

### **Chapter: - Probability, Limits and Derivative**

**Q1.** Evaluate each of the following limits: -

$$(i) \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}. \quad \text{Ans. 1.}$$

$$(ii) \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}. \quad \text{Ans. } 1/2.$$

$$(iii) \lim_{x \rightarrow 0} \frac{\sqrt[3]{8+x} - 2}{x}. \quad \text{Ans. } 1/12$$

$$(iv) \lim_{x \rightarrow 1} \frac{x^4 - 3x^3 + 2}{x^3 - 5x^2 + 3x + 1}. \quad \text{Ans. } 5/4,$$

$$(v) \lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-2x}}{\sin x}. \quad \text{Ans. 2.}$$

$$(vi) \lim_{x \rightarrow 0} \frac{1 - \cos x \sqrt{\cos 2x}}{x^2}. \quad \text{Ans. } 3/2.$$

$$(vii) \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{x - \frac{\pi}{6}}. \quad \text{Ans. 2.}$$

$$(viii) \lim_{x \rightarrow 0} \frac{\cos 3x - \cos x}{x^2}. \quad \text{Ans. -4.}$$

$$(ix) \lim_{x \rightarrow 0} \frac{\tan 2x - \sin 2x}{x^3}. \quad \text{Ans. 4.}$$

$$(x) \lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx}. \quad \text{Ans. } \frac{m^2}{n^2}.$$

$$(xi) \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}. \quad \text{Ans. } 1/2.$$

$$(xii) \lim_{x \rightarrow 0} (1+x)^{\frac{2}{x}}. \quad \text{Ans. } e^2.$$

$$(xiii) \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}. \quad \text{Ans. 2.}$$

$$(xiv) \lim_{x \rightarrow 0} \frac{5^{3x} - 1}{x}. \quad \text{Ans. } 3 \log 5.$$

$$(xv) \lim_{x \rightarrow 0} \frac{\log(1+7x)}{x}. \quad \text{Ans. 7.}$$

**Q2.** Let  $f(x)$  be a function defined by  $f(x) = \begin{cases} 6x - 6 & \text{if } x \leq 3 \\ 2x - k & \text{if } x > 3 \end{cases}$ . Find  $k$ , if  $\lim_{x \rightarrow 3} f(x)$  exists. **Ans. -6**

**Q3.** Let  $f(x)$  be a function defined by  $f(x) = \begin{cases} -2x + 3 & \text{if } x < 1 \\ 1 & \text{if } 1 \leq x \leq 2 \\ 2x - 3 & \text{if } x > 2 \end{cases}$  Find  $\lim_{x \rightarrow 1} f(x)$  and  $\lim_{x \rightarrow 2} f(x)$ . **Ans. 1, 1**

**Q4.(i)** If  $\lim_{x \rightarrow a} \frac{x^7 + a^7}{x + a} = 7$ , find the value of  $a$ . **Ans.  $\pm 1$**  **(ii)** If  $\lim_{\phi \rightarrow 0} (k\phi) \cosec \phi = \lim_{\phi \rightarrow 0} \phi \cosec(k\phi)$ . prove that  $k = \pm 1$

**Q5.** Find the positive integer  $n$ , so that  $\lim_{x \rightarrow 3} \frac{x^n - 3^n}{x - 3} = 108$ , find the value of  $a$ . **Ans. 4.**

**Q6.** Let  $f(x)$  be a function defined by  $f(x) = \begin{cases} \frac{6x}{|x| - 2x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ . Find  $\lim_{x \rightarrow 0} f(x)$ . **Ans.** Does not exist,

**Q7.** Find all the possible values of  $a$ , if  $\lim_{x \rightarrow a} \frac{x^9 - a^9}{x - a} = \lim_{x \rightarrow 5} (4 + x)$ , **Ans.  $\pm 1$**

**Q8.** If  $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2}$ , Find all the possible values of  $k$ . **Ans. 8/3**

**P.T.O.**

**Q9.** Let  $f(x)$  be a function defined by  $f(x) = \begin{cases} x & \text{for } x < 1 \\ 2 & \text{for } x = 1 \\ x + 2 & \text{for } x > 1 \end{cases}$  Find  $\lim_{x \rightarrow 1} f(x)$  **Ans.** Does not exist,

