

	<b>TYPE: INEQUALITY</b>
Q.8)	Show using PMI, $(2n + 1) < (n + 3)^2$
Sol.8)	<p>let <math>P(n)</math>: <math>(2n + 1) &lt; (n + 3)^2</math></p> <p>(i) <math>P(1)</math>: <math>(2 + 7) &lt; (1 + 3)^2</math>  <math>P(1)</math>: <math>9 &lt; 16</math> clearly <math>P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true  i.e., <math>P(k)</math>: <math>2k + 7 &lt; (k + 3)^2</math>  (iii) To prove <math>P(k + 1)</math> is true  <math>P(k + 1)</math>: <math>(2k + 9) &lt; (k + 4)^2</math>  (OR) <math>P(k + 1)</math>: <math>(2k + 9) &lt; k^2 + 8k + 16</math>  We have, <math>2k + 7 &lt; (k + 3)^2</math>  Adding 2 on both sides  <math>\Rightarrow (2k + 7) + 2 &lt; (k + 3)^2 + 2</math>  <math>\Rightarrow (2k + 9) &lt; k^2 + 6k + 9 + 2</math>  <math>\Rightarrow (2k + 9) &lt; k^2 + 6k + 11</math>  <math>\Rightarrow (2k + 9) &lt; k^2 + 6k + 11 &lt; k^2 + 8k + 16 \dots\dots k \in N</math>  <math>\Rightarrow (2k + 9) &lt; k^2 + 8k + 16</math>  <math>\Rightarrow (2k + 9) &lt; (k + 4)^2</math>  <math>\therefore P(k + 1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>
Q.9)	Show that $1^2 + 2^2 + \dots\dots n^2 > \frac{n^3}{3}$ .
Sol.9)	<p>Let <math>P(n)</math>: <math>1^2 + 2^2 + \dots\dots n^2 &gt; \frac{n^3}{3}</math></p> <p>(i) <math>P(1)</math>: <math>1^2 &gt; \frac{1^3}{3}</math>  <math>1 &gt; \frac{1}{3}</math> clearly <math>P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true  <math>P(k)</math>: <math>1^2 + 2^2 + \dots\dots k^2 &gt; \frac{k^3}{3}</math>  (iii) To prove <math>P(k + 1)</math> is true  <math>P(k + 1)</math>: <math>1^2 + 2^2 + \dots\dots k^2 + (k + 1)^2 &gt; \frac{(k+1)^3}{3}</math>  (OR) <math>P(k + 1)</math>: <math>1^2 + 2^2 + \dots\dots k^2 + (k + 1)^2 &gt; \frac{k^3 + 3k^2 + 3k + 1}{3}</math>  We have, <math>1^2 + 2^2 + \dots\dots k^2 &gt; \frac{k^3}{3}</math>  Adding <math>(k + 1)^2</math> on both sides  <math>\Rightarrow 1^2 + 2^2 + 3^2 \dots\dots k^2 + (k + 1)^2 &gt; \frac{k^3}{3} + (k + 1)^2</math>  <math>\Rightarrow 1^2 + 2^2 + 3^2 \dots\dots k^2 + (k + 1)^2 &gt; \frac{k^3 + 3(k^2 + 2k + 1)}{3}</math>  <math>\Rightarrow 1^2 + 2^2 + 3^2 \dots\dots k^2 + (k + 1)^2 &gt; \frac{k^3 + 3k^2 + 6k + 3}{3}</math>  <math>\Rightarrow 1^2 + 2^2 + 3^2 \dots\dots k^2 + (k + 1)^2 &gt; \frac{k^3 + 3k^2 + 6k + 3}{3} &gt; \frac{k^3 + 3k^2 + 3k + 1}{3}</math>  <math>\Rightarrow 1^2 + 2^2 + 3^2 \dots\dots k^2 + (k + 1)^2 &gt; \frac{k^3 + 3k^2 + 3k + 1}{3}</math>  <math>\Rightarrow 1^2 + 2^2 + 3^2 \dots\dots k^2 + (k + 1)^2 &gt; \frac{(k+1)^3}{3}</math>  <math>\therefore P(k + 1)</math> is true</p>



