



<b>MATHEMATICAL INDUCTION(PMI)</b> <b>CLASS XI</b>	
Q.1)	Using PMI, show that $1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{(1+2+3+\dots+n)} = \frac{2n}{n+1}$
Sol.1)	<p>Let <math>P(n): 1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{(1+2+3+\dots+n)} = \frac{2n}{n+1}</math></p> <p>(i) <math>P(1): 1 = \frac{2}{1+1} = \frac{2}{2} = 1</math>  <math>\therefore P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true  <math>P(k): 1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{(1+2+3+\dots+k)} = \frac{2k}{k+1}</math></p> <p>(iii) To prove <math>P(k+1)</math> is true  <math>P(k+1): 1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{(1+2+3+\dots+k)} + \frac{1}{(1+2+3+\dots+k+1)} = \frac{2k+2}{k+2}</math></p> <p>Taking L.H.S., <math>1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{(1+2+3+\dots+k)} + \frac{1}{(1+2+3+\dots+k+1)}</math>  <math>= \frac{2k}{k+1} + \frac{1}{(1+2+3+\dots+k+1)} \dots \{from P(k)\}</math>  <math>= \frac{2k}{k+1} + \frac{1}{\frac{(k+1)(k+2)}{2}}</math>  <math>= \frac{2k}{k+1} + \frac{2}{(k+1)(k+2)}</math>  <math>= \frac{2k^2+4k+2}{(k+1)(k+2)}</math>  <math>= \frac{2(k^2+2k+1)}{(k+1)(k+2)}</math>  <math>= \frac{2(k+1)^2}{(k+1)(k+2)}</math>  <math>= \frac{2(k+1)}{k+2}</math> R.H.S.</p> <p><math>\therefore P(k+1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>
Q2)	By PMI show that, $\left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{(2n+1)}{n^2}\right) = (n+1)^2$
Sol.2)	<p>Let <math>P(n): \left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{(2n+1)}{n^2}\right) = (n+1)^2</math></p> <p>(i) <math>P(1): \left(1 + \frac{3}{1}\right) = (1+1)^2</math>  <math>\Rightarrow 4 = 4</math>  <math>\therefore P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true  <math>P(k): \left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{(2k+1)}{k^2}\right) = (k+1)^2</math></p> <p>(iii) To prove <math>P(k+1)</math> is true  <math>P(k+1): \left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{(2k+1)}{k^2}\right) \cdot \left(1 + \frac{2k+3}{(k+1)^2}\right) = (k+2)^2</math></p> <p>Taking L.H.S., <math>\left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \left(1 + \frac{7}{9}\right) \dots \left(1 + \frac{(2k+1)}{k^2}\right) \cdot \left(1 + \frac{(2k+3)}{(k+1)^2}\right)</math>  <math>= (k+1)^2 \left[1 + \frac{(2k+3)}{(k+1)^2}\right] \dots \{from P(k)\}</math>  <math>= (k+1)^2 \left[\frac{(k+1)^2+(2k+3)}{(k+1)^2}\right]</math>  <math>= k^2 + 2k + 1 + 2k + 3</math>  <math>= k^2 + 4k + 4</math>  <math>= (k+2)^2</math> R.H.S.</p> <p><math>\therefore P(k+1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>
Q.3)	Show by PMI, $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$
Sol.3 )	Let $P(n): \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$



