

J.E.E. Main/ Advanced Foundation - XI Maths Worksheet**Time: 60 min****Chapter#4 : Principle of Mathematical Induction****Full Marks:**Q.1 Let P (n) be the statement $n(n + 1)$ is an even number then find P(6). (1 mark)Q.2 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: (5 marks)

$$1.3 + 2.3^2 + 3.3^3 + \dots + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$$

Q.3 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: (5 marks)

$$1 + \frac{1}{(1+2)} + \frac{1}{(1+2+3)} + \dots + \frac{1}{(1+2+3+\dots+n)} = \frac{2n}{(n+1)}$$

Q.4 Prove by using the principle of mathematical induction $3^{2n} - 1$ is divisible by 8 for $n \in \mathbb{N}$. (3 marks)Q.5 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: (3 marks)

$$1 + 3 + 3^2 + \dots + 3^{n-1} = \frac{(3^n - 1)}{2}$$

Q.6 Prove by using the principle of mathematical induction for all $n \in \mathbb{N}$.

$$1.3 + 3.5 + 5.7 + \dots + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}$$

Q.7 Prove by using the principle of mathematical induction for all $n \in \mathbb{N}$:

$$1 + 3 + 3^2 + \dots + 3^{n-1} = \frac{3^n - 1}{2}$$

Q.8 Prove that the product of two consecutive natural numbers is an even number. (3 marks)

Q.9 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$. (3 marks)

$$(2n+7) < (n+3)^2$$

Q.10 Use the principle of mathematical induction to prove that $1+5+9+13+\dots+(4n-3) = n(2n-1)$, $n \in \mathbb{N}$ Q.11 Prove that : $2.7^n + 3.5^n - 5$ is divisible by 24 for all $n \in \mathbb{N}$.Q.12 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: (3 marks)

$$\left(1 + \frac{3}{1}\right)\left(1 + \frac{5}{4}\right)\left(1 + \frac{7}{9}\right)\dots\left(1 + \frac{(2n+1)}{n^2}\right) = (n+1)^2$$

Q.13 Prove by using the principle of mathematical induction $3^n < 4^n$ for all $n \in \mathbb{N}$. (3 marks)Q.14 For every positive integer n, prove that $7^n - 3^n$ is divisible by 4.Q.15 prove by using the principle of mathematical induction for $n \in \mathbb{N}$: $(2n+7) < (n+3)^2$

Q.16 Prove the following by using the principle of mathematical induction for all $n \in \mathbb{N}$: (3 marks)

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

Q.17 Suppose $P(n)$: $n(n+1)(n+2)$ is divisible by 6. Show that $P(1)$, $P(2)$ and $P(3)$ are true. (1 mark)

Q.18 Let $P(n)$ be the statement $n^2 > 25$, prove that whenever $P(k)$ is true, $P(k + 1)$ is also true. (2 marks)

Q.19 Let $P(n)$ be the statement " $n^2 - n + 41$ is prime". Show that $P(1)$, $P(2)$, $P(3)$ are true whereas $P(41)$ is not true. (2 marks)

Q.20 Explain the principle of mathematical induction. (1 mark)