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PERMUTATIONS & COMBINATIONS

BASICS:-

Permutations: No of ways of arrangement of objects.
Combinations: No of ways of selection of objects.

$^nP_r = \frac{n!}{(n-r)!}$

where, n→ No of items /objects available.
r→ No of items /objects to be arranged.

$$^nC_r = \frac{n!}{r!(n-r)!}$$

where, n→ No of items /objects available.
r→ No of items /objects to be selected.

Relation between ^nP_r and ^nC_r :-
$^nP_r = {}^nC_r \times r!$

Note: Mainly two operations:-
Addition (+): when or/option/cases.
Multiplication (x) : when and/ compulsion/selection or arrangement not completed.
Shortcuts of ^nC_r :-

$^nC_0 = 1$	E.g.	$^7C_0 = 1.$
$^nC_1 = n$	E.g.	$^7C_1=7.$
$^nC_2 = \frac{n(n-1)}{2}$	E.g.	$^7C_2=\frac{7x6}{2}=21$
$^nC_3 = \frac{n(n-1)(n-2)}{6}$	E.g.	$^7C_3=\frac{7x6x5}{6}=35$
$^nC_n = 1$	E.g.	$^7C_7=7$

$^nC_r = {}^nC_{n-r}$
E.g. $^{10}C_8 = {}^{10}C_2$; $^{20}C_{19} = {}^{20}C_1$
If $^nC_x = {}^nC_y$ then x=y = x (or) x=y

Q.1) How many 3 digit even numbers can be made using the digits 1,2,3,4,6,7. If no digit is repeated?

Sol.1) Digits available: 1,2,3,4,6,7
Required: 3-digit even number
(2,4,6)

5	4	3

1. The last place (unit place) can be filled in 3 ways.
2. The first place (hundred) can be filled in 5 ways.
3. The middle place (ten's) can be filled in 4 ways

∴ the required no of 3-digit even numbers that can be formed = 5x4x3 = 60 ans.

Q.2) How many 6-digit numbers can be formed from the digits 0,1,3,5,7,9 . Which are divisible by 10 when:-
(i) Repeat of digits not allowed
(ii) Repeat of digits allowed

Sol.2) Digits available 0,1,3,5,7,9
Required: 6- digit no.s divisible by 10
1. when repeated of digits not allowed:-

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						(0)	
		5	4	3	2	1	1

(i) for no.s divisible by 10, the last digit must be zero
(ii) The last place (unit place) can be filled in 1 ways.
(iii) The first place can be filled in 5 ways.
(iv) The second place can be filled in 4 ways
(v) The third place can be filled in 3 ways
(vi) The fourth place can be filled in 2 ways
(vii) The fifth place can be filled in 1 way

∴ the required 6 digit no.s which are divisible by 10 when repeated not allowed = 5x4x3x2xx1x1= 120 ans.

2. when repeated of digits allowed:-

(0)

5	6	6	6	6	1

The required no.s are = 5x6x6x6x6x1 = 6360 ans.

Q.3) How many 4-digit numbers divisible by 4 can be made with the digits 1,2,3,4,5. If repetitions of digits not allowed?

Sol.3) Digits available : 1,2,3,4,5
Required 4 digits no.s divisible by 4

1. for no.s divisible by 4, the last two digits must be divisible by 4
2. so they can be 12,32,24 and 52
3. so there are 4 cases;
   (i) the numbers ending with 12

		(1)	(2)	
3	2	1	1	= 3 x 2 x 1 x 1 = 6

   (ii) the numbers ending with 24

		(2)	(4)	
3	2	1	1	= 3 x 2 x 1 x 1 = 6

   (iii) the numbers ending with 32

		(3)	(2)	
3	2	1	1	= 3 x 2 x 1 x 1 = 6

   (iv) the numbers ending with 52

		(5)	(2)	
3	2	1	1	= 3 x 2 x 1 x 1 = 6

∴ The required numbers which are divisible by 4 = 6 + 6 + 6 + 6= 24 ans.

Q.4) How many numbers between 100 and 1000 which have exactly one of their digit is 7?

Sol.4) Required 3 digit no.s having its exactly one digit is 7
Digits available 0 to 9
Since its not mention, we take repeated of digits allowed
There are 3 cases:

1. let 7 in the unit place

		(7)
8	9	1

no.s that can be formed = 8 x 9x 1 = 72

2. let 7 in the ten's place

(7)

