## CHAPTER - 2

RELATIONS AND FUNCTIONS

KEY POINTS

- Cartesian Product of two non-empty sets A and B is given by,

  $A \times B = \{(a,b) : a \in A, b \in B\}$

- If $(a,b) = (x, y)$, then $a = x$ and $b = y$

- Relation R from a non-empty set A to a non-empty set B is a subset of $A \times B$.

- Domain of $R = \{a : (a,b) \in R\}$

- Range of $R = \{b : (a,b) \in R\}$

- Co-domain of R = Set B

- Range $\subseteq$ Co-domain

- If $n(A) = p$, $n(B) = q$ then $n(A \times B) = pq$ and number of relations $= 2^{pq}$

- A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

- $D_f = \{x : f(x) \text{ is defined}\}$ $\qquad$ $R_f = \{f(x) : x \in D_f\}$

- Identity function, $f : R \to R$; $f(x) = x \; \forall x \in R$ where R is the set of real numbers.

  $D_f = R \qquad R_f = R$
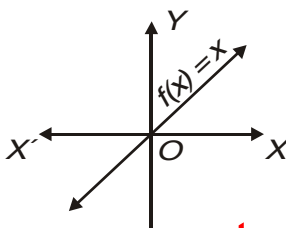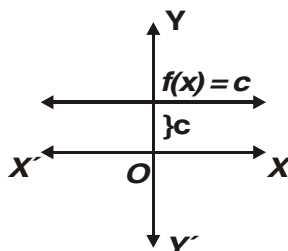
- Constant function, $f : R \to R$; $f(x) = c \ \forall x \in R$ where c is a constant

  $D_f = R \qquad R_f = \{c\}$

- Modulus function, $f : R \to R$; $f(x) = |x| \ \forall x \in R$

  $D_f = R$

  $R_f = R^+ = \{ x \in R: x \geq 0\}$
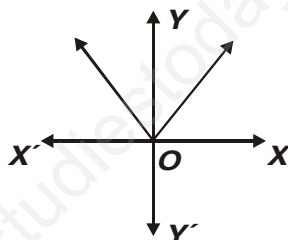
- Signum function, $f : R \to R$; $f(x) = \begin{cases} 1, \text{If } x > 0 \\ 0, \text{ if } x = 0 \\ -1, \text{if } x < 0 \end{cases}$
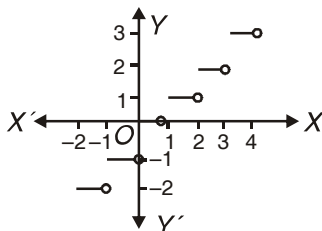
  $D_f = R$

  $R_f = \{-1,0,1\}$
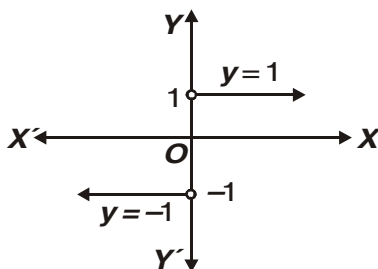
- Greatest Integer function, $f : R \rightarrow R$; $f(x) = [x]$, $x \in R$ assumes the value of the greatest integer, less than or equal to x
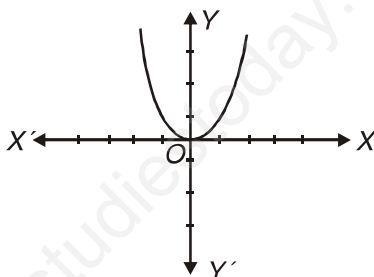
  $D_f = R$ $R_f = Z$

- $f : R \rightarrow R$, $f(x) = x^2$

  $D_f = R$ $R_f = [0, \infty)$

- $f : R \rightarrow R$, $f(x) = x^3$

  $D_f = R$ $R_f = R$
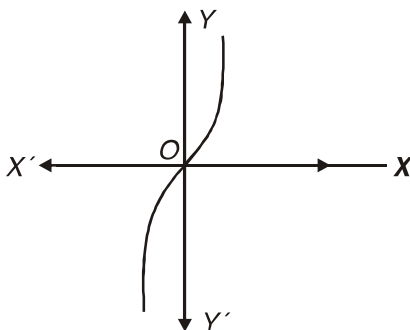
- Let $f : X \rightarrow R$ and $g : X \rightarrow R$ be any two real functions where $x \subset R$ then

  $(f \pm g)(x) = f(x) \pm g(x) \ \forall x \in X$

  $(fg)(x) = f(x) \ g(x) \ \forall x \in X$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad \forall \ x \in X \text{ provided } g(x) \neq 0$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find a and b if $(a - 1, b + 5) = (2, 3)$

   If A = {1,3,5}, B = {2,3} find : (Question-2, 3)

2. A × B

3. B × A

   Let A = {1,2}, B = {2,3,4}, C = {4,5}, find (Question- 4,5)

