

CHAPTER - 9

SEQUENCES AND SERIES

KEY POINTS

- A sequence is a function whose domain is the set N of natural numbers.
- A sequence whose range is a subset of R is called a real sequence.
- General A.P. is,

$$a, a + d, a + 2d, \dots$$

- $a_n = a + (n - 1)d = n^{\text{th}} \text{ term}$
- $S_n = \text{Sum of first } n \text{ terms of A.P.}$

$$= \frac{n}{2} [a + l] \text{ where } l = \text{last term.}$$

$$= \frac{n}{2} [2a + (n - 1)d]$$

- If a, b, c are in A.P. then $a \pm k, b \pm k, c \pm k$ are in A.P., ak, bk, ck are also in A.P., $k \neq 0$
- Three numbers in A.P.

$$a - d, a, a + d$$

- Arithmetic mean between a and b is $\frac{a + b}{2}$.
- If $A_1, A_2, A_3, \dots, A_n$ are inserted between a and b , such that the resulting sequence is A.P. then,

$$A_n = a + n \left(\frac{b - a}{n + 1} \right)$$

- $S_k - S_{k-1} = a_k$
- $a_m = n, a_n = m \Rightarrow a_r = m + n - r$
- $S_m = S_n \Rightarrow S_{m+n} = 0$
- $S_p = q$ and $S_q = p \Rightarrow S_{p+q} = -p - q$
- In an A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last term
- G.P. (Geometrical Progression)

a, ar, ar^2, \dots (General G.P.)

$$a_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

- Geometric mean between a and b is \sqrt{ab}
- Reciprocals of terms in GP always form a G.P.
- If $G_1, G_2, G_3, \dots, G_n$ are n numbers inserted between a and b so that the resulting sequence is G.P., then

$$G_k = a \left(\frac{b}{a} \right)^{\frac{k}{n+1}}, \quad 1 \leq k \leq n$$

- In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last term.
- If each term of a G.P. be raised to some power then the resulting terms are also in G.P.

- Sum of infinite G.P. is possible if $|r| < 1$ and sum is given by $\frac{a}{1-r}$

- $\sum_{r=1}^n r = \frac{n(n+1)}{2}$

- $\sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{r=1}^n r^3 = \left[\frac{n(n+1)}{2} \right]^2$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If n^{th} term of an A.P. is $6n - 7$ then write its 50^{th} term.
2. If $S_n = 3n^2 + 2n$, then write a_2
3. Which term of the sequence,
3, 10, 17, is 136?
4. If in an A.P. 7^{th} term is 9 and 9^{th} term is 7, then find 16^{th} term.
5. If sum of first n terms of an A.P is $2n^2 + 7n$, write its n^{th} term.
6. Which term of the G.P.,
2, 1, $\frac{1}{2}$, $\frac{1}{4}$, is $\frac{1}{1024}$?
7. If in a G.P., $a_3 + a_5 = 90$ and if $r = 2$ find the first term of the G.P.
8. In G.P. 2, $2\sqrt{2}$, 4,, $128\sqrt{2}$, find the 4^{th} term from the end.
9. If the product of 3 consecutive terms of G.P. is 27, find the middle term
10. Find the sum of first 8 terms of the G.P. 10, 5, $\frac{5}{2}$,
11. Find the value of $5^{1/2} \times 5^{1/4} \times 5^{1/8}$ upto infinity.
12. Write the value of $0.\bar{3}$
13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.
14. Write the n^{th} term of the series, $\frac{3}{7.11^2} + \frac{5}{8.12^2} + \frac{7}{9.13^2} + \dots$

15. Find a_5 of the series whose n^{th} term is $2^n + 3$.
16. In an infinite G.P., every term is equal to the sum of all terms that follow it. Find r
17. In an A.P.,
8, 11, 14, find $S_n - S_{n-1}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

18. Write the first negative term of the sequence $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots$
19. Determine the number of terms in A.P. 3, 7, 11, 407. Also, find its 11^{th} term from the end.
20. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.
21. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.
22. Find the sum of the sequence,

$$-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \dots, \frac{10}{3}$$

23. If in an A.P. $\frac{a_7}{a_{10}} = \frac{5}{7}$ find $\frac{a_4}{a_7}$
24. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.
25. Solve : $1 + 6 + 11 + 16 + \dots + x = 148$
26. The ratio of the sum of n terms of two A.P.'s is $(7n - 1) : (3n + 11)$, find the ratio of their 10^{th} terms.
27. If the 1^{st} , 2^{nd} and last terms of an A.P are a , b and c respectively, then find the sum of all terms of the A.P.

28. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{c}$ are in A.P. then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. [Hint : Add 3 to each term]

29. If $A = 1 + r^a + r^{2a} + \dots$ up to infinity, then express r in terms of 'a' & 'A'.
30. Insert 5 numbers between 7 and 55, so that resulting series is A.P.
31. Find the sum of first n terms of the series, $0.7 + 0.77 + 0.777 + \dots$
32. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms.
33. Prove that, $0.03\bar{1} = \frac{7}{225}$

[Hint : $0.03\bar{1} = 0.03 + 0.001 + 0.0001 + \dots$. Now use infinite G.P.]

LONG ANSWER TYPE QUESTIONS (6 MARKS)

34. Prove that the sum of n numbers between a and b such that the resulting series becomes A.P. is $\frac{n(a+b)}{2}$.
35. A square is drawn by joining the mid points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of the first square is 15 cm, then find the sum of the areas of all the squares so formed.
36. If a, b, c are in G.P., then prove that

$$\frac{1}{a^2 - b^2} = \frac{1}{b^2 - c^2} - \frac{1}{b^2}$$

[Hint : Put $b = ar, c = ar^2$]

37. Find two positive numbers whose difference is 12 and whose arithmetic mean exceeds the geometric mean by 2.
38. If a is A.M. of b and c and c, G_1, G_2, b are in G.P. then prove that
- $$G_1^3 + G_2^3 = 2abc$$
39. Find the sum of the series,

$$1.3.4 + 5.7.8 + 9.11.12 + \dots \text{ upto } n \text{ terms.}$$

40. Evaluate $\sum_{r=1}^{10} (2r-1)^2$

ANSWERS

1. 293

2. 11

3. 20^{th}

4. 0

5. $4n + 5$

6. 12^{th}

7. $\frac{9}{2}$

8. 64

9. 3

10. $20 \left(1 - \frac{1}{2^8} \right)$

11. 5

12. $\frac{1}{3}$

13. $\frac{2}{3}$

14. $\frac{2n + 1}{(n + 6)(n + 10)^2}$

15. 35

16. $r = \frac{1}{2}$

17. $3n + 5$

18. $-\frac{1}{4}$

19. 102, 367

20. 33

21. 7999

22. $\frac{63}{2}$

23. $\frac{3}{5}$

24. 11

25. 36

26. $33 : 17$

27. $\frac{(b + c - 2a)(a + c)}{2(b - a)}$

29. $\left(\frac{A - 1}{A} \right)^{1/a}$

30. 15, 23, 31, 39, 47

31. $\frac{7}{81} (9n - 1 + 10^{-n})$

32. $\frac{15}{7} (2^n - 1)$

35. 450 cm²

37. 16, 4

39. $\frac{n(n+1)}{3} (48n^2 - 16n - 14)$

40. 1330

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