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	8 | 1 | 9 no.s that can be formed = 8 x 1 x 9 = 72 3. let 7 in the hundred place (7) 1 | 9 | 9 no.s that can be formed = 1 x 9x 9 = 81 ∴ The required numbers = 72 + 72 + 81 = 225 ans.	
Q.5)	How many numbers are there between 100 and 1000 such that atleast one of the their digit is 7 ?	
Sol.5)	100 < required no < 1000 ∴ it must be a 3-digit numbers digits available : 0 to 9 Repeated of digits allowed:- (i) 3 digit no.s having at least one digit is 7 = (total no of 3 digit numbers) – (3 digit numbers in which 7 does not appear at all) (ii) Total no. of 3 digit numbers:- 9 | 10 | 10 = 9 x 10 x 40 = 900 but it contains the number 100 ∴ Numbers are 900-1 (1 for 100) = 899 (iii) 3 digit numbers in which 7 does not appear at all: 8 | 9 | 9 = 8 x 9x 9 = 648 but it contains the number 100 ∴ Numbers are 648-1 (1 for 100) = 647 ∴ The required 3 digits no.s are = 899-647 = 252 ans.	
Q.6)	How many 3 digit even numbers can be found such that if 5 is one of the digit then 7 must be the next digit?	
Sol.6)	Required: 3 digit (even numbers) Digits available: 0 to 9 There are two cases: Case 1: Let the no. 5 comes: if 5 comes then it can be only in the hundred place, then 7 must be the ten's place. (5) (7) (0,2,4,6,8) 1 | 1 | 5 The last digit must be an even number, ∴ it can be filled in 5 ways ∴ the no.s can be formed = 1x1x5=5 Case 1: Let the no. 5 do not comes: (0,2,4,6,8) 8 | 9 | 5 No.s that can be formed = 8 x 9 x 5= 360 ∴ the required 3-digit no.s are = 5 + 360 = 365 ans.	
Q.7)	Find the sum of all the numbers that can be formed with the digits 2,3,4,5 taken all at a time.?	
Sol.7)	Digits available: 2,3,4,5 Required: 4 digit numbers (∵ we have to use all the available digits) (i) The total four digit numbers that can be formed using the given digits are	

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		8 | 9 | 5 = 4 x 3 x 2 x 1 = 24 (ii) To find the sum of these 24 nos, we will find the sum of digits at unit's place, ten's place , hundred place and thousands place (iii) In units place, each digit (2,3,4,5) occurs 6 times ∴ the sum of these digits in units place = 2 x 6 + 3 x 6 + 5 x 6 = 84 (iv) Similarly, the sum of the all digits in ten's place = 84 the sum of the all digits in hundred place = 84 the sum of the all digits in thousand place = 84 ∴ the sum of all 24 numbers is given by = 84 $(10^3 + 10^2 + 10^1 + 10^0)$ = 84 (1000+100+10+1) = 84 (1111) = 93324 ans.	
Ques.8)	How many numbers greater than 1000000 can be formed using the digits 1,2,0,2,4,2,4,?	
Sol.8)	Required: 7-digit numbers Digits available: 1,2,0,2,4,2,4 (i) The required seven digits no.s = (Total no of 7 digit numbers) – (no.s starting with 0) (ii) The total 7 digits numbers that can be formed are = $\frac{7!}{3!2!} = \frac{5040}{6 \text{ x } 2}$ = 420 (iii) The no. starting with '0': Let '0' is the first place, then remaining 6 digits can be arranged in = $\frac{6!}{3!2!} = \frac{720}{6 \text{ x } 2}$ = 60 ∴ the required no.s are = 420-60 = 360 ans.	
Ques.9)	How many 4 digit numbers divisible by 5 using the digits 0 to 9 when (i) Repeated of digits not allowed (ii) Repeated of digits allowed	
Sol.9)	Digits available 0 t0 9 Required 4 digit no.s divisible by 5 1. Repeated of digits not allowed: For numbers divisible by 5, the last digit can be 0 or 5 Two Cases: (i) Numbers ending with '0' 0 9 | 8 | 7 | (1) No.s that be formed = 9 x 8 x 7 x 1 = 504 (ii) Numbers ending with '5' (5) 8 | 8 | 7 | 1 = 8 x 8 x 7 x 1 = 448 ∴ required 4 digits no.s which are divisible by 5 = 504+ 448 = 952 ans. (iii) Repeated of digit allowed (0,5) 9 | 10 | 10 | 2 a) Unit's place can be filled in 2 ways (0,5) b) Thousand place can be filled 9 ways (0 not allowed) c) Hundred place can be filled in 10 ways d) Ten's place can be filled in 10 ways ∴ required 4 digit no.s which are divisible by 5 = 9 x 10 x 10 x 2 = 1800 ans.	
Q.10)	How many numbers are there between 100 and 100 such that every digit is either 2 or 9 ?	

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Sol.10)

100 < required number < 1000

It must be a 3 digit numbers

Digits available = 2 and 9

(2,9)	(2,9)	(2,9)
2	2	2

(i) The first place (hundred) can be filled in 2 ways

(ii) The second place (ten's) can be filled in 2 ways

(iii) The third place (unit's) can be filled in 2 ways

∴ the required 3-digits no.s = 2 x 2 x 2 = 8 ans.