	<p>(i) <math>P(1): \frac{1}{2} = 1 - \frac{1}{2}</math>  <math>\Rightarrow \frac{1}{2} = \frac{1}{2}</math>  <math>\therefore P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true  <math>P(k): \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^k} = 1 - \frac{1}{2^k}</math></p> <p>(iii) To prove <math>P(k+1)</math> is true  <math>P(k+1): \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^k} + \frac{1}{2^{k+1}} = 1 - \frac{1}{2^{k+1}}</math></p> <p>Taking L.H.S., <math>\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^k} + \frac{1}{2^{k+1}}</math>  <math>= 1 - \frac{1}{2^k} + \frac{1}{2^{k+1}} \dots \{from\ P(k)\}</math>  <math>= 1 - \left\{ \frac{1}{2^k} - \frac{1}{2^{k+1}} \right\}</math>  <math>= 1 - \left\{ \frac{2-1}{2^{k+1}} \right\}</math>  <math>= 1 - \frac{1}{2^{k+1}} R.H.S.</math></p> <p><math>\therefore P(k+1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>
Q.4)	By PMI, show that $(ab)^n = a^n b^n$
Sol.4)	<p>Let <math>P(n): (ab)^n = a^n b^n</math></p> <p>i) <math>P(1): (ab) = ab</math>  <math>\therefore P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true  <math>P(k): (ab)^k = a^k b^k</math></p> <p>(iii) To prove <math>P(k+1)</math> is true  <math>P(k+1): (ab)^{k+1} = a^{k+1} b^{k+1}</math></p> <p>Taking L.H.S., <math>(ab)^{k+1}</math>  <math>= (ab)^k (ab)</math>  <math>= (a^k b^k)(ab) \dots \{from\ P(k)\}</math>  <math>= (a^k a)(b^k b)</math>  <math>= a^{k+1} b^{k+1} R.H.S.</math></p> <p><math>\therefore P(k+1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>
Q.5)	By PMI, show that $\sin \theta + \sin(2\theta) + \dots + \sin(n\theta) = \frac{\sin\left(\frac{n+1}{2}\theta\right) \cdot \sin\left(\frac{n\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}$
Sol.5)	<p>Let <math>P(n): \sin \theta + \sin(2\theta) + \dots + \sin(n\theta) = \frac{\sin\left(\frac{n+1}{2}\theta\right) \cdot \sin\left(\frac{n\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}</math></p> <p>(i) <math>P(1): \sin \theta = \frac{\sin\left(\frac{1+1}{2}\theta\right) \cdot \sin\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}</math>  <math>\Rightarrow \sin \theta = \sin \theta</math>  <math>\therefore P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true  <math>P(k): \sin \theta + \sin(2\theta) + \dots + \sin(k\theta) + \sin(k+1)\theta = \frac{\sin\left(\frac{k+2}{2}\theta\right) \cdot \sin\left(\frac{(k+1)\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}</math></p> <p>(iii) To prove <math>P(k+1)</math> is true  <math>P(k+1): \sin \theta + \sin(2\theta) + \dots + \sin(k\theta) + \sin(k+1)\theta + \sin(k+2)\theta</math></p> <p>Taking L.H.S., <math>\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^k} + \frac{1}{2^{k+1}}</math>  <math>= \frac{\sin\left(\frac{k+1}{2}\theta\right) \cdot \sin\left(\frac{k\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} + \sin(k+2)\theta \dots \{from\ P(k)\}</math></p>



