

<u>Class 11 Limits & Derivatives</u> Class 11th	
Q.1)	If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$. Find $\frac{dy}{dx}$
Sol.1)	We have $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty \quad \dots \quad (i)$ Differentiate both sides w.r.t x $\frac{dy}{dx} = 0 + \frac{1}{1!} + \frac{2x}{2!} + \frac{3x^2}{3!} + \dots \infty$ $\frac{dy}{dx} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots \infty$ $\frac{dy}{dx} = y \quad \dots \quad \text{From eq. (i) ans.}$
Q.2)	Evaluate $\lim_{x \rightarrow 3^+} \left(\frac{x}{[x]} \right)$
Sol.2)	Put $x = 3 + h$ & $h \rightarrow 0$ $= \lim_{h \rightarrow 0} \left(\frac{3+h}{[3+h]} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{3+h}{3} \right) \dots \{ [3.1] = 3 \}$ $= \frac{3}{3} = 1 \text{ ans.}$
Q.3)	If $\lim_{x \rightarrow 0} \left(\sin(mx) \cot \frac{x}{\sqrt{3}} \right) = 2$. Find m
Sol.3)	We have $\lim_{x \rightarrow 0} \left(\sin(mx) \cot \frac{x}{\sqrt{3}} \right) = 2$ $= \lim_{x \rightarrow 0} \left(\frac{\sin(mx)}{\tan \left(\frac{x}{\sqrt{3}} \right)} \right) = 2$ $= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin(mx)}{mx} \times mx}{\frac{\tan \left(\frac{x}{\sqrt{3}} \right)}{\left(\frac{x}{\sqrt{3}} \right)} \times \frac{x}{\sqrt{3}}} \right) = 2$ $= \frac{1 \times m}{1 \times \frac{1}{\sqrt{3}}} = 2$ $= \sqrt{3} m = 2$ $= m = \frac{2}{\sqrt{3}} \text{ ans.}$



Q.4)	If $f(x) = 1 - x + x^2 - x^3 \dots - x^{99} + x^{100}$
Sol.4)	<p>We have $f(x) = 1 - x + x^2 - x^3 \dots - x^{99} + x^{100}$</p> <p>Differentiate both sides w.r.t x</p> $f'(x) = 0 - 1 + 2x - 3x^2 \dots - 99x^{99} + 100x^{100}$ $f'(1) = -1 + 2 - 3 \dots - 99 + 100$ $f'(1) = -(1 - 3 - 5 \dots 99) + (2 + 4 + 6 \dots 100)$ $= -(1 + 3 + 5 \dots 99) + (2 + 4 + 6 \dots 100)$ <p>A.P. $a = 1, d = 2, n = 50$ A.P. $a = 2, d = 2, n = 50$</p> $= -\frac{50}{2}[2 + (49)2] + \frac{50}{2}[4 + 49 \times 2]$ $= -25(100) + 25(102)$ $= -2500 + 2550$ $f'(1) = 50 \text{ ans.}$
Q.5)	<p>Let $f(x) = \begin{cases} x^2 - 1: & a < x < 2 \\ 2x + 3: & 2 \leq x < 3 \end{cases}$ find the quadratic curve whose roots are $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$</p>
Sol.5)	$\lim_{x \rightarrow 2^-} f(x)$ $= \lim_{x \rightarrow 2^-} (x^2 - 1)$ <p>Put $x = 2 - h$ & $h \rightarrow 0$</p> $= \lim_{h \rightarrow 0} ((2 - h)^2 - 1) = 4 - 1 = 3$ <p>Now $\lim_{x \rightarrow 2^+} (2x + 3)$</p> <p>Put $x = 2 + h$ & $h \rightarrow 0$</p> $\lim_{h \rightarrow 0} (2(2 + h) + 3) = 4 + 3 = 7$ <p>Given 3 & 7 are the roots of the quadratic curve $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$</p> $x^2 - (3 + 7)x + 21 = 0$ $x^2 - 10x + 21 \text{ ans.}$
Q.6)	Evaluate $\lim_{x \rightarrow \frac{\pi}{3}} \left(\frac{\sqrt{1-\cos(6x)}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)} \right)$
Sol.6)	We have $\lim_{x \rightarrow \frac{\pi}{3}} \left(\frac{\sqrt{1-\cos(6x)}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)} \right)$

