

<u>Class 11 Limits & Derivatives</u> Class 11th	
	TYPE: 5 TRIGO LIMITS
Q.1)	Evaluate: $\lim_{x \rightarrow 0} \left(\frac{5x+4 \sin(3x)}{4 \sin(2x)+7x} \right)$
Sol.1)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{5x+4 \sin(3x)}{4 \sin(2x)+7x} \right)$</p> $= \lim_{x \rightarrow 0} \left(\frac{5x + \frac{4 \sin(3x)}{3x} \times 3x}{\frac{4 \sin 2x}{2x} \times 2x + 7x} \right)$ $= \lim_{x \rightarrow 0} \left(\frac{x(5 + \frac{4 \sin(3x)}{3x} \times 3)}{x(\frac{4 \sin 2x}{2x} \times 2x + 7)} \right)$ $= \frac{5+12 \lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{3x} \right)}{8 \lim_{x \rightarrow 0} \left(\frac{\sin(2x)}{2x} \right) + 7}$ $= \frac{5+12(1)}{8(1)+7} \quad \left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$ $= \frac{17}{15} \text{ ans.}$
Q.2)	Evaluate: $\lim_{x \rightarrow 0} \left(\frac{1-\cos mx}{1-\cos nx} \right)$
Sol.2)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{1-\cos mx}{1-\cos nx} \right)$</p> $= \lim_{x \rightarrow 0} \left(\frac{2 \sin^2 \frac{mx}{2}}{2 \sin^2 \frac{nx}{2}} \right)$ $= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin^2 \frac{mx}{2}}{\frac{m^2 x^2}{4}} \times \frac{m^2 x^2}{4}}{\frac{\sin^2 \frac{nx}{2}}{\frac{n^2 x^2}{4}} \times \frac{n^2 x^2}{4}} \right)$ $= \frac{m^2 \times \lim_{x \rightarrow 0} \left(\frac{\sin^2 \frac{mx}{2}}{\frac{m^2 x^2}{4}} \right)}{n^2 \times \left(\frac{\sin^2 \frac{nx}{2}}{\frac{n^2 x^2}{4}} \right)}$ $= \frac{m^2(1)^2}{n^2(1)^2} \quad \left\{ \lim_{x \rightarrow 0} \left(\frac{\sin^2 x}{x^2} \right) = 1^2 \right\}$ $= \frac{m^2}{n^2} \text{ ans.}$
Q.3)	Evaluate: $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$
Sol.3)	We have $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$

	$ \begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{1-\cos x}{\sin x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{2 \sin^2 \left(\frac{x}{2} \right)}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \right) \\ &= \lim_{x \rightarrow 0} \left(\tan \frac{x}{2} \right) \\ &= \tan(0) = 0 \text{ ans.} \end{aligned} $
Q.4)	Evaluate: $\lim_{x \rightarrow 0} \left(\frac{x^3 \cot x}{1-\cos x} \right)$
Sol.4)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{x^3 \cot x}{1-\cos x} \right)$</p> $ \begin{aligned} &= \lim_{x \rightarrow 0} \left(x^3 \cdot \frac{1}{\tan x} \cdot \frac{1}{1-\cos x} \right) \\ &= \lim_{x \rightarrow 0} \left(x^3 \cdot \frac{1}{\tan x} \cdot \frac{1}{2 \sin^2 \left(\frac{x}{2} \right)} \right) \\ &= \lim_{x \rightarrow 0} \left(x^3 \cdot \frac{1}{\frac{\tan x}{x} \cdot x} \cdot \frac{1}{\frac{2 \sin^2 \left(\frac{x}{2} \right)}{\frac{x^2}{4}} \times \frac{x^2}{4}} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{x^3}{x^3} \cdot \frac{1}{\frac{\tan x}{x}} \cdot \frac{1}{\frac{1}{1/2} \cdot \frac{1}{\frac{2 \sin^2 \left(\frac{x}{2} \right)}{\frac{x^2}{4}}}} \right) \\ &= \frac{1}{\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)} \cdot \frac{1}{1/2} \cdot \frac{1}{\lim_{x \rightarrow 0} \left(\frac{\sin^2 \left(\frac{x}{2} \right)}{\frac{x^2}{4}} \right)} \\ &= \frac{1}{1} \times \frac{1}{1/2} \times \frac{1}{1^2} \\ &= 2 \text{ ans.} \quad \left\{ \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right\} \end{aligned} $
Q.5)	Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\sin(2x)+\sin(6x)}{\sin(5x)-\sin(3x)} \right)$
Sol.5)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{\sin(2x)+\sin(6x)}{\sin(5x)-\sin(3x)} \right)$</p> $ \begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin(2x)}{2x} \times 2x + \frac{\sin(6x)}{6x} \times 6x}{\frac{\sin(5x)}{5x} \times 5x - \frac{\sin(3x)}{3x} \times 3x} \right) \\ &= \lim_{x \rightarrow 0} \left[\frac{x \left(\frac{\sin(2x)}{2x} \times 2 + \frac{\sin(6x)}{6x} \times 6 \right)}{x \left(\frac{\sin(5x)}{5x} \times 5 - \frac{\sin(3x)}{3x} \times 3 \right)} \right] \\ &= \frac{2 \times \lim_{x \rightarrow 0} \left(\frac{\sin(2x)}{2x} \right) + 6 \times \lim_{x \rightarrow 0} \left(\frac{\sin(6x)}{6x} \right)}{5 \times 2 \times \lim_{x \rightarrow 0} \left(\frac{\sin(5x)}{5x} \right) - 3 \times \lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{3x} \right)} \end{aligned} $

