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| | ***RELATIONS & FUNCTIONS***<br>***Class XI*** |
| | ***FUNCTIONS*** |
| Q.1) | Find the domain of function, $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$. |
| Sol.1) | We have, $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$<br>$f(x)$ is real for all values of $x$ such that<br>$x^2 - 5x + 4 \neq 0$<br>$\Rightarrow (x-4)(x-1) \neq 0$<br>$\therefore$ Domain$=R - \{1,4\}$ |
| Q.2) | Find the domain of the function, $f(x) = \frac{2x-3}{x^2-3x+2}$. |
| SELF | $R - \{1,2\}$ |
| Q.3) | Find the domain of the function, $f(x)\sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$ |
| Sol.3) | $f(x)$is real for all values of $x$ such that<br>$4 - x \geq 0$ and $x^2 - 1 > 0$<br>$\Rightarrow x - 4 \leq 0$ and $(x+1)(x-1) > 0$<br>$\Rightarrow x \leq 4$ and<br>$\Rightarrow x \leq 4$ and $x \in (-00, -1) \cup (1, 00)$<br>The common solution is<br><br>FIG 1<br><br>$\therefore$ Domain is $(-00, -1) \cup (1,4)$ ans. |
| Q.4) | Find the domain and range of function $f(x) = \frac{x-2}{3-x}$ |
| Sol.4) | We have, $f(x) = \frac{x-2}{3-x}$<br>Domain: $f(x)$is real for all values of $x$ such that<br>$3 - x \neq 0 \Rightarrow x + 3$<br>$\therefore$ Domain$=R - \{3\}$<br>Range: let $y = f(x)$<br>$\Rightarrow y = \frac{x-2}{3-x}$<br>$\Rightarrow 3y - xy = x - 2$<br>$\Rightarrow x + xy = 3y + 2$<br>$\Rightarrow x(1+y) = 3y + 2$<br>$\Rightarrow x = \frac{3y+2}{y+1}$<br>$x$ is real for all values of $y$ such that<br>$y + 1 \neq 0$<br>$\Rightarrow y \neq -1$<br>$\therefore$ Range$=R - \{-1\}$ ans. |
| Q.5) | Find the domain & range of the function $f(x) = \sqrt{16 - x^2}$ |
| Sol.5) | We have, $f(x) = \sqrt{16 - x^2}$<br>Domain: $f(x)$is real for all values of $x$ such that<br>$16 - x^2 \geq 0$<br>$\Rightarrow x^2 - 16 \leq 0$<br>$\Rightarrow (x+4)(x-4) \leq 0$<br>$\therefore$ Domain $x \in [-4,4]$<br><br>FIG.2<br>Range: let $y = f(x)$ |

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| | $\Rightarrow y = \sqrt{16 - x^2}$ ............ (1)<br>Squaring,<br>$y^2 = 16 - x^2$<br>$\Rightarrow x^2 = 16 - y^2$<br>$x$ is real for all values of $y$ such that<br><br>Fig.3<br><br>$16 - y^2 \geq 0$<br>$\Rightarrow y^2 - 16 \leq 0$<br>$\Rightarrow (y+4)(y-4) \leq 0$<br>Range $y \in [-4,4]$<br>But $y \geq 0$, from eq.(1)<br>$\therefore$ Range$=R[0,4]$ ans. |
| Q.6) | Find the Domain & Range of $f(x) = \frac{1}{\sqrt{x-5}}$ |
| Sol.6) | We have, $f(x) = \frac{1}{\sqrt{x-5}}$<br>Domain: $f(x)$is real for all values of $x$ such that; $x - 5 > 0$<br>$\Rightarrow x > 5$<br>$\Rightarrow x \in (5,00)$<br>$\therefore$ Domain $(5,00)$<br>Range: let $y = f(x)$<br>$\Rightarrow y = \frac{1}{\sqrt{x-5}}$ ........... (1)<br>Squaring,<br>$y^2 = \frac{1}{x-5}$<br>$\Rightarrow x - 5 = \frac{1}{y^2}$<br>$\Rightarrow x = \frac{1}{y^2} + 5$<br>$\Rightarrow x = \frac{1+5y^2}{y^2} + 5$<br>$x$ is real for all values of $y$ such that<br>$y^2 \neq 0 \Rightarrow y \neq 0$<br>$\therefore y \in R - \{0\}$<br>But $y > 0$, from eq.(1)<br>$\therefore$ Range $[0,00]$ ans. |
| Q.7) | Find Domain & Range of $f(x) = \frac{3}{2-x^2}$ |
| Sol.7) | We have, $f(x) = \frac{3}{2-x^2}$<br>Domain: $f(x)$is real for all values of $x$ such that; $2 - x^2 \neq 0$<br>$\Rightarrow x^2 - 2 \neq 0$<br>$\Rightarrow (x+\sqrt{2})(x-\sqrt{2}) \neq 0$<br>$\Rightarrow x \neq -\sqrt{2}$ and $x \neq \sqrt{2}$<br>$\therefore$ Domain $R - \{-\sqrt{2}, \sqrt{2}\}$<br>Range: let $y = f(x)$<br>$\Rightarrow y = \frac{3}{2-x^2}$<br>$\Rightarrow 2y - x^2 y = 3$<br>$\Rightarrow x^2 y = 2y - 3$<br>$\Rightarrow x^2 = \frac{2y-3}{y}$<br>$\Rightarrow x = \sqrt{\frac{2y-3}{y}}$ |

