	Class 11 Limits & Derivatives ***Class 11th***
	TYPE: 5 TRIGO LIMITS
Q.1)	Evaluate: $\lim\limits_{x\to 0}\left(\frac{\operatorname{cosec} x-\cot x}{x}\right)$
Sol.1)	We have $\lim\limits_{x\to 0}\left(\frac{\operatorname{cosec} x-\cot x}{x}\right)$ $=\lim\limits_{x\to 0}\left(\frac{\frac{1}{\sin x}-\frac{\cos x}{\sin x}}{x}\right)$ $=\lim\limits_{x\to 0}\left(\frac{1-\cos x}{x\sin x}\right)$ $=\lim\limits_{x\to 0}\left(\frac{2\sin^2\left(\frac{x}{2}\right)}{x\sin x}\right)$ $=\lim\limits_{x\to 0}\left(\frac{\frac{2\sin^2\left(\frac{x}{2}\right)}{\frac{x^2}{4}}\times\frac{x^2}{4}}{\frac{x\sin x}{x}\times x}\right)$ $=\frac{2}{4}\frac{\lim\limits_{x\to 0}\left(\frac{\sin^2\left(\frac{x}{2}\right)}{\frac{x^2}{4}}\right)}{\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)}$ $=\frac{1}{2}\times\frac{(1)}{(1)}$ $\quad\left\{\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$ $=\frac{1}{2}$ ans.
	TYPE: 6 TRIGO LIMITS **When** $\lim\limits_{x\to a} f(x)$
Q.2)	Evaluate: $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{1+\cos(2x)}{(\pi-2x)^2}\right)$
Sol.2)	We have : $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{1+\cos(2x)}{(\pi-2x)^2}\right)$ Put $x=\frac{\pi}{2}+h$ and $h\to 0$ $\therefore\ \lim\limits_{h\to 0}\left(\frac{1+\cos\left(2\left(\frac{\pi}{2}+h\right)\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)\right)^2}\right)$ $=\lim\limits_{h\to 0}\left(\frac{1+\cos(\pi+2h)}{(\pi-\pi-2h)^2}\right)$

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	$= \lim_{h\to 0}\left(\frac{1-\cos(2h)}{4h^2}\right)$ $= \lim_{h\to 0}\left(\frac{2\sin^2 h}{4h^2}\right)$ $= \frac{1}{2}\lim_{h\to 0}\left(\frac{\sin^2 h}{h^2}\right)$ $= \frac{1}{2}(1)^2$ $\quad \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1\right\}$ $= \frac{1}{2}$ ans.
Q.3)	Evaluate: $\lim_{x\to\frac{\pi}{6}}\left(\frac{2-\sqrt{3}\cos x-\sin x}{(6x-\pi)^2}\right)$
Sol.3)	We have $\lim_{x\to\frac{\pi}{6}}\left(\frac{2-\sqrt{3}\cos x-\sin x}{(6x-\pi)^2}\right)$ Put $x = \frac{\pi}{6} + h$ and $h \to 0$ $= \lim_{h\to 0}\left(\frac{2-\sqrt{3}\cos\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}+h\right)}{\left(6\left(\frac{\pi}{6}+h\right)-\pi\right)^2}\right)$ $= \lim_{h\to 0}\left(\frac{2-\sqrt{3}\left(\cos\frac{\pi}{6}\cos h-\sin\frac{\pi}{6}\sin h\right)-\left(\sin\frac{\pi}{6}\sin h+\cos\frac{\pi}{6}\cos h\right)}{(\pi+6h-\pi)^2}\right)$ $= \lim_{h\to 0}\left(\frac{2-\sqrt{3}\left(\frac{\sqrt{3}}{2}\cos h-\frac{1}{2}\sin h\right)-\left(\frac{1}{2}\cos h+\frac{\sqrt{3}}{2}\sin h\right)}{36h^2}\right)$ $= \lim_{h\to 0}\left(\frac{2-\frac{3}{2}\cos h+\frac{\sqrt{3}}{2}\sin h-\frac{1}{2}\cos h-\frac{\sqrt{3}}{2}\sin h}{36h^2}\right)$ $= \lim_{h\to 0}\left(\frac{2-2\cos h}{36h^2}\right)$ $= \lim_{h\to 0}\left(\frac{2(1-\cos h)}{36h^2}\right)$ $= \frac{1}{18}\lim_{h\to 0}\left(\frac{2\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}\times 4}\right)$ $= \frac{1}{36}\lim_{h\to 0}\left(\frac{\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}}\right)$ $\quad \left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right) = 1\right\}$ $= \frac{1}{36}\times(1)^2 = \frac{1}{36}$ ans.