**Q10.** Differentiate each of the following functions with respect to  $x$ , from first principle:-

(i)  $f(x) = 15x$  **Ans.** 15

(ii)  $f(x) = x^3 - 1$  **Ans.**  $3x^2$

(iii)  $f(x) = x - \frac{1}{x}$  **Ans.**  $1 + \frac{1}{x^2}$

(iv)  $f(x) = \tan 2x$  **Ans.**  $2\sec^2 2x$

(v)  $f(x) = \cos(2x - 3)$  **Ans.**  $-2\sin(2x - 3)$

(vi)  $f(x) = \sec 3x$  **Ans.**  $3\sec 3x \tan 3x$

(vii)  $f(x) = 120$  **Ans.** 0

(viii)  $f(x) = \sqrt{5x - 7}$  **Ans.**  $\frac{5}{2\sqrt{5x - 7}}$

**Q11.** Evaluate each of the following, from first principle:-

(i)  $f'(4)$  if  $f(x) = x^2$ , **Ans.** 8

(ii)  $f'(16)$  if  $f(x) = 16$ , **Ans.** 0

(iii)  $f'(5)$  if  $f(x) = 2x$ , **Ans.** 2

(iv)  $f'(9)$  if  $f(x) = \sqrt{x}$ , **Ans.**  $1/6$

**Q12.** Find the derivative of each of the following functions, with respect to  $x$ :-

(i)  $f(x) = (3x+7)^5$  **Ans.**  $15(3x+7)^4$

(ii)  $f(x) = x^2 \sin x$  **Ans.**  $x^2 \cos x + 2x \sin x$

(iii)  $f(x) = \frac{x}{1 + \tan x}$  **Ans.**  $\frac{1 + \tan x - x \sec^2 x}{(1 + \tan x)^2}$

(iv)  $f(x) = x \sec 3x$  **Ans.**  $\sec 3x + 3x \sec 3x \tan 3x$

(v)  $f(x) = \frac{x^5 - \cos x}{\sin x}$  **Ans.**  $\frac{1 + x^4(5 \sin x - x \cos x)}{\sin^2 x}$

(vi)  $f(x) = (1-x^2)(3x+4)$  **Ans.**  $-9x^2 - 8x + 3$

(vii)  $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$  **Ans.**  $\frac{-2}{(\sin x - \cos x)^2}$

(viii)  $f(x) = \frac{3x - 4x^3}{x + 1}$  **Ans.**  $\frac{-8x^3 - 12x^2 + 3}{(x + 1)^2}$

**Q13.(i)** If  $y = \cosec x + \cot x$ , show that  $\sin x \frac{dy}{dx} + y = 0$  **(ii)** If  $y = a \sin x + b \cos x$ , show that  $y^2 + \left(\frac{dy}{dx}\right)^2 = a^2 + b^2$

**Q14.** The letter of the word 'SOCIETY' are placed at random in a row. What is the probability that the three vowels come together. **Ans.** 1/7

**Q15.** Four digit numbers are formed by using the digit 1, 2, 3, 4 and 5 without repeating any digit. Find the probability that a number, chosen at random, is an odd number. **Ans.** 3/5

**Q16.** Two dice are thrown simultaneously. Find the probability of getting a doublet. **Ans.** 1/6

**Q17.** What is the chance that a leap year, selected at random, will contain 53 Sundays? **Ans.** 2/7

**Q18.** Find the probability of an event of getting Prime Number or Even number in a single throw of dice. **Ans.** 5/6

**Q19.** Find the probability of an event of getting two head in a tossing of three coins. **Ans.** 3/8

**Q20.** Two unbiased dice are thrown. Find the probability that neither a doublet nor a total of 10 will appear. **Ans.** 7/9

**Q21.** A card is drawn from deck of 52 cards. Find the probability of getting a king or a heart or a red card. **Ans.** 7/13

**Q22.** The probabilities that a student will receive A, B, C or D grade are 0.40, 0.35, 0.15 and 0.10 respectively. Find the probability that a student will receive (i) not an A grade (ii) B or C grade (iii) at most C grade.

**Ans.** 0.60, 0.50, 0.25.

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