	MATHEMATICAL INDUCTION(PMI) ***Class XI***
	TYPE: DIVISIBILITY
Q.1)	By principle of mathematical induction show that $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \in N$.
Sol.1)	Let $P(n): 3^{2n+2} - 8n - 9$ (i)let $P(1): 3^{2+2} - 8n - 9 = 3^4 - 17 = 64$ which is divisible by 8 $\therefore P(1)$ is true. (ii) let $P(k): 3^{2k+2} - 8k - 9 = 8m$ where $m \in N$ (iii) To prove $P(k+1)$ is true $P(k+1): 3^{2(k+1)} - 8(k+1) - 9$ $= 3^{2k+4} - 8k - 17$ $= 3^{2k+2}.3^2 - 8k - 17$ $= [8m + 8k + 9].9 - 8k - 17$ ………. $\{from\ P(k)\}$ $= 72m + 72k + 81 - 8k - 17$ $= 72m + 64k + 64$ $= 8(9 + 8k + 8)$ which is divisible by 8 $\therefore$ by principle of Mathematical Induction $P(n)$ is true for all $n \in N$.
Q.2)	Prove by PMI, 3^{2n} when divided by 8, the remainder is always 1.
Sol.2)	Let $P(n): 3^{2n}$ when divided by 8 leaves remainder 1 (i) let $P(1): 3^2 = 9 = 8 + 1$ Clearly P(1) is true. (ii) let $P(k)$ be true $P(k): 3^{2k} = 8m + 1$ ……… $\{m \in N\}$ (iii) To prove $P(k+1)$ is true $P(k+1): 3^{2(k+2)}$ $= 3^{2k}.3^2$ $= (8m + 1).9$………. $\{from\ P(k)\}$ $= 72m + 9$ $= 8(9m + 1) + 1$ clearly it leaves remainder 1 when divided by 8 $\therefore P(k+1)$ is true $\therefore$ by PMI $P(n)$ is true for all $n \in N$.
Q.3)	By PMI, show $x^{2n} - y^{2n}$ is divisible by $x + y$.
Sol.3)	$P(n): x^{2n} - y^{2n}$ is divisible by $x + y$ (i) $P(1): x^2 - y^2$ $= (x + y)(x - y)$ clearly it is divisible by $x + y$ $\therefore P(1)$ is true (ii) let $P(k)$ be true i.e, $P(k): x^{2k} - y^{2k} = (x + y)m$ ……… $\{m \in N\}$ (iii) To prove $P(k+1)$ is true $P(k+1): x^{2k+2} - y^{2k+2}$ $= x^{2k}.x^2 - y^{2k}.y^2$ $= [(x + y)m + y^{2k}]x^2 - y^{2k}.y^2$………. $\{from\ (1)\}$ $= (x + y)mx^2 + y^{2k}.x^2 - y^{2k}.y^2$ $= (x + y)mx^2 + y^{2k}(x^2 - y^2)$ $= (x + y)[mx^2 + y^{2k}(x - y)]$ clearly it is divisible by $(x + y)$ $\therefore P(k+1)$ is true

	$\therefore$ by PMI $P(n)$ is true for all $n \in N$.	
Q.4)	Show by PMI, $11^{n-12} + 12^{2n+1}$ is multiple of 133.	
Sol.4)	Let $P(n)$: $11^{n-12} + 12^{2n+1}$ (i) $P(1)$: $11^3 + 12^3$ $= 1331 + 1728 = 3059 = 23 \times 133$ which is divisible 133 Clearly, $P(1)$ is true (ii) let $P(k)$ be true $P(k): 11^{k+2} - 12^{2k+1} = 133m$ .... $m \in N$ (iii) To prove $P(k+1)$ is true $P(k+1): 11^{k+3} - 12^{2k+3}$ $= 11^{k+2}.11 + 12^{2k+1}.12^2$ $= (133m - 12).11 + 12^{2k+1}.144$ $= 133m \times 11 - 12^{2k+1}.11 + 12^{2k+1}.144$ $= 133m \times 11 + 12^{2k+1}(144 - 11)$ $= 133m \times 11 + 12^{2k+1}.133$ $= 133[11m + 12^{2k+1}]$ which is divisible by 133 $\therefore P(k+1)$ is true $\therefore$ by PMI $P(n)$ is true for all $n \in N$.	
