## StudiesToday

|  | COMBINATION |
| :---: | :---: |
| Q.36) | 4 cards out of 52 cards are chosen. Find no. of ways in which : |
| Sol.36) | 1. 4 cards are chosen:- <br> (i) 4 cards out of 52 cards can be chosen in $={ }^{52} C_{4}$ ways $=\frac{52!}{4!48!}=270725$ ans. <br> 2. 4 cards out of same suit: <br> (i) There are 4 suits <br> Diamond Club Heart Spade <br> (13) <br> (13) (13) <br> (13) <br> (ii) No. of ways of selecting, 4 diamonds out of 13 diamond cards $={ }^{13} \mathrm{C}_{4}$ <br> (iii) Similarly, ${ }^{13} C_{4}$ ways for selecting 4 spade, 4 clubs $\& 4$ heart <br> (iv) $\therefore$ required no. of ways of selection $={ }^{13} \mathrm{C}_{4}+{ }^{13} \mathrm{C}_{4}+{ }^{13} \mathrm{C}_{4}+{ }^{13} \mathrm{C}_{4}=4 \times{ }^{13} \mathrm{C}_{4}=2860$ ans. <br> 3. 4 cards belong to 4 different suits <br> (i) We have to select 1 card from each suit <br> (ii) 1 diamond out of 13 diamonds can be selected in $={ }^{13} \mathrm{C}_{1}$ ways <br> (iii) Similarly, ${ }^{13} \mathrm{C}_{1}$ is the no. of ways of selecting 1 club, 1 heart \& 1 spade <br> (iv) $\therefore$ required no. of selection $=13 C 1 \times 13 C 1 \times 13 C 1 \times 13 C 1=13 \times 13 \times 13 \times 13=$ $(13)^{4}=28561$ ans. <br> 4. All are face cards <br> (i) There are 12 face cards ( $4 \mathrm{~J}, 4 \mathrm{Q}, 4 \mathrm{~K}) \backslash 4$ face cards out of 12 face cards can be selected in ${ }^{12} \mathrm{C}_{4}$ ways $=\frac{12!}{4!8!}=495$ ans. <br> 5. Two are red \& two are black: <br> (i) Red cards $=26$, black cards $=26$ <br> (ii) 2 red cards out of 26 red cards can be selected in $={ }^{26} \mathrm{C}_{2}$ ways <br> (iii) 2 black cards out of 26 can be selected in $={ }^{26} \mathrm{C}_{2}$ ways <br> (iv) Required no. of selections $={ }^{26} C_{2} \times{ }^{26} C_{2}=\frac{26!}{2!24!} \times \frac{26!}{2!24!}=325 \times 325=105625$ ans. <br> 6. 4 cards are of same colour:- <br> (i) 2 cases: either they all are red \& all are black <br> (ii) 4 red cards out of 26 red cards can be selected in $={ }^{26} \mathrm{C}_{4}$ ways <br> (iii) 4 black cards out of 26 black cards can be selected in ${ }^{26} \mathrm{C}_{4}$ ways <br> (iv) $\therefore$ required no. of ways of selection $={ }^{26} \mathrm{C}_{4}+{ }^{26} \mathrm{C}_{4}=\frac{26!}{4!22!}+\frac{26!}{4!22!}=14950+14950$ $=29900$ ans. |
| Q.37) | A group consisting of 4 girls $\& 7$ boys. In how many way 5 members are selected such that the team consists: |
| Sol.37) | 1. No girls <br> (i) Since no girl are to be selected, the remaining 5 are to selected from 7 boys <br> (ii) Which can be selected in $={ }^{7} C_{5}$ ways $={ }^{7} C_{5}={ }^{7} C_{2}=\frac{7 \times 6}{2}=21$ ans. <br> 2. At least 3 boys <br> Three cases: <br> Case: 1) selecting 3 boys $\& 2$ girls which can be selected in $=7 c_{3} \times 4 c_{2}$ ways $=35 \times 6=$ 210 <br> Case: 2) selecting 4 boys \& 1 girl which can be selected in $7 c_{3} \times 4 c_{2}$ ways $=35 \times 4=180$ <br> Case: 3) selecting 5 boys \& no girl which can selected in $=7 c_{5} \times 4 c_{0}$ ways $=21 \times 1=21$ $\therefore$ required no. of ways of selection $=210+180+21=411$ ans. |