	$= \lim_{x \rightarrow \frac{\pi}{3}} \left(\frac{\sqrt{2 - \sin^2(3x)}}{\sqrt{2} \left(\frac{\pi}{3} - x \right)} \right)$ $= - \lim_{x \rightarrow \frac{\pi}{3}} \left(\frac{\sqrt{2} \sin(3x)}{\sqrt{2} \left(\frac{\pi}{3} - x \right)} \right)$ <p>Put $x = \frac{\pi}{3} + h$ & $h \rightarrow 0$</p> $= - \lim_{h \rightarrow 0} \left(\frac{\sin(3 \left(\frac{\pi}{3} + h \right))}{\frac{\pi}{3} + h - \frac{\pi}{3}} \right)$ $= - \lim_{h \rightarrow 0} \left(\frac{\sin(\pi + 3h)}{h} \right)$ $= - \lim_{h \rightarrow 0} \left(\frac{-\sin(3h)}{h} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\sin(3h)}{3h} \right) \times 3$ $= 1 \times 3 = 3 \text{ ans.}$
Q.7)	Evaluate $\lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\cot^2 x - 3}{\operatorname{cosec} x - 2} \right)$
Sol.7)	<p>We have $\lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\cot^2 x - 3}{\operatorname{cosec} x - 2} \right)$</p> $= \lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\operatorname{cosec}^2 x - 2 - 1 - 3}{\operatorname{cosec} x - 2} \right)$ $= \lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2} \right)$ $= \lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{(\operatorname{cosec} x + 2)(\operatorname{cosec} x - 2)}{\operatorname{cosec} x - 2} \right)$ $= \operatorname{cosec} \left(\frac{\pi}{6} \right) + 2 = 2 + 2 = 4 \text{ ans.}$
Q.8)	Evaluate $\lim_{x \rightarrow 1} \left(\frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2} \right)$
Sol.8)	<p>Since $\lim_{x \rightarrow 1}$ ∵ $(x - 1)$ is the factor of N & D</p> $\begin{array}{r} x^6 + x^5 - x^4 - x^3 - x^2 - x - 1 \\ x - 1 \quad \overline{ \begin{array}{r} x^7 - 2x^5 + 1 \\ -(x^7 - x^6) \\ \hline x^6 - 2x^5 + 1 \\ -(x^6 - x^5) \\ \hline \end{array} } \end{array}$ $\begin{array}{r} x^2 - 2x - 2 \\ x - 1 \quad \overline{ \begin{array}{r} x^3 - 3x^2 + 2 \\ -(x^3 - x^2) \\ \hline -2x^2 + 2 \\ (-2x^2 + 2x) \end{array} } \end{array}$

	$ \begin{array}{r} -x^5 + 1 \\ -(-x^5 - x^4) \\ \hline -x^4 + 1 \\ -(-x^4 + x^3) \\ \hline -x^3 + 1 \\ -(-x^3 + x^2) \\ \hline -x^2 + 1 \\ -(-x^2 + x) \\ \hline -x + 1 \\ -x + 1 \\ \hline x \end{array} $ $ \therefore \lim_{x \rightarrow 1} \frac{(x-1)(x^6+x^5-x^4-x^3-x^2-x-1)}{(x-1)(x^2-2x-2)} \\ = \frac{1+1-1-1-1-1}{1-2-2} = \frac{2-5}{-3} = \frac{-3}{-3} = 1 \text{ ans.} $	$ \begin{array}{r} -2x + 2 \\ -2x + 2 \\ \hline x \end{array} $
Q.9)	Evaluate $\lim_{x \rightarrow 0} \left(\frac{ \sin x }{x} \right)$	
Sol.9)	<p>L.H.L. $\lim_{x \rightarrow 0} \left(\frac{ \sin x }{x} \right)$</p> <p>Put $x = 0 - h = -h$ & $h \rightarrow 0$</p> $\lim_{h \rightarrow 0} \left(\frac{ \sin(-h) }{-h} \right) = -1$ <p>R.H.L. $\lim_{x \rightarrow 0} \left(\frac{ \sin x }{x} \right)$</p> <p>Put $x = 0 + h = h$</p> $\lim_{h \rightarrow 0} \left(\frac{ \sin h }{h} \right) = \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \right)$ <p>$\therefore \lim_{x \rightarrow 0} f(x)$ does not exists</p> <p>L.H.L \neq R.H.L.</p>	