# CHAPTER – 14

# MATHEMATICAL REASONING

## KEY POINTS

- A sentence is called a statement if it is either true or false but not both.
- The denial of a statement p is called its negative and is written as ~p and read as not p.
- Compound statement is made up of two or more simple statements. These simple statements are called component statements.
- 'And', 'or', 'If–then', 'only if' 'If and only if' etc. are connecting words, which are used to form a compound statement.
- Compound statement with **'And'** is
  - (a) true if all its component statements are true
  - (b) false if any of its component statement is false
- Compound statement with **'Or'** is
  - (a) true when at least one component statement is true
  - (b) false when any of its component statement is false.
- A statement with **"If p then q"** can be rewritten as
  - (a) p implies q
  - (b) p is sufficient condition for q
  - (c) q is necessary condition for p
  - (d) p only if q
  - (e) (~q) implies (~p)

- Contrapositive of the statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$
- Converse of the statement $p \Rightarrow q$ is the statement $q \Rightarrow p$
- "For all", "For every" are called universal quantifiers
- A statement is called valid or invalid according as it is true or false.

## VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

**Identify which of the following are statements (Q. No 1 to 7)**

1. Prime factors of 6 are 2 and 3.
2. $x^2 + 6x + 3 = 0$
3. The earth is a planet.
4. There is no rain without clouds.
5. All complex numbers are real numbers.
6. Tomorrow is a holiday.
7. Answer this question.

**Write negation of the following statements (Q. No 8 to 12)**

8. All men are mortal.
9. $\pi$ is not a rational number.
10. Every one in Spain speaks Spanish.
11. Zero is a positive number.

**Write the component statements of the following compound statements**

12. 7 is both odd and prime number.
13. All integers are positive or negative.
14. 36 is a multiple of 4, 6 and 12.
15. Jack and Jill went up the hill.

**Identify the type 'Or' (Inclusive or Exclusive) used in the following statements (Q. No. 16 to 19)**

16. Students can take French or Spanish as their third language.

17. To enter in a country you need a visa or citizenship card.

18. $\sqrt{2}$ is a rational number or an irrational number.

19. 125 is a multiple of 5 or 8.

**Which of the following statements are true or false. Give Reason. (Question No. 20 to 23)**

20. 48 is a multiple of 6, 7 and 8

21. $\pi > 2$ and $\pi < 3$.

22. Earth is flat or it revolves around the moon.

23. $\sqrt{2}$ is a rational number or an irrational number.

**Identify the quantifiers in the following statements (Q. No. 24 to 26)**

24. For every integer p, $\sqrt{p}$ is a real number.

25. There exists a capital for every country in the world.

26. There exists a number which is equal to its square.

**Write the converse of the following statements (Q. No. 27 to 30)**

27. If a number x is even then $x^2$ is also even.

28. If $3 \times 7 = 21$ then $3 + 7 = 10$

29. If n is a prime number then n is odd.

30. Some thing is cold implies that it has low temperature.

**Write contrapositive of the following statements (Q. No. 31 and 32)**

31. If $5 > 7$ then $6 > 7$.

32. x is even number implies that $x^2$ is divisible by 4.

33. Check the validity of the statement 'An integer x is even if and only if $x^2$ is even.

## ANSWERS

1. Statement
2. Not a statement
3. Statement
4. Statement
5. Statement
6. Not a Statement
7. Not a statement
8. All men are not mortal
9. $\pi$ is a rational number.
10. Everyone in Spain doesn't speak Spanish.
11. Zero is not a positive number.
12. 7 is an odd number. 7 is a prime number.
13. All integer are positive. All integers are negative.
14. 36 is a multiple of 4.

    36 is a multiple of 6.

    36 is a multiple of 12.
15. Jack went up the hill.

    Jill went up the hill.
16. Exclusive
17. Inclusive
18. Exclusive
19. Exclusive
20. False, 48 is not a multiple of 7
21. False, $\pi$ lies between 3 and 4
22. False
23. True
24. For every
25. For every, there exists
26. There exists
27. If $x^2$ is even then x is even

28. If $3 + 7 = 10$ then $3 \times 7 = 21$

29. If n is odd then n a prime number.

30. If some thing has low temperature then it is cold.

31. If $6 \leq 7$ then $5 \leq 7$

32. If $x^2$ is not divisible by 4 then x is not even.

33. Valid