4. A × (B ∩ C)

5. A × (B ∪ C)

6. If P = {1,3}, Q = {2,3,5}, find the number of relations from A to B

7. If A = {1,2,3,5} and B = {4,6,9},

   R = {(x, y) : |x − y| is odd, x ∈ A, y ∈ B}

   Write R in roster form

   Which of the following relations are functions. Give reason. (Questions 8 to 10)

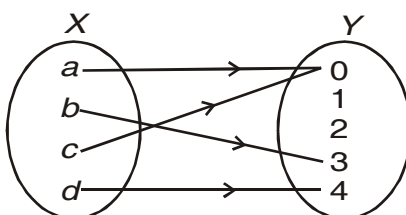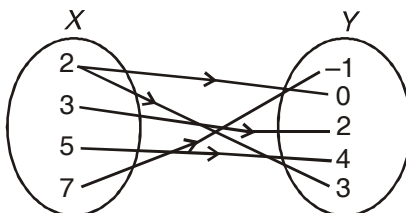8. R = { (1,1), (2,2), (3,3), (4,4), (4,5)}

9. R = { (2,1), (2,2), (2,3), (2,4)}

10. R = { (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)}

    Which of the following arrow diagrams represent a function? Why? (Question- 11,12)

11.

12.

Let f and g be two real valued functions, defined by, $f(x) = x^2$, $g(x) = 3x + 2$, find : (Question 13 to 16)

13. $(f + g)(-2)$

14. $(f - g)(1)$

15. $(fg)(-1)$

16. $\left(\dfrac{f}{g}\right)(0)$

17. If $f(x) = x^3$, find the value of,

$$\dfrac{f(5) - f(1)}{5 - 1}$$

18. Find the domain of the real function,

$$f(x) = \sqrt{x^2 - 4}$$

19. Find the domain of the function, $f(x) = \dfrac{x^2 + 2x + 3}{x^2 - 5x + 6}$

Find the range of the following functions, (Question- 20,21)

20. $f(x) = \dfrac{1}{1 - x^2}$

21. $f(x) = x^2 + 2$

22. Find the domain of the relation,

R = { (x, y) : x, y ∈ Z, xy = 4}

Find the range of the following relations : (Question-23, 24)

23. R = {(a,b) : a, b ∈ N and 2a + b = 10}

24. $R = \left\{ \left(x, \dfrac{1}{x} \right) : x \in z, 0 < x < 6 \right\}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

25. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as,

    R = {(x, y) : x ∈ A, y ∈ B and y = x²}

    (a) Depict this relation using arrow diagram.

    (b) Find domain of R.

    (c) Find range of R.

    (d) Write co-domain of R.

26. Let R = { (x, y) : x, y ∈ N and y = 2x} be a relation on N. Find :

    (i) Domain

    (ii) Codomain

    (iii) Range

        Is this relation a function from N to N?

27. Let $f(x) = \begin{cases} x^2, & \text{when } 0 \le x \le 2. \\ 2x, & \text{when } 2 \le x \le 5 \end{cases}$

    $g(x) = \begin{cases} x^2, & \text{when } 0 \le x \le 3. \\ 2x, & \text{when } 3 \le x \le 5 \end{cases}$

    Show that f is a function while g is not a function.

28. Find the domain and range of,

    f(x) = |2x − 3| − 3

29. Draw the graph of the Greatest Integer function

30. Draw the graph of the Constant function, f : R → R; f(x) = 2 $\forall$ x ∈ R. Also find its domain and range.

ANSWERS

1. a = 3, b = −2

2. A × B = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)}

3. B × A = { (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)}

4. {(1,4), (2,4)}

5. {(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)}

6. 2^6 = 64

7. R = { (1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)}

8. Not a function because 4 has two images.

9. Not a function because 2 does not have a unique image.

10. Function   11. Function

12. Not a function   13. 0

14. −4   15. −1

16. 0   17. 31

18. (−∞, −2] ∪ [2, ∞)   19. R − {2,3}

20. (−∞, 0) ∪ [1, ∞)   21. [2,∞)

22. {−4, −2, −1,1,2,4}   23. {2,4,6,8}

24. $\left\{1, \dfrac{1}{2}, \dfrac{1}{3}, \dfrac{1}{4}, \dfrac{1}{5}\right\}$

25. (a)
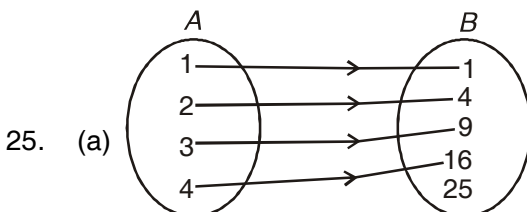
     (b) {1,2,3,4}

     (c) {1,4,9,16}

     (d) {1,4,9,16,25}

26.   (i) N

     (ii) N

     (iii) Set of even natural numbers

     yes, R is a function from N to N.

28.   Domain is R

     Range is $[-3, \infty)$

30.   Domain = R

     Range = {2}