

<u>Class 11 Limits &amp; Derivatives</u>					
<b>Class 11<sup>th</sup></b>					
	<p><b>TYPE: 2 FACTORIZE</b></p> <p><b>Formula: <math>a^2 - b^2</math>, <math>a^3 - b^3</math>, <math>a^4 - b^4</math>, quadratic equation, cubic (hit &amp; trial) L.C.M.</b></p>				
Q.1)	<p>Evaluate: <math>\lim_{x \rightarrow 3} \left[ \frac{x^3 - 7x^2 + 15x - 9}{x^4 - 5x^3 + 27x - 27} \right]</math></p> <p><math>(x - 3)</math> is the factor of both polynomials.</p>				
Sol.1)	<p>We have <math>\lim_{x \rightarrow 3} \left[ \frac{x^3 - 7x^2 + 15x - 9}{x^4 - 5x^3 + 27x - 27} \right]</math></p> <p><math>(x - 3)</math> is the factor of both polynomials</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; text-align: center;"> <math display="block">x^2 - 4x + 3</math> </td> <td style="width: 50%; text-align: center;"> <math display="block">x^3 - 2x^2 - 6x + 9</math> </td> </tr> <tr> <td style="text-align: center;"> <math display="block">x - 3 \left  \begin{array}{r} x^3 - 7x^2 + 15x - 9 \\ -(x^3 - 3x^2) \\ \hline -4x^2 + 15x - 9 \\ -(-4x^2 + 12x) \\ \hline 3x - 9 \\ 3x - 9 \\ \hline x \end{array} \right.</math> </td> <td style="text-align: center;"> <math display="block">x - 3 \left  \begin{array}{r} x^4 - 5x^3 + 27x - 27 \\ -(x^4 - 3x^3) \\ \hline -2x^3 + 27x - 27 \\ -(-2x^3 + 6x^2) \\ \hline -6x^2 + 27x - 27 \\ -(-6x^2 + 18x) \\ \hline 9x - 27 \\ 9x - 27 \\ \hline x \end{array} \right.</math> </td> </tr> </table> <p> <math display="block">= \lim_{x \rightarrow 3} \left[ \frac{(x-3)(x^2-4x+3)}{(x-3)(x^3-2x^2-6x+9)} \right]</math> <math display="block">= \lim_{x \rightarrow 3} \left[ \frac{(x-1)(x-3)}{x^3-2x^2-6x+9} \right]</math> </p> <p>Again <math>(x - 3)</math> is factor of <math>D</math></p> <p> <math display="block">= \lim_{x \rightarrow 3} \left[ \frac{(x-1)(x-3)}{(x-3)(x^2+x-3)} \right]</math> <math display="block">= \lim_{x \rightarrow 3} \left[ \frac{(x-1)}{x^2+x-3} \right]</math> <math display="block">= \frac{3-1}{9+3-3} = \frac{2}{9} \text{ ans.}</math> </p>	$x^2 - 4x + 3$	$x^3 - 2x^2 - 6x + 9$	$x - 3 \left  \begin{array}{r} x^3 - 7x^2 + 15x - 9 \\ -(x^3 - 3x^2) \\ \hline -4x^2 + 15x - 9 \\ -(-4x^2 + 12x) \\ \hline 3x - 9 \\ 3x - 9 \\ \hline x \end{array} \right.$	$x - 3 \left  \begin{array}{r} x^4 - 5x^3 + 27x - 27 \\ -(x^4 - 3x^3) \\ \hline -2x^3 + 27x - 27 \\ -(-2x^3 + 6x^2) \\ \hline -6x^2 + 27x - 27 \\ -(-6x^2 + 18x) \\ \hline 9x - 27 \\ 9x - 27 \\ \hline x \end{array} \right.$
$x^2 - 4x + 3$	$x^3 - 2x^2 - 6x + 9$				
