Assignment Class XI (Chapter 2-Relations and Functions)

1. Find the domain and range of following functions:
   (i) $\sqrt{x-3}$
   (ii) $\sqrt{25-x^2}$
   (iii) $5-|x+1|$
   (iv) $\frac{1}{\sqrt{16-x^2}}$

2. Draw the graph of function f(x) = $\begin{cases} 1+2x & x < 0 \\ 3+5x & x \geq 0 \end{cases}$. Hence find its range.

3. If f(x)= 2x-3 and g(x) = x^2-1 then find:
   (i) 3g+2f
   (ii) g/f
   (iii) 2g+5f

4. Determine a quadratic function f(x)= ax²+bx+c if f(0)=6, f(2)=11 and f(-3) = 6

5. If R ={(x,y): y = x+1 and y $\in$ {0, 1, 2, 3, 4, 5}
   (i) List the elements of R
   (ii) Represent R by an arrow diagram
   (iii) Find R⁻¹.

6. Let f be a subset of QXZ defined by f = ={($\frac{m}{n}$, n): m,n $\in$ z, n $\neq$ 0}. Is f a function from Q to Z? Justify your answer.

7. Let A = {6, 7, 8, 10} and B = {2,4,5} and R is a relation from A to B defined as aRb if and only if a is divisible by b. Write R in roster form.

8. If A = {1,2,3} and B = {4,5} and C = {5,6} verify that
   (i) A X (BUC) = (AXB) U (AXC)
   (ii) AX(B-C) = (AXB) - (AXC)

9. If f(X)=$\frac{x-1}{x+1}$ then show that $f\left(\frac{1}{x}\right) = -f(x)$.

10. Find The domain of following functions:
    (i) f(x) = $\frac{1}{\sqrt{|x|-x}}$
    (ii) f(x) = $\frac{x}{x^2+3x+2}$
    (iii) f(x) = $\frac{|x|-x}{2x}$

Assignment on Limits and Derivatives

I) Evaluate the limits :

1. $\displaystyle \lim_{x \to 2} \frac{x^3 - 3x^2 + 4}{x^4 - 8x^2 + 16}$

2. $\displaystyle \lim_{x \to 3} \left(\frac{1}{x-3} - \frac{2}{x^2 - 4x + 3} \right)$

3. $\displaystyle \lim_{x \to \sqrt{3}} \frac{x^4 - 9}{x^4 + 4\sqrt{3}x - 15}$

4. $\displaystyle \lim_{x \to 3} \frac{x - 3}{\sqrt{x-2} - \sqrt{4-x}}$

5. $\displaystyle \lim_{x \to 4} \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}}$

6. $\displaystyle \lim_{x \to 0} \frac{\cos Ax - \cos Bx}{x^2}$

7. $\displaystyle \lim_{x \to 0} \frac{\cot 2x - \cos ec 2x}{x}$

8. $\displaystyle \lim_{x \to \pi} \frac{1 + \cos^3 x}{\sin^2 x}$

9. $\displaystyle \lim_{x \to \pi} \frac{1 - \sin \frac{x}{2}}{\cos \frac{x}{2} \left(\cos \frac{x}{4} - \sin \frac{x}{4} \right)}$

10. $\displaystyle \lim_{x \to \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}}$

II) If $\displaystyle \lim_{x \to 2} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$, then find the value of K.

III) Differentiate the following by the method of First Principle:

1. $\sqrt{\sin x}$ 2. $\tan \sqrt{x}$ 3. $\cos^2 x$ 4. $\tan x^2$ 5. $\dfrac{2x + 3}{x - 2}$

IV) i) If $y = \sqrt{\dfrac{\sec x - \tan x}{\sec x + \tan x}}$, show that $\dfrac{dy}{dx} = \sec x(\tan x + \sec x)$.

ii) If $y = \dfrac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$, then find $\dfrac{dy}{dx}$.

iii) If $y = \sqrt{\dfrac{1-x}{1+x}}$, prove that $(1 - x^2)\dfrac{dy}{dx} + y = 0$.

iv) If $y = \dfrac{\cos x - \sin x}{\cos x + \sin x}$, prove that $\dfrac{dy}{dx} + y^2 + 1 = 0$.

v) If $y = \dfrac{x}{x+5}$, prove that $x\dfrac{dy}{dx} = y(y-1)$.

vi) If $y = \sqrt{\dfrac{x}{a}} + \sqrt{\dfrac{a}{x}}$, prove that $2xy\dfrac{dy}{dx} = \dfrac{x}{a} - \dfrac{a}{x}$