##  StudiesToday

|  | 3. At most 2 boys: <br> Case:1) selecting 2 boys $\& 3$ girls which can be selected in $=7 c_{2} \times 4 c_{3}$ ways $=21 \times 4=$ 84 <br> Case: 4) selecting 1 boy 4 girls which can be selected in $7 c_{1} \times 4 c_{4}$ ways $=7 \times 1=7$ <br> $\therefore$ required no. of ways of selection $=84+7=91$ ans. <br> 4. At least 1 boy \& 1 girl <br> Case:1) selecting 1 boy 4 girls which can be selected in $=7 c_{1} \times 4 c_{4}$ ways $=7 \times 1=$ 7 <br> Case:2) selecting 2 boys 3 girls which can be selected in $=7 c_{2} \times 4 c_{3}$ ways $=21 \times 4$ $=84$ <br> Case:3) selecting 3 boys \& 2 girls which can be selected in $=7 c_{3} \mathrm{x} 4 c_{2}$ ways $=35 \mathrm{x}$ $6=210$ <br> Case: 4 ) selecting 4 boys $\& 1$ girl which can be selected in $7 c_{4} \times 4 c_{1}$ ways $=35 \times 4=$ 180 <br> $\therefore$ required no. of ways of selection $=$ case $: 1+$ case $: 2+$ case $: 3+$ case $: 4$ <br> $=7+84+210=180=481$ ans. <br> 5. At most 1 girl is chosen <br> (7) (4) <br> Case:1) selecting 4 boys \& 1 girl which can be selected in $=4 c_{1} \times 7 c_{4}$ ways $=4 \mathrm{x}$ $35=180$ <br> Case:2) selecting NO girl \& 5 boys which can be selected in $=4 c_{6} \times 7 c_{5}$ ways $=1 \mathrm{x}$ $21=21$ <br> $\therefore$ required no. of ways of selections $=180+21=201$ <br> 6. A particular boy \& a particular girl is always chosen:- <br> (i) Let the particular boy is A girl is B <br> (ii) They are selected only in 1 way (as they are always selected) <br> (iii) Now we have to select 3 persons from the remaining 11 persons <br> (iv) Which can be selected in $=11 C_{3}$ ways $=165$ ans. |
| :---: | :---: |
| Q.38) | A polygon has $n$ sides. Find the number of diagonals? |
| Sol.38) | (i) A polygon having $n$ sides has $n$ vertices <br> (ii) Total number of lines that can be drawn using $n$ vertices (points) $={ }^{n} C_{2}$ <br> (iii) Then ${ }^{\mathrm{n}} \mathrm{C}_{2}$ lines also contain n -sides <br> (iv) $\therefore$ the number of diagonals ${ }^{\mathrm{n}} \mathrm{C}_{2}-\mathrm{n}=\frac{n(n-1)}{2}-n=\frac{n^{2}-n-2 n}{2}=\frac{n^{2}-3 n}{2}$ ans. |
| Q.39) | A polygon has 44 diagonals. Find the number of sides? |
| Sol.39) | (i) We know that the total no. of diagonals having n -sides $=\frac{n^{2}-3 n}{2}$ (from q.38) <br> (ii) Given: no. of diagonals 44 $\therefore \frac{n^{2}-3 n}{2}=44$ |