$x - 3 \left  \begin{array}{r} x^3 - 7x^2 + 15x - 9 \\ -(x^3 - 3x^2) \\ \hline -4x^2 + 15x - 9 \\ -(-4x^2 + 12x) \\ \hline 3x - 9 \\ 3x - 9 \\ \hline x \end{array} \right.$	$x - 3 \left  \begin{array}{r} x^4 - 5x^3 + 27x - 27 \\ -(x^4 - 3x^3) \\ \hline -2x^3 + 27x - 27 \\ -(-2x^3 + 6x^2) \\ \hline -6x^2 + 27x - 27 \\ -(-6x^2 + 18x) \\ \hline 9x - 27 \\ 9x - 27 \\ \hline x \end{array} \right.$				
Q.2)	<p>Evaluate: <math>\lim_{x \rightarrow \sqrt{2}} \left[ \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8} \right]</math></p>				
Sol.2)	<p>We have <math>\lim_{x \rightarrow \sqrt{2}} \left[ \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8} \right]</math></p>				

	$= \lim_{x \rightarrow \sqrt{2}} \left[ \frac{(x^2)^2 - (2)^2}{x^2 + 4\sqrt{2}x - \sqrt{2}x - 8} \right]$ $= \lim_{x \rightarrow \sqrt{2}} \left[ \frac{(x^2+2)(x^2-2)}{x(x+4\sqrt{2}) - \sqrt{2}(x+4\sqrt{2})} \right]$ $= \lim_{x \rightarrow \sqrt{2}} \left[ \frac{(x^2+2)(x+\sqrt{2})(x-\sqrt{2})}{(x+4\sqrt{2})(x-\sqrt{2})} \right]$ $= \frac{((\sqrt{2})^2+2)(\sqrt{2}+\sqrt{2})}{(\sqrt{2}+4\sqrt{2})}$ $= \frac{(4)(2\sqrt{2})}{(5\sqrt{2})}$ $= \frac{8}{5} \text{ ans.}$
	<p><b>TYPE: 3 RATIONALIZE</b></p> <p>When rationalize: <math>(\sqrt{\quad} - \sqrt{\quad})</math>; <math>(\sqrt{\quad} - \text{function})</math>; <math>(\text{function} - \sqrt{\quad})</math></p>
Q.3)	<p>Evaluate: <math>\lim_{x \rightarrow 4} \left[ \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right]</math></p> <p>Rationalize both <math>N</math> &amp; <math>D</math> simultaneously</p>
Sol.3)	<p>We have <math>\lim_{x \rightarrow 4} \left[ \frac{3 - \sqrt{5+x}}{1 - \sqrt{5-x}} \right]</math></p> $= \lim_{x \rightarrow 4} \left[ \frac{(3 - \sqrt{5+x})(1 + \sqrt{5-x})(3 + \sqrt{5+x})}{(1 - \sqrt{5-x})(1 + \sqrt{5-x})(3 + \sqrt{5+x})} \right]$ $= \lim_{x \rightarrow 4} \left[ \frac{(9 - 5 - x)(1 + \sqrt{5-x})}{(1 - \sqrt{5-x})(3 + \sqrt{5+x})} \right]$ $= \lim_{x \rightarrow 4} \left[ \frac{(4-x)(1 + \sqrt{5-x})}{(x-4)(3 + \sqrt{5+x})} \right]$ $= \lim_{x \rightarrow 4} \left[ \frac{-(x-4)(1 + \sqrt{5-x})}{(x-4)(3 + \sqrt{5+x})} \right]$ $= \frac{-(1 + \sqrt{5-4})}{(3 + \sqrt{5+4})}$ $= \frac{-(1+1)}{3+3} = -\frac{2}{6}$ $= -\frac{1}{3} \text{ ans.}$
Q.4)	<p>Evaluate: <math>\lim_{x \rightarrow a} \left[ \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]</math></p>
