow n(M \cap P \cap C) \leq 14</math><br/> <math>\therefore</math> largest possible number that could have passed all the three exams is 14 ans.</p>   |  |
| Q.14)   | <p>Suppose <math>A_1, A_2, \dots, A_{30}</math> are thirty sets each having 5 elements and <math>B_1, B_2, \dots, B_n</math> are <math>n</math> sets with each 3 elements. Let <math>\bigcup_{i=1}^{30} A_i = \bigcup_{i=1}^n B_i = S</math> and each element of <math>S</math> belongs to exactly 10 of the <math>A_i</math>'s and exactly 9 of the <math>B_i</math>'s then find the value of <math>n</math>.</p>   |  |
| Sol.14) | <p><math>n(A_1), n(A_2), n(A_3) \dots (A_{30}) = 5</math><br/> <math>\bigcup_{i=1}^{30} A_i = S \therefore</math> we get <math>n(S) = 150</math><br/>         But each element of <math>S</math> belongs to exactly 10 of <math>A_i</math>'s<br/> <math>\therefore \frac{150}{10} = 15</math> are the number of distinct elements in <math>S</math><br/>         Also each element of <math>S</math> belong to exactly 9 of the <math>B_i</math>'s and each <math>B_i</math> contains 3 elements<br/> <math>\therefore</math> and <math>\bigcup_{i=1}^n B_i = S</math><br/> <math>\Rightarrow \frac{3n}{9} = 15</math><br/> <math>\Rightarrow n = 45</math><br/> <math>\therefore n(B) = 45</math> ans.</p>  |  |



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| Q.15)   | Using properties show that, $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$  |
| Sol.15) | $(A \cup B) - (A \cap B)$ $= (A \cup B) \cap (A \cap B)^1 \dots\dots\dots \{A - B = A \cap B^1\}$ $= (A \cup B) \cap (A^1 \cup B^1) \dots\dots\dots \{De - morgan's law\}$ $= [(A \cup B) \cap A^1] \cup [(A \cup B) \cap B^1] \dots\dots\dots \{Distributive law\}$ $= [(A \cap A^1) \cup (B \cap A^1)] \cup [(A \cap B^1) \cup (B \cap B^1)]$ $= [\emptyset \cup (B - A)] \cup [(A - B) \cup \emptyset]$ $= (B - A) \cup (A - B) \dots\dots\dots \{\emptyset \cup A = A\}$ $= (A - B) \cup (B - A) \dots\dots\dots \{A \cup B = B \cup A\} \text{ R.H.S. (proved)}$   |
| Q.16)   | For any three sets show that $A \times (B \cup C) = (A \times B) \cup (A \times C)$   |
| Sol.16) | <p>Let <math>(a, b) \in A \times (B \cup C)</math></p> $\Rightarrow a \in A \text{ and } b \in (B \cup C)$ $\Rightarrow a \in A \text{ and } b \in B \text{ or } b \in C$ $\Rightarrow (a \in A \text{ and } b \in B) \text{ or } (a \in A \text{ and } b \in C)$ $\Rightarrow (a, b) \in A \times B \text{ or } (a, b) \in A \times C$ $\Rightarrow (a, b) \in (A \times B) \cup (A \times C)$ $\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C) \dots\dots\dots (1)$ <p>Now, let <math>(x, y) \in (A \times B) \cup (A \times C)</math></p> $\Rightarrow (x, y) \in A \times B \text{ or } (x, y) \in A \times C$ $\Rightarrow (x \in A \text{ and } y \in B) \text{ or } (x \in A, y \in C)$ $\Rightarrow x \in A \text{ and } (y \in B \text{ or } y \in C)$ $\Rightarrow x \in A \text{ and } y \in (B \cup C)$ $\Rightarrow (x, y) \in A \times (B \cup C)$ $\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C) \dots\dots\dots (2)$ <p>From (1) and (2), <math>A \times (B \cup C) = (A \times B) \cup (A \times C)</math></p> |