## Downloaded from www.studiestoday.com

## CHAPTER - 16

## PROBABILITY

- Random Experiment : If an experiment has more than one possible out come and it is not possible to predict the outcome in advance then experiment is called random experiment.
- Sample Space : The collection of all possible outcomes of a random experiment is called sample space associated with it. Each element of the sample space(set) is called a sample point.
- Some examples of random experiments and their sample spaces
(i) A coin is tossed

$$
S=\{H, T\}, \quad n(S)=2
$$

Where $n(S)$ is the number of elements in the sample space $S$.
(ii) A die is thrown

$$
S=\{1,2,3,4,5,6], \quad n(S)=6
$$

(iii) A card is drawn from a pack of 52 cards
$\mathrm{n}(\mathrm{S})=52$.
(iv) Two coins are tossed

$$
S=\{H H, H T, T H, T T\}, \quad n(S)=4 .
$$

(v) Two dice are thrown

$$
S=\left\{\begin{array}{l}
11,12,13,14,15,16, \\
21,22,-----, 26, \\
\vdots \\
61,62,-----, 66
\end{array}\right\}
$$

## Downloaded from www.studiestoday.com

(vi) Two cards are drawn from a well shuffled pack of 52 cards
(a) with replacement $\mathrm{n}(\mathrm{S})=52 \times 52$
(b) without replacement $\mathrm{n}(\mathrm{S})={ }^{52} \mathrm{C}_{2}$

- Event : A subset of the sample space associated with a random experiment is called an event.
- Simple Event : Simple event is a single possible outcome of an experiment.
- Compound Event : Compound event is the joint occurrence of two or more simple events.
- Sure Event : If event is same as the sample space of the experiment, then event is called sure event.
- Impossible Event : Let $S$ be the sample space of the experiment, $\phi \subset S$, $\phi$ is an event called impossible event.
- Exhaustive and Mutually Exclusive Events : Events $\mathrm{E}_{1}, \mathrm{E}_{2}, \mathrm{E}_{3}-----\mathrm{E}_{\mathrm{n}}$ are mutually exclusive and exhaustive if
$E_{1} U E_{2} U E_{3} U \cdots----E_{n}=S$ and $E_{i} \cap E_{j}=\phi$ for all $i \neq j$
- Probability of an Event : For a finite sample space $S$ with equally likely outcomes, probability of an event $A$ is $P(A)=\frac{n(A)}{n(S)}$, where $n(A)$ is number of elements in $A$ and $n(S)$ is number of elements in set $S$ and $0 \leq P(A) \leq 1$.
(a) If $A$ and $B$ are any two events then

$$
\begin{aligned}
P(A \text { or } B) & =P(A \cup B)=P(A)+P(B)-P(A \cap B) \\
& =P(A)+P(B)-P(A \text { and } B)
\end{aligned}
$$

(b) If $A$ and $B$ are mutually exclusive events then

$$
P(A \cup B)=P(A)+P(B)
$$

(c)

$$
P(A)+P(\bar{A})=1
$$

## Downloaded from www.studiestoday.com

(d) $\quad P$ (Sure event) $=1$
(e) P (impossible event) $=0$

- $P(A-B)=P(A)-P(A \cap B)=P(A \cap \bar{B})$
- If $S=\left\{w_{1}, w_{2}, \ldots . . . . ., w_{n}\right\}$ then
(i) $0 \leq P\left(w_{i}\right) \leq 1$ for each $w_{i} \in S$
(ii) $P\left(w_{1}\right)+P\left(w_{2}\right)+\ldots \ldots \ldots+P\left(w_{n}\right)=1$
(iii) $P(A)=\Sigma P\left(w_{i}\right)$ for any event $A$ containing elementary events $w_{i}$.
- $P(\bar{A} \cap \bar{B})=1-P(A \cup B)$


## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

## Describe the Sample Space for the following experiments (Q. No. 1 to 4)

1. A coin is tossed twice and number of heads is recorded.
2. A card is drawn from a deck of playing cards and its colour is noted.
3. A coin is tossed repeatedly until a tail comes up for the first time.
4. A coin is tossed. If it shows head we draw a ball from a bag consisting of 2 red and 3 black balls. If it shows tail, coin is tossed again.
5. Write an example of an impossible event.
6. Write an example of a sure event.
7. Three coins are tossed. Write three events which are mutually exclusive and exhaustive.
8. A coin is tossed $n$ times. What is the number of elements in its sample space?

If $E, F$ and $G$ are the subsets representing the events of a sample space S. What are the sets representing the following events? (Q No 9 to 12).
9. Out of three events atleast two events occur.
10. Out of three events only one occurs.

Downloaded from www.studiestoday.com

## Downloaded from www.studiestoday.com

11. Out of three events only E occurs.
12. Out of three events exactly two events occur.
13. If probability of event $A$ is 1 then what is the type of event 'not $A$ '?
14. One number is chosen at random from the numbers 1 to 21 . What is the probability that it is prime?
15. What is the probability that a given two digit number is divisible by 15 ?
16. If $P(A \cup B)=P(A)+P(B)$, then what can be said about the events $A$ and $B$ ?
17. If $A$ and $B$ are mutually exclusive events then what is the probability of $A \cap B$ ?
18. If $A$ and $B$ are mutually exclusive and exhaustive events then what is the probability of $A \cup B$ ?

