| | SEQUENCE AND SERIES<br>Class XI |
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| Q.11) | The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11. Find the number of terms. |
| Sol.11) | Given, $a_1 + a_2 + a_3 + a_4 = 56$ and $a_1 = 11$<br>$\Rightarrow a + (a + d) + (a + 2d) + (a + 3d) = 56$<br>$\Rightarrow 4a + 6d = 56$<br>$\Rightarrow 44 + 6d = 56$<br>$\Rightarrow 6d = 12$<br>$\Rightarrow d = 12$<br>Now, sum of least four terms is 42<br>$\Rightarrow a + (n-1)d + a + (n-2)d + a + (n-3)d + a + (n-4)d = 112$<br>$\Rightarrow 4a + d(n - 1 + n - 2 + n - 3 + n - 4) = 112$<br>$\Rightarrow 44 + 2(4n - 10) = 112$<br>$\Rightarrow 44 + 8n - 20 = 112$<br>$\Rightarrow 8n = 112 - 24$<br>$\Rightarrow 8n = 88$<br>$n = 11$ ans. |
| Q.12) | Find the sum of integers from 1 to 100 which are divisible by 2 or 5. |
| Sol.12) | The no.s which are divisible by 2 or 5 from 1 to 100 are The no.s which are divisible by 2 or 5 from 1 to 100 are 2, 4, 5, 6, 8, 10, 12, ……… 100<br>There are two sequences in above equation<br>1st 2, 4, 6, 8, 10, 12, ………… 100<br>And 2nd 5, 15, 25 ……….. 95<br>1st is an A.P. with $a = 2, d = 2, n = 50$<br>∴ its sum $= \frac{50}{2}[2(2) + (50 - 1)2]$<br>$= 25(4 + 98)$<br>$= 25(102)$<br>$= 2550$<br>2nd is also an AP with $a = 5, d = 10, n = 10$<br>∴ 1st sum $= \frac{10}{2}[2(5) + (10 - 1)10]$<br>$= 5(10 + 90)$<br>$= 500$<br>∴ sum $= 2550 + 500 = 3050$ ans. |
| | **G.P.** |
| Q.13) | The sum of first three terms of a G.P. is $\frac{13}{12}$ & their product is $-1$. Find the common ratio & their terms. |
| Sol.13) | Let the terms are $\frac{a}{r}, a, ar$<br>Product $= -1$<br>$\Rightarrow \frac{a}{r}.a.ar = -1$<br>$\Rightarrow a^3 = -1$<br>$\Rightarrow a = -1$<br>Sum $= \frac{13}{12}$ |

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| | $\Rightarrow \frac{a}{r} + a + ar = \frac{13}{12}$<br>$\Rightarrow a\left(\frac{1}{r} + 1 + r\right) = \frac{13}{12}$<br>$\Rightarrow (-1)\left[\frac{1+r+r^2}{r}\right] = \frac{13}{12}$<br>$\Rightarrow \frac{1+r+r^2}{r} = \frac{-13}{12}$<br>$\Rightarrow 12 + 12r + 12r^2 = -13r$<br>$\Rightarrow 12r^2 + 25r + 12 = 0$<br>$\Rightarrow 12r^2 + 16r + 9r + 12 = 0$<br>$\Rightarrow 4r[3r + 4] + 3(3r + 4) = 0$<br>$\Rightarrow (3r + 4)(4r + 3) = 0$<br>$\Rightarrow r = \frac{-4}{3}$ and $r = \frac{-3}{4}$<br>For $a = -1$ and $r = \frac{-4}{3}$ the terms are $\frac{3}{4}, -1, \frac{4}{3}$<br>For $a = -1$ and $r = \frac{-3}{4}$<br>The term are $\frac{4}{3}, -1, \frac{3}{4}$<br>$\therefore$ required term are $\frac{3}{4}, -1, \frac{4}{3}$ or $\frac{4}{3}, -1, \frac{3}{4}$ | |
| Q.14) | The sum of three numbers in G.P. is 56. If we subtract 1, 7, 21 from these numbers, we obtain an A.P. find the numbers. | |
| Sol.14) | Let the no.s in G.P. are $a + ar + ar^2$<br>Given, $a + ar + ar^2 = 56$<br>$\Rightarrow a(1 + r + r^2) = 56$ ..................... (i)<br>We have, $a - 1, ar - 7, ar^2 - 21$ are in A.P.<br>$\Rightarrow 2(ar - 7) = (a - 1) + (ar^2 - 21)$<br>$\Rightarrow 2ar - 14 = a + ar^2 - 22$<br>$\Rightarrow ar^2 - 2ar + a = 8$<br>$\Rightarrow a(r^2 - 2r + 1) = 8$ .................. (ii)<br>Dividing (i) by (ii)<br>$\Rightarrow \frac{a(1+r+r^2)}{a(r^2-2r+1)} = \frac{56}{8} = 7$<br>$\Rightarrow 1 + r + r^2 = 7r^2 - 14r + 7$<br>$\Rightarrow 6r^2 - 15r + 6 = 0$<br>$\Rightarrow 2r^2 - 5r + 2 = 0$<br>$\Rightarrow 2r^2 - 4r - r + 2 = 0$<br>$\Rightarrow 2r(r - 2) - 1(r - 2) = 0$<br>$\Rightarrow (2r - 1)(r - 2) = 0$<br>$\Rightarrow r = \frac{1}{2}$ & $r = 2$<br>Put $r = \frac{1}{2}$ in eq. (i)<br>$\therefore a\left(1 + \frac{1}{2} + \frac{1}{4}\right) = 56$<br>$\Rightarrow a\left(\frac{7}{4}\right) = 56$<br>$\Rightarrow a = \frac{4 \times 56}{1}$<br>$\Rightarrow a = 32$<br>For $r = 2$<br>$\Rightarrow a(1 + 2 + 4) = 56$<br>$\Rightarrow a(7) = 56$ | |

