<table>
<tr><td></td><td>Class 11 Limits & Derivatives
Class XI</td></tr>
<tr><td></td><td>Limits:-</td></tr>
<tr><td>Q.1)</td><td>If $f(x) = \begin{cases} 5x-4\,; 0 < x < 1 \\ 4x^3-3x\,; 1 < x < 2 \end{cases}$
Evaluate $\lim\limits_{x\to a} f(x)$</td></tr>
<tr><td>Sol.1)</td><td>For L.H.L. : $f(x) = 5x-4$
For R.H.L : $f(x) = 4x^3-3x$
L.H.L. = $\lim\limits_{x\to 1^-}(5x-4)$
Put $x = 1-h$ & $h \to 0$
∴ L.H.L. = $\lim\limits_{h\to 0}(5(1-h)-4)$
⇒ L.H.L = $5-4 = 1$
∴ L.H.L.= 1
Now, R.H.L.= $\lim\limits_{x\to 1^+}(4x^3-3x)$
Put $x = 1+h$ & $h \to 0$
∴ R.H.L. = $\lim\limits_{h\to 0}[4(1+h)^3-3(1+h)]$
⇒ R.H.L = $4(1)^3-3(1) = 1$
∴ R.H.L.= 1
Since, L.H.L. = R.H.L. = 1
∴ $\lim\limits_{x\to 1} f(x)$ Exists & $\lim\limits_{x\to 1} f(x) = 1$ ans.</td></tr>
<tr><td>Q.2)</td><td>If $f(x) = \begin{cases} \frac{x-|x|}{x}\,; x \neq 0 \\ 2\,; x = 0 \end{cases}$
Show that $\lim\limits_{x\to 0} f(x)$ does not exists.</td></tr>
<tr><td>Sol.2)</td><td>Here, for L.H.L. & R.H.L.
$f(x) = \begin{cases} \frac{x-|x|}{x} \end{cases}$
L.H.L. = $\lim\limits_{x\to 0^-}\left[\frac{x-|x|}{x}\right]$
Put $x = 0-h = -h$ & $h \to 0$
∴ L.H.L. = $\lim\limits_{h\to 0}\left(\frac{-h-|-h|}{-h}\right)$</td></tr>
</table>

	$\Rightarrow$ L.H.L. $= \lim_{h\to 0}\left(\frac{-h-h}{-h}\right) = \lim_{h\to 0}\left(\frac{-2h}{-h}\right)$ $\Rightarrow \lim_{h\to 0}(2)$ $\therefore$ L.H.L.$= 2$ Now, R.H.L. $= \lim_{x\to 0^+}\left[\frac{x-\|x\|}{x}\right]$ Put $x = 0 + h = h$ & $h \to 0$ $\therefore$ R.H.L. $= \lim_{h\to 0}\left(\frac{h-\|-h\|}{-h}\right) = \lim_{h\to 0}\left(\frac{h-h}{h}\right)$ $\Rightarrow$ R.H.L. $= \lim_{h\to 0}\left(\frac{0}{h}\right) = \lim_{h\to 0}(0) = 0$ $\therefore$ R.H.L.$= 0$ Clearly L.H.L. $\neq$ R.H.L. $\therefore \lim_{x\to 0} f(x)$ does not exists ans.
Q.3)	$f(x) = \begin{cases} \frac{4x-5}{x-4} & ; x \le 2 \\ x-\pi ; x > 2 \end{cases}$ Find value of π if $\lim_{x\to 2} f(x)$ exists.
Sol.3)	For L.H.L. $f(x) = 4x - 5$ For R.H.L. $f(x) = x - \pi$ L.H.L. $= \lim_{x\to 2^+}(4x-5)$ Put $x = 2 - h$ & $h \to 0$ $\Rightarrow$ L.H.L. $= \lim_{h\to 0}(4(2-h)-5) = \lim_{h\to 0}(8-5)$ $\Rightarrow$ L.H.L. $= \lim_{h\to 0}(3)$ $\Rightarrow$ L.H.L. $= 3$ Now, R.H.L. $= \lim_{x\to 2^+}(x-\pi)$ Put $x = 2 + h$ and $h \to 0$ $\Rightarrow$ R.H.L. $= \lim_{h\to 0}(2+h-\pi) = \lim_{h\to 0}(2-\pi)$ $\therefore$ R.H.L.$= 2 - \pi$ Since, L.H.L. = R.H.L. $= 1$ $\therefore \lim_{x\to 2} f(x)$ exists (given) $\therefore$ L.H.L. $=$ R.H.L. $\Rightarrow 3 - 2\pi \Rightarrow \pi = -1$ ans.
Q.4)	Show that $\lim_{x\to 0}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ does not exists.

