

## CHAPTER - 6

# LINEAR INEQUALITIES

### KEY POINTS

- Two real numbers or two algebraic expressions related by the symbol ' $<$ ', ' $>$ ', ' $\leq$ ' or ' $\geq$ ' form an inequality.
- The inequalities of the form  $ax + b > 0$ ,  $ax + b < 0$ ,  $ax + b \geq 0$ ,  $ax + b \leq 0$  ;  $a \neq 0$  are called linear inequalities in one variable  $x$
- The inequalities of the form  $ax + by + c > 0$ ,  $ax + by + c < 0$ ,  $ax + by + c \geq 0$ ,  $ax + by + c \leq 0$ ,  $a \neq 0$ ,  $b \neq 0$  are called linear inequalities in two variables  $x$  and  $y$
- Rules for solving inequalities :
  - (i)  $a \geq b$  then  $a \pm k \geq b \pm k$   
where  $k$  is any real number.
  - (ii) but if  $a \geq b$  then  $ka$  is not always  $\geq kb$ .  
If  $k > 0$  (i.e. positive) then  $a \geq b \Rightarrow ka \geq kb$   
If  $k < 0$  (i.e. negative) then  $a \geq b \Rightarrow ka \leq kb$
- **Solution Set** : A solution of an inequality is a number which when substituted for the variable, makes the inequality true. The set of all solutions of an inequality is called the solution set of the inequality.
- The graph of the inequality  $ax + by > c$  is one of the half planes and is called the solution region
- When the inequality involves the sign  $\leq$  or  $\geq$  then the points on the line are included in the solution region but if it has the sign  $<$  or  $>$  then the points on the line are not included in the solution region and it has to be drawn as a dotted line.

**VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)**

1. Solve  $5x < 24$  when  $x \in \mathbb{N}$
2. Solve  $3x < 11$  when  $x \in \mathbb{Z}$
3. Solve  $3 - 2x < 9$  when  $x \in \mathbb{R}$
4. Show the graph of the solution of  $2x - 3 > x - 5$  on number line.
5. Solve  $5x - 8 \geq 8$  graphically
6. Solve  $\frac{1}{x-2} \leq 0$
7. Solve  $0 < \frac{-x}{3} < 1$

**Write the solution in the form of intervals for  $x \in \mathbb{R}$ . for Questions 8 to 10**

8.  $\frac{2}{x-3} < 0$
9.  $-3 \leq -3x + 2 < 4$
10.  $3 + 2x > -4 - 3x$
11. Draw the graph of the solution set of  $x + y \geq 4$ .
12. Draw the graph of the solution set of  $x \leq y$

**SHORT ANSWER TYPE QUESTIONS (4 MARKS)**

**Solve the inequalities for real  $x$**

13.  $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$
14.  $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2$
15.  $-5 \leq \frac{2-3x}{4} \leq 9$

16.  $|x - 2| \geq 5$
17.  $|4 - x| + 1 < 3$
18.  $\frac{3}{x - 2} < 1$
19.  $\frac{x}{x - 5} > \frac{1}{2}$
20.  $\frac{x + 3}{x - 2} > 0$
21.  $x + 2 \leq 5, 3x - 4 > -2 + x$
22.  $3x - 7 > 2(x - 6), 6 - x > 11 - 2x$
23. The water acidity in a pool is considered normal when the average PH reading of three daily measurements is between 7.2 and 7.8. If the first two PH readings are 7.48 and 7.85, find the range of PH value for the third reading that will result in the acidity level being normal.
24. While drilling a hole in the earth, it was found that the temperature ( $T$  °C) at  $x$  km below the surface of the earth was given by  

$$T = 30 + 25(x - 3), \text{ when } 3 \leq x \leq 15.$$

Between which depths will the temperature be between 200°C and 300°C?

Solve the following systems of inequalities graphically : (Questions 25, 26)
25.  $x + y > 6, 2x - y > 0$
26.  $3x + 4y \leq 60, x + 3y \leq 30, x \geq 0, y \geq 0$

### LONG ANSWER TYPE QUESTIONS (6 MARKS)

**Solve the system of inequalities for real  $x$**

$$27. \quad \frac{5x}{4} + \frac{3x}{8} > \frac{39}{8} \text{ and}$$

$$\frac{2x - 1}{12} - \frac{x - 1}{3} < \frac{3x + 1}{4}$$

**Solve the following system of inequalities graphically (Questions 28 to 30)**

28.  $3x + 2y \leq 24$ ,  $x + 2y \leq 16$ ,  $x + y \leq 10$ ,  $x \geq 0$ ,  $y \geq 0$

29.  $2x + y \geq 4$ ,  $x + y \leq 3$ ,  $2x - 3y \leq 6$

30.  $x + 2y \leq 2000$ ,  $x + y \leq 1500$ ,  $y \leq 600$ ,  $x \geq 0$ ,  $y \geq 0$

## ANSWERS

1.  $\{1, 2, 3, 4\}$

2.  $\{ \dots, -2, -1, 0, 1, 2, 3 \}$

3.  $x > -3$

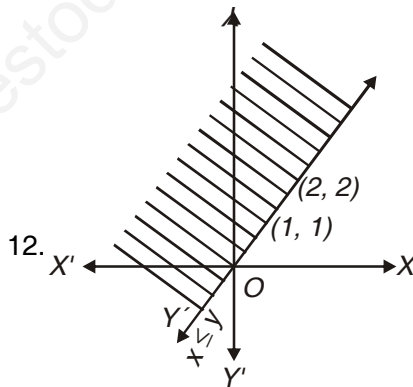
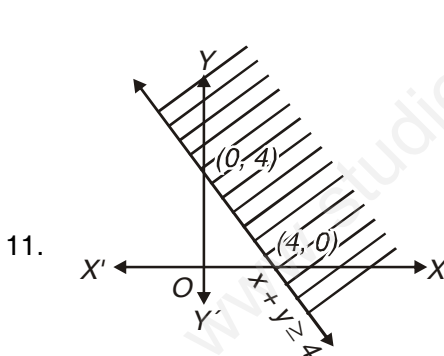
6.  $x < 2$

7.  $-3 < x < 0$

8.  $(-\infty, 3)$

9.  $\left[ \frac{-2}{3}, \frac{5}{3} \right]$

10.  $\left( \frac{-7}{5}, \infty \right)$



13.  $\left[ -\infty, \frac{63}{10} \right]$

14.  $\left[ -\infty, \frac{-13}{2} \right]$

15.  $\left[ \frac{-34}{3}, \frac{22}{3} \right]$

16.  $(-\infty, -3] \cup [7, \infty)$

17.  $(2, 6)$

18.  $(-\infty, 2) \cup (5, \infty)$

19.  $(-\infty, -5) \cup (5, \infty)$

20.  $(-\infty, -3) \cup (2, \infty)$

21.  $(1, 3]$

22.  $(5, \infty)$

23. Between 6.27 and 8.07

24. Between 9.8 m and 13.8 m

27.  $(3, \infty)$