

Class: XI

Subject : Mathematics

Assignment No. 3

1. Prove by the principle of Mathematical induction for all natural numbers:

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

2. Prove by the principle of Mathematical induction for all natural numbers:

$$\sin \theta + \sin 2\theta + \sin 3\theta + \dots + \sin n\theta = \frac{\sin\left(\frac{n+1}{2}\theta\right) \sin \frac{n\theta}{2}}{\sin \frac{\theta}{2}}$$

3. Prove by the principle of Mathematical induction for all natural numbers:

$$(11^{n+2} + 12^{2n+1}) \text{ is divisible by } 133.$$

4. (i) If $P(2n-1, n) : P(2n+1, n-1) = 22 : 7$, find n
 (ii) If $P(2n, 3) : P(n, 3) = 11 : 1$, find n
5. How many numbers are there between 100 and 1000 which have exactly one of their digits as 7?
6. How many words beginning and ending with a consonant can be formed by using the letters of the word EQUATION?
7. In how many arrangements of the word 'GOLDEN' will the vowels never occur together?
8. If all the letters of the word 'GUAVA' be arranged as in a dictionary, what is the rank of word GUAVA?
9. There are 8 men and 6 ladies to dine at a round table. In how many ways can they seat themselves so that no two ladies are together?
10. Out of 6 teachers and 8 students a committee of 11 is to be formed. In how many ways can this be done, if the committee contains:
 (i) exactly 4 teachers (ii) at least 4 teachers.
11. A card is drawn from a pack of 52 cards. A gambler bets that it is a heart or a queen. What are the odds against his winning the bet?
12. The letters of the word 'SOCIETY' are placed at random in a row. What is the probability that the three vowels do not come together?
13. A and B are two events such that $P(A) = 0.3$, $P(B) = 0.48$ and $P(A \cap B) = 0.16$. Find (i) $P(A')$
 (ii) $P(B')$ (iii) $P(\text{neither } A \text{ nor } B)$

14. A die is thrown twice. What is the probability that at least one of the two throws comes up with number 3?
15. A box contains 80 apples and 120 oranges. One tenth of the each fruits are rotten. Mohan takes out a fruit from the box. What is the probability that the fruit chosen is rotten or an apple.
16. Using binomial theorem, expand the following:

(i) $\left(\frac{2x}{3} - \frac{3}{2x}\right)^6$ (ii) $(1 + 2x - 3x^2)^5$ (iii) $(\sqrt[3]{x} - \sqrt[3]{y})^6$ (iv) $(x^2 - 2/x)^7$

17. Find the coefficient of

(i) x^{18} in the expansion of $(x^2 + 3a/x)^{15}$.

(ii) x^6 in the expansion of $(3x^2 - 1/3x)^9$.

18. Find the middle terms in the expansion of

(i) $(3a - a^3/6)^9$ (ii) $(\frac{1}{x^3} - x^4)^{11}$ (iii) $\left(\sqrt{x} + \frac{1}{3x^2}\right)^{10}$ (iv) $\left(x - \frac{1}{x^2}\right)^{16}$

19. Find the term independent of x in the expansion of:

(i) $\left(x - \frac{1}{x^2}\right)^3$ (ii) $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ (iii) $\left(x^2 + \frac{1}{x}\right)^9$ (iv) $\left(x - \frac{2}{x}\right)^{10}$

20. Using binomial theorem, prove that $(3^n - 2n - 1)$ is divisible by 4, Where n is a natural number.

21. Show that the co-efficient of the middle term in the expansion of $(1+x)^{2n}$ is the sum of the co-efficient of the two middle terms in the expansion of $(1+x)^{2n-1}$.

22. If the co-efficient of $(r-1)^{\text{th}}$, r^{th} , and $(r+1)^{\text{th}}$ terms in the expansion of $(x+1)^n$ are in the ratio of 1: 3: 5, Find n and r .

23. Solve: $\frac{3n+1}{3} - \frac{4x+5}{6} \leq \frac{4x+1}{6} - \frac{2x+3}{2}$.

24. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 liters of the 8% solution, how many liters of the 2% solution will have to be added?

25. Find the region of the graph where all the inequalities $(x + 2y) \geq 0$, $(2x + y) \leq 4$, $x \geq 0$ and $y \leq 2$ hold good. Also find the ordered pairs of the vertices of the region.

26. Solve the system of linear inequations graphically:-

$x + 2y \leq 8$, $2x + y \geq 2$, $x - y \leq 1$, $x \geq 0$, $y \geq 0$

27. Calculate the mean deviation about mean deviation about mean and median for the given data:

x_i	6	8	10	12	14	16	18	20
fi	1	14	25	27	18	9	4	2

28. Calculate the mean deviation about mean deviation about mean and median for the given data:

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	5	10	20	5	10

29. Calculate standard deviation for the following set of numbers:

45, 30, 40, 35, 25, 60, 45, 20. Also calculate the standard deviation if each number is divided by 5.

30. Calculate the co-efficient of variation, mean and standard deviation for the given data:

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	1	2	3	4	5

31. The mean of 5 observations is 4.4 and variance is 8.24. If three observations are 1, 2 and 6. Find the other two observations.

32. Find the converse of each of the statements:

(i) P : "If x is a natural number then $3x$ is an odd number"

(ii) P : "If $x = 1$ or $x = 3$, then $x^2 - 4x + 3 = 0$ "

33. Write the contra positive of the following statements:

(i) "If two triangles are congruent then they are similar"

(ii) "If the opposite angles of a quadrilateral are equal then it is a parallelogram".

34. Find the image of the point (4, -3) in the line $x + y + 1 = 0$.

35. Find the equation of the line passing through the point of intersection of $3x - y - 15 = 0$ and $x - y = 3$ which is perpendicular to the line $5x - 4y + 1 = 0$.

36. Find the ratio in which the line $x + y = 0$ divides the line segment joining the points (3, 1) and (6, -3).

37. Find the equation of the altitude through the vertex A (2,2) of the triangle whose vertices are A (2,2), B (0,1) and C (3,-1).

38. Find area of triangle formed by lines $6x + 4y = 1$ and the co-ordinate axes

39. Find the equation the circle concentric with the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ and which touches the y-axis.

40. Find the equation of the circle passing through the points (1,3), (5,3) and (1,-2).

41. Find the image of the point (-4, -3) in the line $x + y + 1 = 0$ also find the co-ordinates of foot of perpendicular from point to the line.

42. Find the equation of the circle passing through the points (4,1) and (6,5) and whose centre is on the line $4x + y = 16$

43. Find the area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 12y$ to the ends of its latus rectum.

44. An arc is in the shape of a parabola with its axis vertical. The arc is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex of the parabola?

45. Find the co-ordinates of the foci, the vertices, length of the major axis, the length of the minor axis, the eccentricity and the length of the latus rectum of the ellipse $x^2/9 + y^2/4 = 1$.
46. Find the equation of the hyperbola whose foci are $(\pm 4, 0)$ and eccentricity is 8.
47. Find the equation of the locus of the point P (x,y,z) whose distance from the point A(5,1,0) is k times that of the distance from the point B(2,-1,4).
48. The midpoints of the sides of a triangle are (2,3,-1), (0,8,5) and (5,7,11). Find its vertices.
49. Three vertices of a parallelogram ABCD are A (3, -1, 2), B (1, 2, -4), C (-1, 1, 2). Find the co-ordinates of the fourth vertex.
50. Find the equation of the locus of the point P (x,y,z) whose distance from the point A(-5,1,0) is k times that of the distance from the point B(2,1,-4).

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