## StudiesToday

|  | $\begin{aligned} & \Rightarrow n^{2}-3 n-88=0 \\ & \Rightarrow(n-11)(n+8)=0 \\ & \Rightarrow n=11 \\ & \Rightarrow n=-8 \text { (no. of sides can never be }-n) \end{aligned}$ <br> $\therefore$ there are 11 sides in the polygon |  |
| :---: | :---: | :---: |
| Q.40) | There are 10 points in a plane, out of which 4 points are collinear. Find no. of straight lines \& no. of triangles? |  |
| Sol.40) | 1. Total no. of straight lines using 10 points $=10 c_{2}$ <br> (i) No. of straight line using 4 points $=4 c_{2}$ <br> (ii) But 4 collinear points, when join pair wise gives only 1 straight line <br> (iii) $\therefore$ required no. of straight lines $=10 c_{2}-4 c_{2}+1=45-6+1=40$ ans. <br> 2. Total no. of triangles using 10 points $=10 c_{3}$ <br> (i) No. of triangles using 4 points $=4 c_{3}$ <br> (ii) But 4 collinear points cannot form a triangle <br> (iii) $\therefore$ required no. of triangles $=10_{3}-4 c_{3}=120-4=116$ ans. |  |
| Q.41) | There are ' $m$ ' no. of horizontal parallel lines \& ' $n$ ' no. of vertical parallel lines. How many no. of parallelogram can be formed? |  |
| Sol.41) | (i) To form a parallelogram, we require two horizontal lines \& two vertical lines <br> (ii) Now two horizontal lines out of 'm horizontal' lines can be selected in = $m c_{2}$ ways <br> (iii) Two vertical lines out of ' $n$ vertical' lines can be selected $\mathrm{in}=n c_{2}$ ways <br> (iv) $\therefore$ the required no. of triangles $=m c_{2} \times n c_{2}$ |  |
| Q.42) | From a class of 25 students, 10 are to be chosen for a party. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can they be chosen? |  |
| Sol.42) | There are two cases <br> Case:1) three particular students join the party:- <br> (i) Now we have to select 7 student from the remaining 22 students <br> (ii) Which can be selected in ${ }^{22} \mathrm{C}_{7}$ ways <br> Case:2) three particular students do not join the party:- <br> (i) Now we have to choose 10 students from the remaining 22 students <br> (ii) Which can be selected in $={ }^{22} \mathrm{C}_{10}$ ways <br> $\therefore$ required no. of ways of selection $=$ case: $1+$ case: $2={ }^{22} \mathrm{C}_{7}+{ }^{22} \mathrm{C}_{10}$ $=\frac{22!}{7!15!}+\frac{22!}{10!12!}=817190 \mathrm{ans}$ |  |
| Q.43) | A boy has 3 library tickets and 8 books of his interest in the library of these 8 books; he does not want to borrow chemistry part 2 , unless chemistry part 1 is also borrowed. In how many ways can he choose the three books? |  |
| Sol.43) | There are 2 cases:- <br> Case:1) when chemistry part 1 is borrowed :- <br> (i) Now, he has to select 2 books out of the remaining 7 books <br> (ii) Which can be selected in $=7 c_{2}$ ways <br> Case:2) when chemistry part 1 is not borrowed:- <br> (i) Then, he does not want to borrow chemistry part 2 <br> (ii) Now, he has to select 3 books out of the remaining 6 books <br> (iii) Which can be selected in $6 c_{3}$ ways <br> $\therefore$ required no. of ways of selection $=$ case: $1+$ case: 2 <br> $=7 c_{2}+6 c_{3}=21+20=41$ ans. |  |
| Q.44) | A box contains 5 red balls \& 5 black balls. In how many ways 6 balls be selected such that: |  |
| Sol.44) | 1. There are exactly 2 red balls <br> 2. At least 3 red balls |  |


|  | 3. At least 2 red balls <br> 4. At least 2 balls from each colour <br> 5. No. of black balls \& no. of white balls are equal <br> 6. Red balls are in majority |  |
| :---: | :---: | :---: |
| Q.45) | If $2 n_{C_{3}}: n_{C_{3}}=11: 1$, find $n$ ? |  |
| Sol.45) | We have, $\frac{2 n_{C_{3}}}{n_{C_{3}}}=\frac{11}{1}$ $\begin{aligned} & \Rightarrow \frac{\frac{(2 n)!}{3!(2 n-3)!}}{\frac{n!}{3!(n-2)!}}=\frac{11}{1} \ldots \ldots . . . . . . . . . . . . . . . . . . . . . .\left\{n_{C_{3}}=\frac{n!}{\mathrm{r}!(\mathrm{n}-\mathrm{r})!}\right\} \\ & \Rightarrow \frac{(2 \mathrm{n})!(\mathrm{n}-3)!}{(2 \mathrm{n}-3)!\mathrm{n}!}=11 \\ & \Rightarrow \frac{(2 \mathrm{n})(2 \mathrm{n}-1)(2 \mathrm{n}-2)(2 \mathrm{n}-3)!(\mathrm{n}-3)!}{(2 \mathrm{n}-3)!\mathrm{n}(\mathrm{n}-1)(\mathrm{n}-2)(\mathrm{n}-3)!}=11 \\ & \Rightarrow \frac{2(2 n-1)(2 n-2)}{(n-1)(n-2)}=11 \\ & \Rightarrow \frac{4(2 n-1)(n-1)}{(n-1)(n-2)}=11 \\ & \Rightarrow 8 \mathrm{n}-4=11 \mathrm{n}-22 \\ & \Rightarrow 3 \mathrm{x}=18 \\ & \Rightarrow \mathrm{n}=6 \text { ans. } \end{aligned}$ |  |
| Q.46) | If $2 n_{C_{3}}: n_{C_{2}}=44: 3$, find $n$ ? |  |
| Sol.46) |  | n $=$ 6 |