Sol.4)	For L.H.L. & R.H.: $f(x) = (\frac{e^{1/x}-1}{e^{1/x}+1})$ L.H.L. $= \lim\limits_{x\to 0^-} (\frac{e^{1/x}-1}{e^{1/x}+1})$ Put $x = 0 - h = -h$ & $h \to 0$ $\therefore$ L.H.L. $= \lim\limits_{h\to 0} \left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ Put directly $h = 0$ $\Rightarrow$ L.H.L $= \frac{e^{-\infty}-1}{e^{-\infty}+1} = \frac{0-1}{0+1}$ $e^{-\infty} = 0$ $\Rightarrow$ L.H.L.$= -1$ Now, R.H.L. $= \lim\limits_{x\to 0^+} \left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ Put $x = 0 + h = h$ & $h \to 0$ $\therefore$ R.H.L. $= \lim\limits_{h\to 0} \left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ (don't put directly $h = 0$) $\frac{\infty}{\infty}$ form $\Rightarrow$ R.H.L. $= \lim\limits_{h\to 0} \left(\frac{1-\frac{1}{e^{1/h}}}{1+\frac{1}{e^{1/h}}}\right)$ divide by $e^{1/h}$ $\Rightarrow$R.H.L. $= \lim\limits_{h\to 0} \left(\frac{1-e^{-1/h}}{1+e^{-1/h}}\right)$ Put $h = 0$ $\Rightarrow$ R.H.L. $= \lim\limits_{h\to 0} \left(\frac{1-e^{-\infty}}{1+e^{-\infty}}\right) = \frac{1-0}{1+0}$ $e^{-\infty} = 0$ $\Rightarrow$ R.H.L. $= 1$ Since, L.H.L. $\neq$ R.H.L. $= 1$ $\therefore \lim\limits_{x\to 0} f(x)$ does not Exists ans.
Q.5)	$f(x) = \begin{cases} a + bx \,; x < 1 \\ 4 \,; x = 1 \\ b - ax \,; x > 1 \end{cases}$ and if $\lim\limits_{x\to 1} f(x) = f(1)$, what are possible values of a & b ?
Sol.5)	For L.H.L. $f(x) = a + bx$ For R.H.L. $f(x) = b - ax$ and $f(1) = 4$ (when $x = 1; f(x) = 4$) given, $\lim\limits_{x\to 1} f(x) = f(1)$ $\Rightarrow \lim\limits_{x\to 1} f(x) = 4$

	$\Rightarrow$ L.H.L. = R.H.L. = 4 $\Rightarrow \lim_{x\to 1^-}(a+bx) = \lim_{x\to 1^+}(b-ax) = 4$ Put $x = 1-h$ Put $x = 1+h$ & $h \to 0$ & $h \to 0$ $\Rightarrow \lim_{h\to 0}\big(a+b(1-h)\big) = \lim_{h\to 0}\big(b-a(1+h)\big) = 4$ $\Rightarrow a+b = b-a = 4$ $\Rightarrow a+b = 4$ & $b-a = 4$ Solving we get $a = 0$ & $b = 4$ ans.
Q.6)	$f(x) = \begin{cases} mx^2+n; x<0 \\ nx+m\,; 0 \le x \le 1 \\ nx^3+m\,; x>1 \end{cases}$ For what integers m and n does the $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ exists.
Sol.6)	Given that $\lim_{x\to 0} f(x)$ exists For L.H.L. $f(x) = nx+m$ L.H.L=R.H.L. $\Rightarrow \lim_{x\to 0^-}(mx^2+n) = \lim_{x\to 0^+}(nx+m)$ Put $x = 0-h = -h$ Put $x = 0+h = h$ & $h \to 0$ & $h \to 0$ $\therefore \lim_{h\to 0}(m(-h)^2+n) = \lim_{h\to 0}(n(h)+m)$ $\Rightarrow 0+n = 0+m$ $\Rightarrow = m = n$ ………… (i) Given that $\lim_{x\to 0} f(x)$ exists For L.H.L. $f(x) = nx+m$ For R.H.L. $f(x) = nx^3+m$ L.H.L. = R.H.L. $\Rightarrow \lim_{x\to 1^-}(nx+m) = \lim_{x\to 1^+}(nx^3+m)$ Put $x = 1-h$ Put $x = 1+h$ & $h \to 0$ & $h \to 0$ $\Rightarrow \lim_{h\to 0}(n(1-h)+m) = \lim_{h\to +}(n(1+h)^3+m)$ (put directly $h = 0$) $\Rightarrow n+m = n+m$ ……….(ii) From (i) & (ii)