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| | $\Rightarrow a = 8$<br>$\therefore$ for $a = 8$ & $r = 2$<br>No.s are 8,16,32<br>For $a = 32$ & $r = \frac{1}{2}$<br>No.s are 32,16,8<br>$\therefore$ required no.s are 8,16,32 or 32,16,8 ans. |
| Q.15) | A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying odd places. Find the common ratio. |
| Sol.15) | Let the G.P. contains $(2n)$ no. of terms<br>We have $a_1 + a_2 + a_3 + \ldots\ldots\ldots\ldots a_{2n} = 5(a_1 + a_3 + a_5 + \cdots \ldots..n\ terms)$<br>$a + ar + ar^2 + \cdots \ldots \ldots \ldots (2n)terms = 5(a + ar^2 + ar^4 + \cdots \ldots..n\ terms)$<br>$\leftarrow$ G.P. 1st term $= a$     $\leftarrow$ G.P. 1st term $= a$<br>Ratio$= r$     Ratio$= r^2$<br>No. of term$= 2n$     No. of term$= n$<br>$\Rightarrow a\left(\frac{r^{2n}-1}{r-1}\right) = 5a\left(\frac{(r^2)^{n-1}}{r^2-1}\right)$<br>$\Rightarrow \frac{r^{2n}-1}{r-1} = 5\left[\frac{r^{2n}-1}{(r+1)(r-1)}\right]$<br>$\Rightarrow 1 = \frac{5}{r+1}$<br>$\Rightarrow r + 1 = 5$<br>$\Rightarrow r = 4$ ans. |
| Q.16) | If $\frac{a+bx}{ab-x} = \frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$, then show that $a, b, c$ & $d$ are in G.P. |
| Sol.16) | Consider,<br>$\Rightarrow \frac{a+bx}{ab-x} = \frac{b+cx}{b-cx}$<br>$\Rightarrow ab + b^2x - acx - bcx^2 = ab + acx - b^2x - bc$<br>$\Rightarrow 2b^2x = 2acx$<br>$\Rightarrow b^2 = ac$<br>$\therefore a, b, c$ are in G.P. ............. (i)<br>Now consider, $\frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$<br>$\Rightarrow bc + c^2x - bdx - cd = bc + bdx - c^2x - cdx^2$<br>$\Rightarrow 2c^2x = 2bdx$<br>$\Rightarrow c = bd$<br>$\therefore b, c, d$ are in G.P. ............. (ii)<br>From (i) & (ii)<br>$a, b, c, d$ are in G.P. |
| Q.17) | If $a, b, c, d$ are in G.P. then show that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P. |
| Sol.17) | Given, $a, b, c, d$ are in G.P.<br>Let $a = a, b = ar, c = ar^2, d = ar^3$<br>To prove, $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.<br>i.e., $(b^n + c^n)^2 = (a^n + b^n).(c^n + d^n)$<br>Taking L.H.S. $(b^n + c^n)^2$<br>= $[(ar)^n + (ar^2)^n]^2$<br>= $[a^n r^n + a^n r^{2n}]^2$<br>= $a^{2n}.r^{2n}[1 + r^n]^2$<br>Taking RHS $(a^n + b^n).(c^n + d^n)$<br>= $(a^n + (ar)^n).((ar^2)^n + (ar^3)^n)$ |