	$\therefore$ by PMI $P(n)$ is true for all $n \in N$ .	
Q.10)	Show by PMI, $1 + 2 + 3 + \dots + n < \frac{1}{8}(2n + 1)^2$	
Sol.10)	<p>let <math>P(n)</math>: <math>1 + 2 + 3 + \dots + n &lt; \frac{1}{8}(2n + 1)^2</math></p> <p>(i) <math>P(1)</math>: <math>1 &lt; \frac{1}{8}(2 + 1)^2</math>  <math>\Rightarrow 1 &lt; \frac{9}{8} = 1.1</math> clearly <math>P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true  <math>P(k)</math>: <math>1 + 2 + 3 + \dots + k &lt; \frac{1}{8}(2k + 1)^2</math></p> <p>(iii) To prove <math>P(k + 1)</math> is true  <math>P(k + 1)</math>: <math>1 + 2 + 3 + \dots + k + (k + 1) &lt; \frac{1}{8}(2k + 3)^2</math>  (OR) <math>P(k + 1)</math>: <math>1 + 2 + 3 + \dots + k(k + 1) &lt; \frac{4k^2+9+12k}{8}</math>  We have, <math>1 + 2 + 3 + \dots + k &lt; \frac{1}{8}(2k + 1)^2</math>  Adding <math>(k + 1)</math> on both sides  <math>\Rightarrow 1 + 2 + 3 + \dots + k + (k + 1) &lt; \frac{1}{8}(2k + 1)^2 + (k + 1)</math>  <math>\Rightarrow 1 + 2 + 3 + \dots + k + (k + 1) &lt; \frac{4k^2+1+4k+8k+8}{8}</math>  <math>\Rightarrow 1 + 2 + 3 + \dots + k + (k + 1) &lt; \frac{4k^2+12k+9}{8}</math>  <math>\Rightarrow 1 + 2 + 3 + \dots + k + (k + 1) &lt; \frac{(2k+3)^2}{8}</math>  Clearly <math>P(k + 1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>	
Q.11)	Prove that $(1 + x)^n \geq (1 + nx)$ for all natural no. $n$ , where $x > -1$ .	
Sol.11)	<p>Let <math>P(n)</math>: <math>(1 + x)^n \geq (1 + nx)</math>, <math>x &gt; -1</math></p> <p>(i) <math>P(1)</math>: <math>(1 + x) \geq (1 + x)</math>  clearly <math>P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true  <math>P(k)</math>: <math>(1 + x)^k \geq (1 + kx)</math></p> <p>(ii) To prove <math>P(k + 1)</math> is true  <math>P(k + 1)</math>: <math>(1 + x)^{k+1} \geq (1 + (k + 1)x)</math>  (OR) <math>(1 + x)^{k+1} \geq (1 + kx + x)</math>  We have, <math>(1 + x)^k \geq (1 + kx)</math>  multiply <math>(1 + x)</math> on both sides  <math>\Rightarrow (1 + x)^k(1 + x) \geq (1 + kx)(1 + x)</math>  <math>\Rightarrow (1 + x)^{k+1} \geq (1 + x + kx + kx^2)</math>  <math>\Rightarrow (1 + x)^{k+1} \geq (1 + x + kx + kx^2) \geq (1 + kx + x) \dots \{\because x &gt; -1 \text{ when } x = 0, kx^2 = 0\}</math>  <math>\Rightarrow (1 + x)^{k+1} \geq (1 + kx + x)</math>  <math>\Rightarrow (1 + x)^{k+1} \geq [1 + (k + 1)x]</math>  Clearly <math>P(k + 1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>	
Q.12)	Prove that $2^n > n$ for all $n \in N$	
Sol.12)	<p>Let <math>P(n)</math>: <math>2^n &gt; n</math></p> <p>(i) <math>P(1)</math>: <math>2^1 &gt; 1</math>  clearly <math>P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true</p>	



$P(k): 2^k > k$ (ii) To prove $P(k + 1)$ is true $P(k + 1): 2k^{k+1} > k + 1$ We have, $2^k > k$ multiply 2 on both sides $\Rightarrow 2^k \cdot 2 > 2k$ $\Rightarrow 2^{k+1} > k + k$ $\Rightarrow 2^{k+1} > k + k \geq k + 1$ $\Rightarrow 2^{k+1} > k + 1$ Clearly $P(k + 1)$ is true $\therefore$ by PMI $P(n)$ is true for all $n \in N$ .	
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