	$ \begin{aligned} &= \frac{2(1)+6(1)}{5(1)-3(3)} \\ &= \frac{8}{2} = 4 \text{ ans.} \end{aligned} $	$\left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$
Q.6)	Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{x^3} \right)$	
Sol.6)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{x^3} \right)$</p> $ \begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin x}{\cos x} - \sin x}{x^3} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x - \sin x \cdot \cos x}{x^3 \cdot \cos x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x(1-\cos x)}{x^3 \cdot \cos x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x \cdot 2 \sin^2 \left(\frac{x}{2} \right)}{x^3 \cdot \cos x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{2 \sin^2 \left(\frac{x}{2} \right)}{x^2} \cdot \frac{1}{\cos x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \cdot \frac{2 \sin^2 \left(\frac{x}{2} \right)}{\frac{x^2}{4} \times 4} \cdot \frac{1}{\cos x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \times \frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{\sin^2 \left(\frac{x}{2} \right)}{\frac{x^2}{4}} \right) \lim_{x \rightarrow 0} \left(\frac{1}{\cos x} \right) \\ &= 1 \times \frac{1}{2} \times (1)^2 \times \frac{1}{1} \\ &= \frac{1}{2} \text{ ans.} \end{aligned} $	$\left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$
Q.7)	Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{\sin^3 x} \right)$	
Sol.7)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{\tan x - \sin x}{\sin^3 x} \right)$</p> $ \begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x - \sin x \cdot \cos x}{\sin^3 x \cdot \cos x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x(1-\cos x)}{\sin^3 x \cdot \cos x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin x \cdot 2 \sin^2 \left(\frac{x}{2} \right)}{\sin^3 x \cdot \cos x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{2 \sin^2 \left(\frac{x}{2} \right)}{\sin^2 x \cdot \cos x} \right) \end{aligned} $	

	$ \begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{\frac{2 \sin^2(\frac{x}{2}) \times x^2}{\frac{x^2}{4}}}{\frac{\sin^2 x}{x^2} \times x^2 \cdot \cos x} \right) \\ &= \frac{\frac{1}{2} \lim_{x \rightarrow 0} \left(\frac{\sin^2(\frac{x}{2})}{\frac{x^2}{4}} \right)}{\lim_{x \rightarrow 0} \left(\frac{\sin^2 x}{x^2} \right) \lim_{x \rightarrow 0} (\cos x)} \\ &= \frac{\frac{1}{2}(1)^2}{(1)(1)} \quad \left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\} \\ &= \frac{1}{2} \text{ ans.} \end{aligned} $
Q.8)	Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\sec(4x) - \sec(2x)}{\sec(3x) - \sec x} \right)$
Sol.8)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{\sec(4x) - \sec(2x)}{\sec(3x) - \sec x} \right)$</p> $ \begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{\frac{1}{\cos(4x)} - \frac{1}{\cos(2x)}}{\frac{1}{\cos(3x)} - \frac{1}{\cos x}} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\frac{\cos(2x) - \cos(4x)}{\cos(4x) \cdot \cos(2x)}}{\frac{\cos x - \cos(3x)}{\cos(3x) \cdot \cos x}} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\cos(2x) - \cos(4x)}{\cos x - \cos(3x)} \cdot \frac{\cos x \cdot \cos(3x)}{\cos(4x) \cdot \cos(2x)} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{-2 \sin(3x) \cdot \sin(-x)}{-2 \sin(2x) \cdot \sin(-x)} \cdot \frac{\cos(3x) \cdot \cos x}{\cos(4x) \cdot \cos(2x)} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\frac{\sin(3x)}{3x} \times 3x}{\frac{\sin(2x)}{2x} \times 2x} \cdot \frac{\cos(3x) \cdot \cos x}{\cos(4x) \cdot \cos(2x)} \right) \\ &= \frac{3 \lim_{x \rightarrow 0} \left(\frac{\sin(3x)}{3x} \right)}{2 \lim_{x \rightarrow 0} \left(\frac{\sin(2x)}{2x} \right)} \cdot \lim_{x \rightarrow 0} \left(\frac{\cos(3x) \cdot \cos x}{\cos(4x) \cdot \cos(2x)} \right) \\ &= \frac{3 \times 1}{2 \times 1} \cdot \left(\frac{(1)(1)}{(1)(1)} \right) \quad \left\{ \begin{array}{l} \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \\ \text{and} \\ \cos \theta = 1 \end{array} \right\} \\ &= \frac{3}{2} \text{ ans.} \end{aligned} $
Q.9)	Evaluate: $\lim_{y \rightarrow 0} \left(\frac{(x+y) \sec(x+y) - x \sec x}{y} \right)$
Sol.9)	<p>We have $\lim_{y \rightarrow 0} \left(\frac{(x+y) \sec(x+y) - x \sec x}{y} \right)$</p> <p>Here y is the variable not x</p> $ \begin{aligned} &= \lim_{y \rightarrow 0} \left(\frac{x \sec(x+y) + y \sec(x+y) - x \sec x}{y} \right) \\ &= \lim_{y \rightarrow 0} \left(\frac{x \{\sec(x+y) - \sec x\} + y \sec(x+y)}{y} \right) \end{aligned} $