## SHORT ANSWER TYPE QUESTIONS (4 MARKS)

19. The letters of the word EQUATION are arranged in a row. Find the probability that
(i) all vowels are together
(ii) the arrangement starts with a vowel and ends with a consonant.
20. An urn contains 5 blue and an unknown number $x$ of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find $x$.
21. Out of 8 points in a plane 5 are collinear. Find the probability that 3 points selected at random form a triangle.
22. Find the probability of almost two tails or atleast two heads in a toss of three coins.
23. A, B and $C$ are events associated with a random experiment such that $\mathrm{P}(\mathrm{A})=0.3, \mathrm{P}(\mathrm{B})=0.4, \mathrm{P}(\mathrm{C})=0.8, \mathrm{P}(\mathrm{A} \cap \mathrm{B})=0.08 \mathrm{P}(\mathrm{A} \cap \mathrm{C})=0.28$ and $P(A \cap B \cap C)=0.09$. If $P(A \cup B \cup C) \geq 0.75$ then prove that $P(B \cap C)$ lies in the interval $[0.23,0.48]$

## Downloaded from www.studiestoday.com

$$
\begin{aligned}
{[\text { Hint }: P(A \cup B A \cup C)=} & P(A)+P(B)+P(C)-P(A \cap B)-P(B \cap C) \\
& -P(A \cap C)+P(A \cap B \cap C)] .
\end{aligned}
$$

24. For a post three persons $A, B$ and $C$ appear in the interview. The probability of $A$ being selected is twice that of $B$ and the probability of $B$ being selected is twice that of C . The post is filled. What are the probabilities of $A, B$ and $C$ being selected?
25. $A$ and $B$ are two candidates seeking admission in college. The probability that $A$ is selected is 0.5 and the probability that both $A$ and $B$ are selected is utmost 0.3 . Show that the probability of $B$ being selected is utmost 0.8 .
26. $S=\{1,2,3,----, 30\}, A=\{x: x$ is multiple of 7$\} B=\{x: x$ is multiple of $5\}, C=\{x: x$ is a multiple of 3$\}$. If $x$ is a member of $S$ chosen at random find the probability that
(i) $x \in A \cup B$
(ii) $x \in B \cap C$
(iii) $x \in A \cap C^{\prime}$
27. A number of 4 different digits is formed by using 1, 2, 3, 4, 5, 6, 7. Find the probability that it is divisible by 5 .
28. A bag contains 5 red, 4 blue and an unknown number of $m$ green balls. Two balls are drawn. If probability of both being green is $\frac{1}{7}$ find m .
29. A ball is drawn from a bag containing 20 balls numbered 1 to 20 . Find the probability that the ball bears a number divisible by 5 or 7 ?
30. What is the probability that a leap year selected at random will contain 53 Tuesdays?

## ANSWERS

1. $\{0,1,2\}$
2. \{Red, Black\}
3. $\{\mathrm{T}, \mathrm{HT}, \mathrm{HHT}, \mathrm{HHHT} . . . . . . .$.
4. $\left\{\mathrm{HR}_{1}, \mathrm{HR}_{2}, \mathrm{HB}_{1}, \mathrm{HB}_{2}, \mathrm{HB}_{3}, \mathrm{TH}, \mathrm{TT}\right\}$
5. Getting a number 8 when a die is rolled

## Downloaded from www.studiestoday.com

6. Getting a number less then 7 when a die is rolled
7. $A=\{H H H, H H T, H T H, T H H\}$
$B=\{H T T, T H T, H T T\}$
$C=\{T T T\}$
8. $2^{n}$
9. $(E \cap F \cap G) \cup\left(E^{\prime} \cap F \cap G\right) \cup\left(E \cap F^{\prime} \cap G\right) \cap\left(E \cap F \cap G^{\prime}\right)$
10. $\left(E \cap F^{\prime} \cap G\right) \cup\left(E^{\prime} \cap F \cap G^{\prime}\right) \cup\left(E^{\prime} \cap F^{\prime} \cap G\right)$
11. $\left(E \cap F^{\prime} \cap G^{\prime}\right)$
12. $\left(E \cap F \cap G^{\prime}\right) \cup\left(E \cap F^{\prime} \cap G\right) \cup\left(E^{\prime} \cap F \cap G\right)$
13. Impossible event
14. $\frac{8}{21}$
15. $\frac{1}{15}$
16. Mutually exclusive events.
17. 0
18. 1
19. (i) $\frac{1}{14}$ (ii) $\frac{15}{56}$
20. 3
21. $\frac{23}{28}$
22. $\frac{7}{8}$
23. $0.23 \leq P(B) \leq 0.48$
24. $\frac{4}{7}, \frac{2}{7}, \frac{1}{7}$
25. 

(i) $\frac{1}{3}$, (ii) $\frac{1}{15}$,
(iii) $\frac{1}{10}$
27. $\frac{1}{7}$
28. 6
29. $\frac{3}{10}$.
30. $\frac{2}{7}$.