	$  \begin{aligned}  &= \frac{\sin\left(\frac{k+1}{2}\theta\right) \cdot \sin\left(\frac{k\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} + 2\sin\left(\frac{k+1}{2}\theta\right) \theta \cdot \cos\left(\frac{k+1}{2}\theta\right) \\  &= \sin\left(\frac{k+1}{2}\theta\right) \theta \left[ \frac{\sin\left(\frac{k\theta}{2}\right) + 2\sin\left(\frac{\theta}{2}\right) \cos\left(\frac{k+1}{2}\theta\right)}{\sin\left(\frac{\theta}{2}\right)} \right] \dots \{2\sin A \cos B = \sin(A+B) + \sin(A-B)\} \\  &= \frac{\sin\left(\frac{k+1}{2}\theta\right) \theta}{\sin\left(\frac{\theta}{2}\right)} \left[ \sin\left(\frac{k\theta}{2}\right) + \sin\left(\frac{k+2}{2}\theta\right) - \sin\left(\frac{-k\theta}{2}\right) \right] \\  &= \frac{\sin\left(\frac{k+1}{2}\theta\right) \theta}{\sin\left(\frac{\theta}{2}\right)} \left[ \sin\left(\frac{k\theta}{2}\right) + \sin\left(\frac{k+2}{2}\theta\right) - \sin\left(\frac{k\theta}{2}\right) \right] \\  &= \frac{\sin\left(\frac{k+1}{2}\theta\right) \theta \cdot \sin\left(\frac{k+2}{2}\theta\right)}{\sin\left(\frac{\theta}{2}\right)} \text{ R.H.S.}  \end{aligned}  $ <p><math>\therefore P(k+1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>
Q.6)	Show by using PMI, $\cos \alpha \cdot \cos(2\alpha) \cdot \cos(4\alpha) \dots \dots \cos(2^{n-1}\alpha) = \frac{\sin(2^n\alpha)}{2^n \sin \alpha}$
Sol.6)	<p>Let <math>P(n): \cos \alpha \cdot \cos(2\alpha) \cdot \cos(4\alpha) \dots \dots \cos(2^{n-1}\alpha) = \frac{\sin(2^n\alpha)}{2^n \sin \alpha}</math></p> <p>(i) <math>P(1): \cos \alpha = \frac{\sin(2\alpha)}{2 \sin \alpha}</math>  <math>\Rightarrow \cos \alpha = \frac{2 \sin \alpha \cos \alpha}{2 \sin \alpha}</math>  <math>\Rightarrow \cos \alpha = \cos \alpha</math>  <math>\therefore P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true</p> $P(k): \cos \alpha \cdot \cos(2\alpha) \cdot \cos(4\alpha) \dots \dots \cos(2^{k-1}\alpha) = \frac{\sin(2^k\alpha)}{2^k \sin \alpha}$ <p>(iii) To prove <math>P(k+1)</math> is true</p> $P(k+1): \cos \alpha \cdot \cos(2\alpha) \cdot \cos(4\alpha) \dots \dots \cos(2^{k-1}\alpha) \cdot \cos(2^k\alpha) = \frac{\sin(2^{k+1}\alpha)}{2^{k+1} \sin \alpha}$ <p>Taking L.H.S., <math>\cos \alpha \cdot \cos(2\alpha) \dots \dots \cos(2^{k-1}\alpha) \cdot \cos(2^k\alpha)</math>  <math>= \frac{\sin(2^k\alpha)}{2^k \sin \alpha} \cdot \cos(2^k\alpha) \dots \dots \{from P(k)\}</math></p> <p>Multiply &amp; divide by 2</p> $  \begin{aligned}  &= \frac{1}{2 \cdot 2^k \sin \alpha} + [2 \sin(2^k\alpha) \cdot \cos(2^k\alpha)] \\  &= \frac{1}{2^{k+1} \sin \alpha} + [\sin(2 \cdot 2^k\alpha)] \\  &= \frac{1}{2^{k+1} \sin \alpha} + \sin(2^{k+1}\alpha) \text{ R.H.S.}  \end{aligned}  $ <p><math>\therefore P(k+1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>
Q.7)	By PMI show that $7 + 77 + 777 \dots \dots (777 \dots \dots 7) = \frac{7}{81} [10^{n+1} - 9n - 10]$
Sol.7)	<p>Let <math>P(n): 7 + 77 + 777 \dots \dots (777 \dots \dots 7) = \frac{7}{81} [10^{n+1} - 9n - 10]</math></p> <p>(i) <math>P(1): 7 = \frac{7}{81} [10^2 - 9 - 10]</math>  <math>\Rightarrow 7 = \frac{7}{81} (81) \Rightarrow 7 = 7</math>  <math>\therefore P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true</p> $P(k): 7 + (77) + (777) \dots \dots (777 \dots \dots 7) = \frac{7}{81} [10^{k+1} - 9k - 10]$ <p style="text-align: center;">k-digits</p> <p>(iii) To prove <math>P(k+1)</math> is true</p> <p><math>P(k+1):</math></p>