	$ \begin{aligned} &= \lim_{y \rightarrow 0} \left(\frac{x \left\{ \frac{1}{\cos(x+y)} - \frac{1}{\cos x} \right\}}{y} + \frac{y \sec(x+y)}{y} \right) && \left\{ \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \right\} \\ &= \lim_{y \rightarrow 0} \left(\frac{x \cdot \{\cos(x) - \cos(x+y)\}}{y \cdot \cos(x+y) \cdot \cos x} + \sec(x+y) \right) \\ &= \lim_{y \rightarrow 0} \left(\frac{x \cdot \{-2 \sin\left(\frac{2x+y}{2}\right) - \sin\left(\frac{-y}{2}\right)\}}{2 \times \frac{y}{2} \cdot \cos(x+y) \cdot \cos x} + \sec(x+y) \right) \\ &= \lim_{y \rightarrow 0} \left(\frac{\sin \frac{y}{2}}{\frac{y}{2}} \right) \cdot \lim_{y \rightarrow 0} \left(\frac{x \cdot \sin\left(\frac{2x+y}{2}\right)}{\cos(x+y) \cdot \cos x} \right) + \lim_{y \rightarrow 0} \sec(x+y) \\ &= 1 \times x \frac{\sin x}{\cos x \cdot \cos x} + \sec x && \left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\} \\ &= x \tan x \cdot \sec x + \sec x \text{ ans.} \end{aligned} $
Q.10)	Evaluate: $\lim_{x \rightarrow 0} \left(\frac{1-\cos x \sqrt{\cos 2x}}{x^2} \right)$
Sol.10)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{1-\cos x \sqrt{\cos 2x}}{x^2} \right)$</p> <p>Rationalize</p> $ \begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{(1-\cos x \sqrt{\cos 2x})(1+\cos x \sqrt{\cos 2x})}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{1-\cos^2 x (\cos 2x)}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{1-\cos^2 x (2 \cos^2 x - 1)}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{1-2\cos^4 x + \cos^2 x}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\ &= -\lim_{x \rightarrow 0} \left(\frac{2\cos^4 x - \cos^2 x - 1}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\ &= -\lim_{x \rightarrow 0} \left(\frac{2\cos^4 x - (\cos^2 x - 1) + 1(\cos^2 x - 1)}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\ &= -\lim_{x \rightarrow 0} \left(\frac{(2\cos^2 x + 1)(\cos^2 x - 1)}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{(2\cos^2 x + 1)(1-\cos^2 x)}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{(2\cos^2 x + 1) \sin^2 x}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{\sin^2 x}{x^2} \right) \times \lim_{x \rightarrow 0} \left(\frac{(2\cos^2 x + 1)}{1+\cos x \sqrt{\cos 2x}} \right) \\ &= (1)^2 \times \left(\frac{2(1)+1}{1+(1)(1)} \right) \\ &= \frac{3}{2} \text{ ans.} \end{aligned} $