$= (a^n + a^n r^n).(a^n r^{2n} + a^n r^{3n})$
$= a^n(1 + r^n).a^n r^{2n}(1 + r^n)$
$= a^{2n} r^{2n}(1 + r^n)^2$
$\therefore$ LHS = RHS
$\therefore (a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

**Q.18)** If $a$ and $b$ are the roots of $x^2 - 3x + p = 0$ and $c, d$ are the roots of $x^2 - 12x + q = 0$, where $a, b, c, d$ form a G.P. show that $(q + p):(q - p) = 17:15$.

**Sol.18)** Given, $a$ & $b$ are roots of $x^2 - 3x + p = 0$

$\Rightarrow a + b = 3$ ............... $\left\{\begin{matrix} \because\ \alpha + \beta = \frac{-b}{a} \\ \alpha\beta = \frac{c}{d} \end{matrix}\right\}$

And $ab = p$
Also $c$ and $d$ are the roots of $x^2 - 12x + q = 0$
$\Rightarrow c + d = 12$
And $cd = q$
$a, b, c$ and $d$ are in G.P.
$\Rightarrow a = a, b = ar, c = ar^2, d = ar^3$
to prove $\frac{q+p}{q-p} = \frac{17}{15}$
taking LHS $\frac{q+p}{q-p}$
$= \frac{cd+ab}{cd-ab}$ ............... $\left\{\begin{matrix} \because\ cd = q \\ ab = p \end{matrix}\right\}$
$= \frac{(a^2)(a^3)+(a)(ar)}{(a^2)(a^3)-(a)(ar)}$
$= \frac{a^2r^5+a^2r}{a^2r^5-a^2r}$
$= \frac{a^2r+(r^4+1)}{a^2r-(r^4-1)}$
$\therefore \frac{q+p}{q-p} = \frac{r^4+1}{r^4-1}$ .............. (i)
Now we have,

$\Rightarrow a + b = 3$
$\Rightarrow a + ar = 3$
$\Rightarrow ar^2(1 + r) = 3$............ (iii)

$\Rightarrow c + d = 12$
$\Rightarrow ar^2 + ar^3 = 12$
$\Rightarrow ar^2(1 + r) = 12$............ (iv)

Dividing (iv) by (iii)
$\therefore \frac{ar^2(1+r)}{ar^2(1+r)} = \frac{12}{3}$
$\Rightarrow r^2 = 4$ put in eq.(i)
$\therefore \frac{q+p}{q-p} = \frac{(4)^2+1}{(4)^2-1}$
$= \frac{17}{15}$
$\therefore (q + p):(q - p) = 17:15$ ans.

**Q.19)** The ratio of the A.M. and G.M. of two possible numbers $a$ and $b$ is $m:n$. Show that $a:b = \left(m + \sqrt{m^2 - n^2}\right):\left(m - \sqrt{m^2 - n^2}\right)$.

**Sol.19)** Given, $\frac{A.M.}{G.M.} = \frac{m}{n}$
$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$
Apply componendo and dividendo $\left(\frac{N+D}{N-D}\right)$