Sol.4)	<p>We have, <math>\lim_{x \rightarrow a} \left[ \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]</math></p> <p>Rationalize both <math>N</math> &amp; <math>D</math></p> $= \lim_{x \rightarrow a} \left[ \frac{(\sqrt{a+2x} - \sqrt{3x})(\sqrt{3a+x} + 2\sqrt{x})(\sqrt{a+2x} + \sqrt{3x})}{(\sqrt{3a+x} - 2\sqrt{x})(\sqrt{3a+x} + 2\sqrt{x})(\sqrt{a+2x} + \sqrt{3x})} \right]$

	$= \lim_{x \rightarrow a} \left[ \frac{(a + 2x - 3x)(\sqrt{3a + x} + 2\sqrt{x})}{(3a + x - 4x)(\sqrt{a + 2x} + \sqrt{3x})} \right]$ $= \lim_{x \rightarrow a} \left[ \frac{(a - x)(\sqrt{3a + x} + 2\sqrt{x})}{(3a + 3x)(\sqrt{a + 2x} + \sqrt{3x})} \right]$ $= \lim_{x \rightarrow a} \left[ \frac{(a - x)(\sqrt{3a + x} + 2\sqrt{x})}{3(a - x)(\sqrt{a + 2x} + \sqrt{3x})} \right]$ $= \frac{(\sqrt{3a + a} + 2\sqrt{a})}{3(\sqrt{a + 2a} + \sqrt{3a})}$ $= \frac{2\sqrt{a} + 2\sqrt{a}}{3(\sqrt{3a} + \sqrt{3a})}$ $= \frac{4\sqrt{a}}{3(2\sqrt{3a})}$ $= \frac{4\sqrt{a}}{6\sqrt{3}\sqrt{a}} = \frac{2}{3\sqrt{3}} \text{ ans.}$
Q.5)	Evaluate: $\lim_{x \rightarrow 1} \left[ \frac{(2x-3)-(\sqrt{x}-1)}{2x^2+x-3} \right]$
Sol.5)	<p>We have, <math>\lim_{x \rightarrow 1} \left[ \frac{(2x-3)-(\sqrt{x}-1)}{2x^2+x-3} \right]</math></p> <p>Rationalize both <math>N</math> &amp; <math>D</math></p> $= \lim_{x \rightarrow 1} \left[ \frac{(2x - 3)(\sqrt{x} - 1)(\sqrt{x} + 1)}{(2x^2 + 3x - 2x - 3)(\sqrt{x} + 1)} \right]$ $= \lim_{x \rightarrow 1} \left[ \frac{(2x - 3)(x - 1)}{(2x + 3)(x - 1)(\sqrt{x} + 1)} \right]$ $= \lim_{x \rightarrow 1} \left[ \frac{(2x - 3)}{(2x + 3)(\sqrt{x} + 1)} \right]$ $= \frac{(2x - 3)}{(2x + 3)(1 + 1)}$ $= \frac{-1}{(5)(2)}$ $= \frac{-1}{10} \text{ ans.}$
	<b>TYPE: 4</b> $\lim_{x \rightarrow a} \left( \frac{x^n - a^n}{x - a} \right) = na^{n-1}$
Q.6)	Evaluate: $\lim_{x \rightarrow 2} \left( \frac{x^{10} - 1024}{x^5 - 32} \right)$
Sol.6)	We have $\lim_{x \rightarrow 2} \left( \frac{x^{10} - 1024}{x^5 - 32} \right)$

	$= \lim_{x \rightarrow 2} \left[ \frac{x^{10} - 2^{10}}{x^5 - 2^5} \right]$ <p>Divide <math>N</math> &amp; <math>D</math> by <math>(x - 2)</math></p> $= \lim_{x \rightarrow 2} \left[ \frac{\frac{x^{10} - 2^{10}}{x - 2}}{\frac{x^5 - 2^5}{x - 2}} \right]$ $= \frac{\lim_{x \rightarrow 2} \left[ \frac{x^{10} - 2^{10}}{x - 2} \right]}{\lim_{x \rightarrow 2} \left[ \frac{x^5 - 2^5}{x - 2} \right]}$ $= \frac{10(2)^{10-1}}{5(2)^{5-1}} \quad \left\{ \lim_{x \rightarrow a} \left( \frac{x^n - a^n}{x - a} \right) = na^{n-1} \right\}$ $= 2(2)^5$ $= 2 \times 32 = 64 \text{ ans.}$
Q.7)	Evaluate: $\lim_{x \rightarrow a} \left( \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x - a} \right)$