	$7 + (77) + (777) \dots \dots (777 \dots \dots 7) + (7777 \dots \dots 7) = \frac{7}{81} [10^{k+2} - 9(k+1) - 10]$ <p style="text-align: center;"><i>k digits</i>      <i>(k+1) digits</i></p> <p>Taking L.H.S., <math>7 + 77 + 7 \dots \dots (777 \dots \dots 7) + (7777 \dots \dots 7)</math></p> $= \frac{7}{81} [10^{k+1} - 9k - 10] + \frac{7}{9} [9999 \dots \dots (k+1) \text{times}]$ $= \frac{7}{81} [10^{k+1} - 9k - 10] + \frac{7}{9} [10^{k+1} - 1]$ $= \frac{7}{81} [10^{k+1} - 9k - 10 + 9 \cdot 10^{k+1} - 9]$ $= \frac{7}{81} [10 \cdot 10^{k+1} - 9k - 9 - 10]$ $= \frac{7}{81} [10^{k+1} - 9(k+1) - 10] \text{ R.H.S.}$ <p><math>\therefore P(k+1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>
Q.8)	Prove by Induction $P(n): 1 \times 1! + 2 \times 2! + 3 \times 3! + \dots \dots n \times n! = (n+1)! - 1$
Sol.8)	<p>Let <math>P(n): 1 \times 1! + 2 \times 2! + 3 \times 3! + \dots \dots n \times n! = (n+1)! - 1</math></p> <p>(i) <math>P(1): 1 \times 1! = (1+1)! - 1</math>  <math>\Rightarrow 1 = 2! - 1</math>  <math>\Rightarrow 1 = 2 - 1 \Rightarrow 1 = 1</math></p> <p><math>\therefore P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true</p> <p><math>P(k): 1 \times 1! + 2 \times 2! + 3 \times 3! + \dots \dots k \times k! = (k+1)! - 1</math></p> <p>(iii) To prove <math>P(k+1)</math> is true</p> <p><math>P(k+1): 1 \times 1! + 2 \times 2! + 3 \times 3! + \dots \dots k \times k! + (k+1)(k+1)! = (k+2)! - 1</math></p> <p>Taking L.H.S., <math>1 \times 1! + 2 \times 2! + 3 \times 3! + \dots \dots k \times k! + (k+1)(k+1)!</math>  <math>= (k+1)! - 1 + (k+1)(k+1)! \dots \dots \{from P(k)\}</math>  <math>= (k+1)! \{k+1+1\} - 1</math>  <math>= (k+1)! (k+2) - 1</math>  <math>= (k+2)! - 1 \text{ R.H.S.}</math></p> <p><math>\therefore P(k+1)</math> is true  <math>\therefore</math> by PMI <math>P(n)</math> is true for all <math>n \in N</math>.</p>
Q.9)	<p>Prove by PMI <math>\sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots \dots \sin(\alpha + (n-1)\beta) =</math></p> $\frac{\sin(\alpha + \frac{(n-1)}{2}\beta) \cdot \sin(\frac{n\beta}{2})}{\sin(\frac{\beta}{2})}$
Sol.9)	<p>Let <math>P(n): \sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots \dots \sin(\alpha + (n-1)\beta)</math></p> $= \frac{\sin\left(\alpha + \frac{(n-1)}{2}\beta\right) \cdot \sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$ <p>(i) <math>P(1): \sin \alpha = \frac{\sin(\alpha+0\beta) \cdot \sin\left(\frac{\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}</math>  <math>\Rightarrow \sin \alpha = \sin \alpha</math></p> <p><math>\therefore P(1)</math> is true</p> <p>(ii) let <math>P(k)</math> be true</p> <p><math>P(k): \sin \alpha + \sin(\alpha + \beta) + \dots \dots \sin(\alpha + (k-1)\beta) = \frac{\sin\left(\alpha + \frac{(k-1)}{2}\beta\right) \cdot \sin\left(\frac{k\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}</math></p> <p>(iii) To prove <math>P(k+1)</math> is true</p>



$$P(k+1): \sin \alpha + \sin(\alpha + \beta) + \dots + \sin(\alpha + (k-1)\beta) + \sin(\alpha + k\beta)$$

$$= \frac{\sin\left(\alpha + \frac{k\beta}{2}\right) \cdot \sin\left((k+1)\frac{\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

Taking L.H.S.,  $\sin \alpha + \sin(\alpha + \beta) + \dots + \sin(\alpha + (k-1)\beta) + \sin(\alpha + k\beta)$

$$= \frac{\sin\left(\alpha + \frac{(k-1)\beta}{2}\right) \cdot \sin\left(\frac{k\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} + \sin(\alpha + k\beta)$$

$$= \frac{\sin\left(\alpha + \frac{(k-1)\beta}{2}\right) \cdot \sin\left(\frac{k\beta}{2}\right) + \sin(\alpha + k\beta) \cdot \sin\left(\frac{\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

$$= \frac{2\sin\left(\alpha + \frac{(k-1)\beta}{2}\right) \cdot \sin\left(\frac{k\beta}{2}\right) + 2\sin(\alpha + k\beta) \cdot \sin\left(\frac{\beta}{2}\right)}{2\sin\left(\frac{\beta}{2}\right)}$$

$$= \frac{\cos\left(\alpha + \frac{(k-1)\beta - k\beta}{2}\right) - \cos\left(\alpha + \frac{(k-1)\beta + k\beta}{2}\right) + \cos\left(\alpha + k\beta - \frac{\beta}{2}\right) - \cos\left(\alpha + k\beta + \frac{\beta}{2}\right)}{2\sin\left(\frac{\beta}{2}\right)}$$