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| | $\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$<br>$\Rightarrow \frac{(\sqrt{a})^2+(\sqrt{b})^2+2\sqrt{a}\sqrt{b}}{(\sqrt{a})^2+(\sqrt{b})^2-2\sqrt{a}\sqrt{b}} = \frac{m+n}{m-n}$<br>$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{m+n}{m-n}$<br>$\Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$<br>Apply componendo and dividendo<br>$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})+(\sqrt{a}-\sqrt{b})}{(\sqrt{a}+\sqrt{b})-(\sqrt{a}-\sqrt{b})} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$<br>$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$<br>Squaring both sides<br>$\Rightarrow \frac{a}{b} = \frac{(m+n)+(m-n)+2\sqrt{m+n}\sqrt{m-n}}{(m+n)+(m-n)-2\sqrt{m+n}\sqrt{m-n}}$<br>$\Rightarrow \frac{a}{b} = \frac{2m+2\sqrt{m^2-n^2}}{2m-2\sqrt{m^2-n^2}}$<br>$\Rightarrow \frac{a}{b} = \frac{2(m+\sqrt{m^2-n^2})}{2(m-\sqrt{m^2-n^2})}$<br>$\therefore a:b = \left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$ ans. | |
| Q.20) | If $a, b, c$ are in A.P., $b, c, d$ are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P. prove that $a, c, e$ are in G.P. | |
| Sol.20 | Given, $a, b, c$ are in A.P.<br>$\Rightarrow 2b = ac$ ..................... (i)<br>Given, $b, c, d$ are in G.P.<br>$\Rightarrow c^2 = bd$ ........... (ii)<br>Given, $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P.<br>$\Rightarrow \frac{2}{d} = \frac{1}{c} + \frac{1}{e}$<br>$\Rightarrow \frac{2}{d} = \frac{e+c}{ce}$<br>$\Rightarrow \frac{d}{2} = \frac{ec}{e+c}$<br>$\Rightarrow d = \frac{2ec}{e+c}$ ............. (iii)<br>To prove, $a, c, e$ are in G.P.<br>i.e., $c^2 = ae$<br>we have, $c^2 = bd$...........from (ii)<br>put value of $b$ $and$ $d$ from eq. (i) and (ii)<br>$\Rightarrow c^2 = \left(\frac{a+c}{2}\right)\left(\frac{2ec}{e+c}\right)$<br>$\Rightarrow c^2(e+c) = (a+c)(ec)$<br>$\Rightarrow c^3 = ace$<br>$\Rightarrow c^2 = ae$<br>$\therefore a, c, e$ are in G.P. (proved) | |
| Q.21) | Find the sum to $n$ terms of given series $5 + 55 + 555 +$............. | |
| Sol.21) | Let $S_n = 5 + 55 + 555 +$......$n$ terms<br>$S_n = 5[1 + 1 + 111 + \cdots \ldots n$ terms$]$<br>Multiply & divide by 9<br>$= \frac{5}{9}[9 + 99 + 999 + \cdots \ldots n\ terms]$ | |

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| | $= \frac{5}{9}(10 + 10^2 + 10^3 + \cdots \ldots n\, terms) - (1 + 1 + 1 + \cdots \ldots n\, terms)$<br>$\leftarrow G.P\ a = 1; r = 10 \rightarrow$<br>$= \frac{5}{9}\left[10\left(\frac{10^n-1}{10-1}\right) - n\right]$<br>$= \frac{5}{9}\left[\frac{10^{n+1}-10}{9} - n\right]$<br>$\therefore S_n = \frac{5}{81}[10^{n+1} - 10 - 9n]$ ans. | |
| Q.22) | Find the sum of the series to $n$ terms $0.6 + 0.66 + 0.666 + \ldots\ldots\ldots\ldots n$ terms. | |
| Sol.22) | Let $S_n = 0.6 + 0.66 + 0.666 + \ldots\ldots\ldots\ldots n$ terms<br>$S_n = 6[0.1 + 0.11 + 0.111 + \cdots \ldots \ldots \ldots . n \text{ terms}]$<br>Multiply & divide by 9<br>$= \frac{6}{9}[0.9 + 0.99 + 0.999 + \cdots \ldots n\, terms]$<br>$= \frac{2}{3}((1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \cdots \ldots n\, terms)$<br>$= \frac{2}{3}(1 + 1 + 1 + \cdots \ldots n\, terms) - (0.1 + 0.11 + 0.111 + \cdots \ldots n\, terms)$<br>$= \frac{2}{3}\left[n - \left(\frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + \cdots \ldots n\, terms\right)\right]$<br>$\leftarrow G.P: a = \frac{1}{10}; r = \frac{1}{10} \rightarrow$<br>$= \frac{2}{3}\left[n - \frac{1}{10}\left(\frac{1-\frac{1}{10^n}}{1-\frac{1}{10}}\right)\right]$<br>$= \frac{2}{3}\left[n - \frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{\frac{9}{10}}\right]$<br>$= \frac{2}{3}\left[\frac{9n-1+\frac{1}{10^n}}{9}\right]$<br>$\therefore S_n = \frac{2}{27}\left[\frac{9n-1+\frac{1}{10^n}}{9}\right]$ ans. | |