Sol.7)	<p>We have <math>\lim_{x \rightarrow a} \left( \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x - a} \right)</math></p> <p>Substitution : put <math>x + 2 = y</math></p> <p>Limits change: when <math>x \rightarrow a</math> then <math>y \rightarrow a + 2</math></p> $\therefore \lim_{y \rightarrow a+2} \left[ \frac{y^{5/3} - (a+2)^{5/3}}{y - 2 - a} \right]$ $= \lim_{y \rightarrow a+2} \left[ \frac{y^{5/3} - (a+2)^{5/3}}{y - (a+2)} \right]$ $= \frac{5}{3} (a+2)^{5/3-1} \quad \left\{ \lim_{x \rightarrow a} \left( \frac{x^n - a^n}{x - a} \right) = na^{n-1} \right\}$ $= \frac{5}{3} (a+2)^{2/3} \text{ ans.}$
Q.8)	Evaluate: $\lim_{x \rightarrow 0} \left( \frac{(1+x)^6 - 1}{(1+x)^2 - 1} \right)$
Sol.8)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{(1+x)^6 - 1}{(1+x)^2 - 1} \right)</math></p> <p>put <math>1 + x = y</math>, when <math>x \rightarrow 0</math> then <math>y \rightarrow 1</math></p> $\therefore \lim_{y \rightarrow 1} \left[ \frac{y^6 - 1}{y^2 - 1} \right]$ <p>Divide <math>N</math> &amp; <math>D</math> by <math>(y - 1)</math></p>

	$= \lim_{y \rightarrow 1} \left[ \frac{y^6 - 1}{y - 1} \right]$ $= \frac{\lim_{y \rightarrow 1} [y^6 - 1]}{\lim_{y \rightarrow 1} [y - 1]}$ $= \frac{6(1)^5}{5(1)^1}$ $= \frac{6}{2} = 3 \text{ ans.}$
Q.9)	Evaluate : $\lim_{x \rightarrow 1} \left\{ \frac{(x+x^2+x^3 \dots x^n) - n}{x-1} \right\}$
Sol.9)	<p>We have <math>\lim_{x \rightarrow 1} \left\{ \frac{(x+x^2+x^3 \dots x^n) - n}{x-1} \right\}</math></p> $= \lim_{x \rightarrow 1} \left\{ \frac{(x+x^2+x^3 \dots x^n) - (1+1+1 \dots n \text{ terms})}{x-1} \right\}$ $= \lim_{x \rightarrow 1} \left\{ \frac{(x-1) + (x^2-1) + (x^3-1) \dots (x^n-1)}{x-1} \right\}$ $= \lim_{x \rightarrow 1} \left[ \frac{x-1}{x-1} \right] + \lim_{x \rightarrow 1} \left( \frac{x^2-1^2}{x-1} \right) + \lim_{x \rightarrow 1} \left( \frac{x^3-1^3}{x-1} \right) + \dots + \lim_{x \rightarrow 1} \left( \frac{x^n-1^n}{x-1} \right)$ $= 1 + 2(1)^1 + 3(1)^2 + \dots + n(1)^{n-1}$ $= 1 + 2 + 3 + \dots + n$ $= \frac{n(n+1)}{2} \text{ ans.}$
Q.10)	If $\lim_{x \rightarrow -a} \left( \frac{x^9+a^9}{x+a} \right) = 9$ . Find the value of $a$ .
Sol.10)	<p>We have <math>\lim_{x \rightarrow -a} \left( \frac{x^9 - (-a^9)}{x - (-a)} \right) = 9</math></p> $\Rightarrow 9(-a)^{9-1} = 9$ $\Rightarrow 9(-9)^8 = 9$ $\Rightarrow (-a)^8 = 1$ $\Rightarrow a^8 = 1$ $\Rightarrow a = \pm 1 \quad \text{ans.}$ <div style="text-align: right;"> <math display="block">\left\{ \lim_{x \rightarrow a} \left( \frac{x^n - a^n}{x - a} \right) = na^{n-1} \right\}</math> </div>