Q.5)	By PMI, show that $2.7^n + 3.5^n - 5$ is divisible by 24.	
Sol.5)	Let $P(n): 2.7^n + 3.5^n - 5$ is divisible by 24 (i) $P(1): 2.7 + 3.5 - 5$ $= 14 + 15 - 5 = 24$ which is divisible by 24 $\therefore P(1)$ is true (ii) let $P(k)$ be true $P(k): 2.7^k + 3.5^k - 5 = 24m$......... $m \in N$ (iii) To prove $P(k+1)$ is true $P(k+1): 2.7^{k+1} + 3.5^{k+1} - 5$ $= 2.7^k.7 + 3.5^k.5 - 5$ $= \left(24m - 3.5^k + 5\right).7 + 15.5^k - 5$ $= 24m \times 7 - 21.5^k + 35 + 15.5^k - 5$ $= 24m \times 7 + 30 - 6.5^k$ $= 24m \times 7 - 6\left(5^k - 5\right)$ Now, $\left(5^k - 5\right)$ is always divisible / multiple of 4 for all values of $k \in N$ e.g., $k = 1: 5^1 - 5 = 0, 4 \times 0$ $k = 2: 5^2 - 5 = 25 - 5 = 20 = 4 \times 5$ $k = 3: 5^3 - 5 = 125 - 5 = 120$ *and so on* Clearly for all values $k, 5^k - 5$ is always divisible by 24 $P(k+1): 24m \times 7 - 6(4p)$ .... Where $p \in N$ $= 24(7m - p)$ which is divisible by 24 $\therefore P(k+1)$ is true $\therefore$ by PMI $P(n)$ is true for all $n \in N$.	
Q.6)	Show by PMI, $n(n+1)(n+5)$ is multiple of 3.	
Sol.6)	Let $P(n): n(n+1)(n+5)$ is multiple of 3 (i) $P(1) = 1(1+1)(1+5) = (2)(6) = 12$ which is multiple of 3 $\therefore P(1)$ is true (ii) let $P(k)$ be true $P(k): k(k+1)(k+5) = 3m$......... $m \in N$	

	$\Rightarrow P(k): k(k^2+6k+5)=3m$ $\Rightarrow P(k): k^3+6k^2+5k=3m$ .... (1) (ii) To prove $P(k+1)$ is true $P(k+1): (k+1)(k+2)(k+6)$ $= (k+1)(k^2+8k+12)$ $= k^3+8k^2+12k+k^2+8k+12$ $= k^3+9k^2+20k+12$ $= (k^3+6k^2+5k)+(15k+12)$ $= 3m+15k+12$ .... $\{from\ (1)\}$ $= 3(m+5k+4)$ which is a multiple of 4 $\therefore P(k+1)$ is true $\therefore$ by PMI $P(n)$ is true for all $n \in N$.	
Q.7)	Prove by induction that the sum of the cubes of the consecutive natural numbers is divisible by 9.	
Sol.7)	Let three consecutive natural no.s are $n, (n+1), (n+2)$ Let $P(n): n^3+(n+1)^3+(n+2)^3$ is divisible by 9 (i) $P(1): 1^3+(1+1)^3+(1+2)^3 = 1+8+27 = 36$ which is divisible by 9 $\therefore P(1)$ is true (ii) let $P(k)$ be true i.e., $P(k): k^3+(k+1)^3+(k+2)^3 = 9m$ .... $m \in N$ (OR) $P(k): 3k^3+9k^2+15k+9=9m$ (iii) To prove $P(k+1)$ is true $P(k+1): (k+1)^3+(k+2)^3+(k+3)^3$ $= k^3+3k^2+3k+1+k^3+8k^2+12k+8+k^3+9k^2+27k+27$ $= 3k^3+18k^2+42k+36$ $= (3k^3+9k^2+15k+9)+(9k^2+27k+27)$ $= 9m+9k^2+27k+27$ .... $\{from\ (1)\}$ $= 9(m+k^2+3k+3)$ which is divisible by 9 $\therefore P(k+1)$ is true $\therefore$ by PMI $P(n)$ is true for all $n \in N$.	