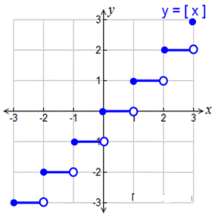
	RELATIONS & FUNCTIONS
Q.1)	Draw the graph of the function $f(x) = [x - 1]$.
Sol.1)	$f(x) = [x - 1]$ e.g., when $x = 1$ then $f(x) = [0] = 0$ when $x = 2$ then $f(x) = [1] = 1$  Domain = R Range = Z
Q.2)	Let R be relation on N defined by $R = \{(a, b): a = b \in N \text{ and } a = b^2\}$ Are the following true? (i) $(a, a) \in R$ for all $a \in N$, (ii) $(a, b) \in R \Rightarrow (b, a) \in R$, (iii) $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$.
Sol.2)	$R = \{(a, b): a = b^2\}$ (i)$2 \in N$ But $(2,2) \in R$ Since $2 \neq 2^2$ $\therefore$ false (ii) $(4,2) \notin R$ as $4 = 2^2$ but $(2,4) \notin R$ Since $2 \neq 4^2$ $\therefore$ false (iii) $(16,4) \in R \text{ and } (4,2) \in R$ as $16 = 4^2$ and $4 = 2^2$ but $(16,2) \notin \mathrm{R}$ since $16 \neq 2^2$ $\therefore$ false
Q.3)	Let R be a relation on Q defined by $= \{(a, b): a, b \in Q \text{ and } 4 - b \in Z \text{ i.e., } a - b \text{ is an integer}\}$. Show that, (i) $(a, a) \in R$ for all $a \in Q$, (ii) $(a, b) \in R \Rightarrow (b, a) \in R$, (iii) $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.
Sol.3)	We have, $R = \{(a, b): a - b \text{ is an integer}\}$ $Q \rightarrow$ set of rational number (i) for any $a \in Q, a - a = 0$ which is an integer $\therefore (a, a) \in R$ (ii) let $(a, b) \in R$ $\Rightarrow a - b$ is an integer $\Rightarrow a - b = m$ where $m \in Z$ $\Rightarrow b - a = -m$ which is also an integer $\Rightarrow (b, a) \in R$ (iii) let $(a, b) \in R$ and $(b, c) \in R$ $\Rightarrow a - b = m \text{ and } b - c = n$ (where $m, n \in Z$) Now, $(a - c) = (a - b) + (b - c)$ $= m + n$ $\{\because \text{sum of two integers is also an integer}\}$ $= integer$ $\therefore (a, c) \in R$
Q.4)	If $A = (2,3)$ and $B = (1,2,3)$. Find the no. of relations.
Sol.4)	$A = (2,3)$ and $B = (1,2,3)$ Here $m = 2$ and $n = 3$ No. of relations = 2^{mn} = $2^{2 \times 3}$

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	$= 2^6 = 64$ ans.	
Q.5)	If $A \times A$ has 9 elements, among which two are $(1,0)$ and $(0,1)$. Find Set A & remaining elements of $A \times A$	
Sol.5)	Given, $n(A \times A) = 9$ $\Rightarrow n(A) = 3$ Now, $(-1,0)$ and $(0,1) \in A \times A$ $\Rightarrow -1,0 \in A\ and\ 0.1 \in A$ $\Rightarrow -1,0.1 \in A$ Also $n(A) = 3$ $\therefore A = \{-1,0,1\}$ $A \times A = \{(-1,-1),(-1,0),(-1,1),(0,1),(0,0),(0,1),(1,-1),(1,0),(1,1)\}$	
Q.6)	Find the domain of $f(x) = \frac{1}{\sqrt{x-\lvert x \rvert}}$	
Sol.6)	We have, $f(x) = \frac{1}{\sqrt{x-\lvert x \rvert}}$ $\lvert x \rvert = \begin{cases} x : x \geq 0 \\ -x : x < 0 \end{cases}$ $-\lvert x \rvert = \begin{cases} -x : x \geq 0 \\ x : x < 0 \end{cases}$ $x - \lvert x \rvert = \begin{cases} x - x : x \geq 0 \\ x + x : x < 0 \end{cases}$ $\Rightarrow x - \lvert x \rvert = \begin{cases} 0 : x \geq 0 \\ 2x : x < 0 \end{cases}$ .... (1) But we have, $f(x) = \frac{1}{\sqrt{x-\lvert x \rvert}}$ $\therefore x - \lvert x \rvert > 0$ From (1) $x - \lvert x \rvert = 0$ or $x - \lvert x \rvert$ is $-$ve $\therefore \frac{1}{\sqrt{x-\lvert x \rvert}}$ does not take real values for $\therefore$ Domain = $\emptyset$	
Q.7)	Find the domain of $f(x) = \frac{1}{\sqrt{x-[x]}}$	
Sol.7)	$f(x)$ will be defined when $x - [x] > 0$ We know, $0 \leq x - [x] < 1$ for all $x \in R$ But $x - [x] = 0$ for all $x \in Z$ $\therefore x - [x] > 0$ when $x \in R - Z$ $\therefore$ Domain $R - Z$ ans.	
Q.8)	find the domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal	
Sol.8)	we have, $f(x) = g(x)$ $\Rightarrow 2x^2 - 1 = 1 - 3x$ $\Rightarrow 2x^2 + 3x - 2 = 0$ $\Rightarrow 2x^2 + 4x - x - 2 = 0$ $\Rightarrow 2x(x+2) - 1(x+2) = 0$ $\Rightarrow (2x-1)(x+2) = 0$ $\Rightarrow x = \frac{1}{2}\ or\ x = -2$ Domain all the values of x $\therefore$ Domain = $\left\{\frac{1}{2}, -2\right\}$ ans.	
Q.9)	Redefine the function $f(x) = \lvert x - 2 \rvert + \lvert 2 + x \rvert$ where $-3 \leq x \leq 3$.	
Sol.9)	Critical point of $\lvert x - 2 \rvert$ is 2 Critical point of $\lvert 2 + x \rvert$ is -2	

	First arrange in ascending order $\therefore f(x) = \lvert 2+x \rvert + \lvert x-2 \rvert$ There will be 3 cases: (i) when $-3 \le x < -2$, (ii) when $-2 \le x < 2$, (iii) when $2 < x \le 3$ $\therefore f(x) = \begin{cases} -(2+x)-(x-2): -3 \le x < -2 \\ (2+x)-(x-2): -2 \le x < 2 \\ (2+x)+(x-2): 2 < x \le 3 \end{cases}$ $f(x) = \begin{cases} -2x: -3 \le x < -2 \\ 4: (x-2): -2 \le x < 2 \\ 2x: (x-2): 2 < x \le 3 \end{cases}$ ans.	
Q.10)	Find the Range of $f(x) = \frac{1}{1-2\cos x}$	
Sol.10)	We have, $-1 \le 2\cos x \le 1$ $-2 \le 2\cos x \le 2$ $2 \ge -2\cos x \ge -2$ $3 \ge 1 - 2\cos x \ge -1$ $\Rightarrow \frac{1}{3} \le \frac{1}{1-2\cos x} \le \frac{-1}{1}$ $\Rightarrow \frac{1}{3} \le f(x) \le \frac{-1}{1}$ $\Rightarrow f(x) \le -1 \ or \ f(x) \ge \frac{1}{3}$ $x \in [-00, -1] \cup \left[\frac{1}{3}, 00\right]$ ans.	