	m and n can be any integers such that $m = n$ ans.
Q.7)	$f(x) = \begin{cases} \lvert x\rvert + 1\,; x < 0 \\ 0\,; x = 0 \\ \lvert x\rvert - 1\,; x > 0 \end{cases}$ For what value(s) of a does the $\lim\limits_{x\to 0} f(x)$ exists.
Sol.7)	For L.H.L. $f(x) = \lvert x\rvert + 1$ For R.H.L. $f(x) = \lvert x\rvert - 1$ L.H.L. $= \lim\limits_{x\to 0^-} (\lvert x\rvert + 1)$ Put $x = 0 - h = -h$ & $h \to 0$ $\therefore$ L.H.L. $= \lim\limits_{h\to 0}(\lvert -h\rvert + 1) = \lim\limits_{h\to 0}(h + 1) = 0 + 1$ $\Rightarrow$ L.H.L. $= 1$ Now, R.H.L. $= \lim\limits_{x\to 0^+}(\lvert x\rvert - 1)$ Put $x = 0 + h = h$ & $h \to 0$ $\therefore$ R.H.L. $\lim\limits_{h\to 0}(\lvert -h\rvert - 1) = \lim\limits_{h\to 0}(h - 1) = 0 - 1$ $\Rightarrow$ R.H.L. $= -1$ Since L.H.L $\neq$ R.H.L. $\therefore \lim\limits_{x\to 0} f(x)$ Does not exist. .... (i) But we are given, $\lim\limits_{x\to a} f(x)$ exists. .... (ii) From (i) & (ii) We conclude that a can be any real no. except $a = 0$ $\therefore a \in R - \{0\}$ ans.
Q.8)	$f(x) \begin{cases} 2x + 3\,; x \leq 0 \\ 3(x+1); x > 0 \end{cases}$ Evaluate $\lim\limits_{x\to 1} f(x)$
Sol.8)	For L.H.L. $f(x) = 3(x + 1)$ Also for R.H.L. $f(x) = 3(x + 1)$ L.H.L. $= \lim\limits_{x\to 1^-} 3(x + 1)$ Put $x = 1 - h$ & $h \to 0$ $\therefore$ L.H.L. $= \lim\limits_{h\to 0}(3(1 - h + 1)) = 3(2) = 6$ $\Rightarrow$ L.H.L. $= 6$ Now, R.H.L. $= \lim\limits_{x\to 1^+}(3(x + 1))$ Put $x = 1 + h = h$ & $h \to 0$

	∴ R.H.L. $\lim_{h\to 0}(3(1+h+1)=3(2)=6$ $\Rightarrow$ R.H.L. $=6$ Since L.H.L $=$ R.H.L. ∴ $\lim_{x\to 0} f(x)$ Exist and $\lim_{x\to 1} f(x)=6$ ans.
Q.9)	a_1, a_2, a_3..........a_n are any real numbers $f(x)=(x-a_1)(x-a_2)(x-a_3)$........$(x-a_n)$. What is $\lim_{x\to a} f(x)$? Also compute $\lim_{x\to a} f(x)$.
Sol.9)	$\lim_{x\to a_1} f(x)=\lim_{x\to a_1}[(x-a_1)(x-a_2)(x-a_3)\ldots\ldots\ldots.(x-a_n)]$ $=(a_1-a_1)(a_1-a_2)(a_1-a_3)\ldots\ldots\ldots.(a_1-a_n)$ $=0(a_1-a_2)(a_1-a_3)$..........a_1-a_n $=0$ ans. $\lim_{x\to a} f(x)=\lim_{x\to a}[(x-a_1)(x-a_2)\ldots\ldots\ldots.(x-a_n)]$ $=(a-a_1)(a-a_2)$.........$(a-a_n)$ ans.
	TYPE: 2 FACTORIZE **Formula: a^2-b^2, a^3-b^3, a^4-b^4, quadratic equation, cubic (hit & trial) L.C.M.**
Q.10)	Evaluate : $\lim_{x\to 1}\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]$
Sol.10)	We have $\lim_{x\to 1}\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right]$ $=\lim_{x\to 1}\left[\frac{x-2}{x(x-1)}-\frac{1}{x(x^2+3x+2)}\right]$ $=\lim_{x\to 1}\left[\frac{x-2}{x(x-1)}-\frac{1}{x(x-1)(x-2)}\right]$ $=\lim_{x\to 1}\left[\frac{(x-2)^2-1}{x(x-1)(x-2)}\right]$ $=\lim_{x\to 1}\left[\frac{x^2-4x+4-1}{x(x-1)(x-2)}\right]$ $=\lim_{x\to 1}\left[\frac{(x-3)(x-1)}{x(x-1)(x-2)}\right]$ $=\lim_{x\to 1}\left[\frac{(x-3)}{x(x-2)}\right]$ $=\frac{(1-3)}{(1)(1-2)}=\frac{-2}{-1}=2$ ans.