$x$ is real for all values of $y$ such that

$\frac{2y-3}{y} \geq 0$ and $y \neq 0$

$\Rightarrow \frac{y(2y-3)}{y^2} \geq 0$ and $y \neq 0$

$\{multiply\ \&\ divide\ by\ y\}$

$\Rightarrow y(2y-3) \geq 0$ and $y \neq 0$

FIG.4

$y \in R[-00,0] \cup \left[\frac{3}{2}, 00\right]$ but $y \neq 0$

$\therefore$ Range $(-00,0) \cup \left[\frac{3}{2}, 00\right]$ ans.

**Q.8)** Let $f = \left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R to R. Determine Domain & Range.

**Sol.8)** We have, $f(x) = \frac{x^2}{1+x^2}$

Domain: $f(x)$is real for all values of $x$ such that; $x \in R$ ……… $\{\because 1 + x^2 \neq 0$ for any $x \in R\}$

$\therefore$ Domain $= R$

Range: let $y = f(x)$

$\Rightarrow y = \frac{x^2}{1+x^2}$

$\Rightarrow y + x^2 y = x^2$

$\Rightarrow x^2 y - x^2 = -y$

$\Rightarrow x^2(y-1) = -y$

$\Rightarrow x^2 = \frac{-y}{y-1}$

$\Rightarrow x = \sqrt{\frac{-y}{y-1}}$

$x$ is real for all values of $y$ such that

$\frac{-y}{y-1} \geq 0$ and $y - 1 \neq 0$

$\Rightarrow \frac{-y}{y-1} < 0$ and $y \neq 1$

$\Rightarrow y\frac{(y-1)}{(y-1)^2} < 0$ and $y \neq 0$

$\{multiply\ \&\ divide\ by\ (y-1)\}$

$\Rightarrow y(y-1) \leq 0$

FIG.5

$\Rightarrow y \in [0,1]$ but $y \neq 1$

$\therefore$ Range $(0,1)$ ans.

**Q.9)** Find Domain & Range of $f(x) = \frac{x}{1+x^2}$

**Sol.9)** We have, $f(x) = \frac{x}{1+x^2}$

Domain: $f(x)$is real for all values of $x$ such that; $x \in R$

$\therefore$ Domain=R

Range: let $y = f(x)$

$\Rightarrow y = \frac{x}{1+x^2}$

$\Rightarrow y + x^2 y = x$

$\Rightarrow x^2 y - x + y = 0$

Here, $a = y, b = -1, c = y$

By quadratic formula

$\Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

$x$ is real for all values of y such that

$1 - 4y^2 \geq 0$ and $2y \neq 0$

$\Rightarrow 4y^2 - 1 \leq 0$ and $y \neq 0$

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| | $\Rightarrow (2y+1)(2y-1) \leq 0$<br><br>FIG.5<br><br>$\Rightarrow y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ but $y \neq 0$<br>$\therefore$ Range $= \left[-\frac{1}{2}, 0\right] \cup \left[0, \frac{1}{2}\right]$<br>OR $\left[-\frac{1}{2}, \frac{1}{2}\right], -\{0\}$ ans. | |
| Q.10) | Find the Domain and Range of $f(x) = \frac{x^2-9}{x-3}$ | |
| Sol.10) | We have, $f(x) = \frac{x^2-9}{x-3}$<br>Domain: $f(x)$is real for all values of x such that;<br>$x - 3 \neq 0$<br>$x \neq 3$<br>$\therefore$ Domain$=R - \{3\}$<br>Range: let $y = f(x)$<br>$\Rightarrow y = \frac{x^2-9}{x-3}$<br>$\Rightarrow y = \frac{(x+3)(x-3)}{(x-3)}$<br>$\Rightarrow y = x + 3$<br>$\Rightarrow x = y - 3$<br>Clearly x is real for all values of y such that $y \in R$<br>But y $\neq$ 6, since when y = 6 then x = 3 and x = 3 is not in the Domain<br>$\therefore$ Range $= R - \{0\}$ ans. | |