$$= \frac{\cos\left(\alpha - \frac{\beta}{2}\right) - \cos\left(\alpha + k\beta + \frac{\beta}{2}\right) + \cos\left(\alpha + k\beta - \frac{\beta}{2}\right) - \cos\left(\alpha + k\beta + \frac{\beta}{2}\right)}{2\sin\left(\frac{\beta}{2}\right)}$$

$$= \frac{\cos\left(\alpha - \frac{\beta}{2}\right) - \cos\left(\alpha + k\beta + \frac{\beta}{2}\right)}{2\sin\left(\frac{\beta}{2}\right)}$$

$$= \frac{\sin\left(\frac{\alpha - \frac{\beta}{2} + \alpha + k\beta + \frac{\beta}{2}}{2}\right) \cdot \sin\left(\frac{\alpha - \frac{\beta}{2} - \alpha - k\beta - \frac{\beta}{2}}{2}\right)}{2\sin\left(\frac{\beta}{2}\right)}$$

$$= \frac{-\sin\left(\alpha + \frac{k\beta}{2}\right) \cdot \sin\left(-\frac{\beta}{2}(k+1)\right)}{\sin\left(\frac{\beta}{2}\right)}$$

$$= \frac{\sin\left(\frac{\alpha + k\beta}{2}\right) \cdot \sin\left(\frac{(k+1)\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \text{ R.H.S.}$$

$\therefore P(k+1)$  is true

$\therefore$  by PMI  $P(n)$  is true for all  $n \in N$ .

Q.10) Prove by Induction that  $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$  is a natural number for all  $n \in N$

Sol.10) Let  $P(n): \frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$  is natural number

$$\text{(i) } P(1): \frac{1^5}{5} + \frac{1^3}{3} + \frac{7}{15} = \frac{1}{5} + \frac{1}{3} + \frac{7}{15}$$

$$\Rightarrow \frac{3+5+7}{15} = \frac{15}{15}$$

$$= 1 \text{ which is a natural number}$$

$\therefore P(1)$  is true

(ii) let  $P(k)$  be true

$$P(k): \frac{k^5}{5} + \frac{k^3}{3} + \frac{7k}{15} = \pi \dots \quad (\pi \in N)$$

(iii) To prove  $P(k+1)$  is true

$$\begin{aligned} P(k+1): & \frac{(k+1)^5}{5} + \frac{(k+1)^3}{3} + \frac{7(k+1)}{15} \\ &= \frac{1}{5}[5c_0 k^5 1^0 + 5c_1 k^4 (1)^1 + 5c_2 k^3 + 5c_3 k^2 + 5c_4 k^1 + 5c_5 k^0] + \frac{1}{3}[3c_0 k^3 + \\ & 3c_1 k^2 + 3c_2 k^1 + 3c_3 k^0] + \frac{7k+7}{15} \\ &= \frac{1}{5}[k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1] + \frac{1}{3}[k^3 + 3k^2 + 3k + 1] + \frac{7k}{15} + \frac{7}{15} \\ &= \left(\frac{k^5}{5} + \frac{k^3}{3} + \frac{7}{15} k\right) [k^4 + 2k^3 + 3k^2 + 2k] \frac{1}{5} + \frac{1}{3} + \frac{7}{15} \\ &= \pi + k^4 + 2k^3 + 3k^2 + 2k + 1 \end{aligned}$$

Which is a natural number



	$\therefore P(k + 1)$ is true $\therefore$ by PMI $P(n)$ is true for all $n \in N$ .	
Q.11)	If $P(n): 2 \cdot 4^{2n+1} + 3^{3n+1}$ is divisible by $\pi$ for all $n \in N$ is true then the value of $\pi$	
Sol.11)	$P(n): 2 \cdot 4^{2n+1} + 3^{3n+1}$ For $n = 1$ $P(1): 2 \cdot 4^3 + 3^4 = 2 \times 64 + 81 = 209$ For $n = 2$ $P(2): 2 \times 4^5 + 3^7 = 2048 + 2187 = 4235$ HCF of $P(1)$ & $P(2)$ i.e., 209 & 4235 is 11 $\therefore P(n)$ is divisible by 11 $\therefore \pi = 11$	
Q.12)	If $P(n): 49^n + 16^n + k$ is divisible by 64 is true, then find the least negative integral value of $k$	
Sol.12)	$P(n): 49^n + 16^n + k$ For $n = 1$ $P(1): 49 + 16 + k = 65 + k$ $k$ must be equal to $-1$ Since $65 - 1 = 64$ which is divisible by 64 ans.	