Q.5)	Evaluate: $\lim_{x\to\pi}\left(\frac{\sin(3x)-3\sin x}{(\pi-x)^3}\right)$
Sol.5)	We have $\lim_{x\to\pi}\left(\frac{\sin(3x)-3\sin x}{(\pi-x)^3}\right)$ $=\lim_{x\to\pi}\left(\frac{3\sin x-4\sin^3 x-3\sin x}{(\pi-x)^3}\right)$ $=\lim_{x\to\pi}\left(\frac{-4\sin^3 x}{(\pi-x)^3}\right)$ Put $x=\pi+h$ and $h\to 0$ $=\lim_{h\to 0}\left(\frac{-4\sin^3(\pi+h)-3\sin(\pi+h)}{(\pi-\pi+h)^3}\right)$ $=\lim_{h\to 0}\left(\frac{+4\sin^3 h}{-h^3}\right)$ $\{\sin(\pi+\theta)=-\sin\theta\}$ $=-4\lim_{h\to 0}\left(\frac{\sin^3 h}{h^3}\right)$ $=-4(1)^3=-4$ ans. $\left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$
Q.6)	Evaluate: $\lim_{x\to\frac{\pi}{2}}\left(\frac{\cot x-\cos x}{(\pi-2x)^3}\right)$
Sol.6)	We have $\lim_{x\to\frac{\pi}{2}}\left(\frac{\cot x-\cos x}{(\pi-2x)^3}\right)$ Put $x=\frac{\pi}{2}+h$ and $h\to 0$ $=\lim_{h\to 0}\left(\frac{\cot\left(\frac{\pi}{2}+h\right)-\cos\left(\frac{\pi}{2}+h\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)\right)^3}\right)$ $=\lim_{h\to 0}\left(\frac{-\tan h+\sin h}{(\pi-\pi+2h)^3}\right)$ $=\lim_{h\to 0}\left(\frac{-\tan h+\sin h}{-8h^3}\right)$ $=\lim_{h\to 0}\left(\frac{\tan h-\sin h}{8h^3}\right)$ $=\frac{1}{8}\lim_{h\to 0}\left(\frac{\frac{\sin h}{\cos h}-\sin h}{h^3}\right)$ $=\frac{1}{8}\lim_{h\to 0}\left(\frac{\sin h-\sin h.\cos h}{h^3.\cos h}\right)$ $=\frac{1}{8}\lim_{h\to 0}\left(\frac{\sin h\,(1-\cos h)}{h^3.\cos h}\right)$ $=\frac{1}{8}\lim_{h\to 0}\left(\frac{\sin h.\,2\sin^2\left(\frac{h}{2}\right)}{h^3.\cos h}\right)$

	$= \frac{1}{8}\lim_{h\to 0}\left(\frac{\sin h}{h}.\frac{2\sin^2\left(\frac{h}{2}\right)}{h^2}.\frac{1}{\cos h}\right)$ $= \frac{1}{8}\lim_{h\to 0}\left(\frac{\sin h}{h}.\frac{2\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}\times 4}.\frac{1}{\cos h}\right)$ $= \frac{1}{8}\lim_{h\to 0}\left(\frac{\sin h}{h}\right)\times\frac{2}{4}.\lim_{h\to 0}\left(\frac{\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}}\right)\times\lim_{h\to 0}\left(\frac{1}{\cos h}\right)$ $= \frac{1}{8}\times 1\times\frac{1}{2}\times 1^2\times 1$ $\left\{\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right\}$ $= \frac{1}{16}$ ans.
Q.7)	Evaluate: $\lim_{x\to\frac{\pi}{2}}\left(\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}\right)$
Sol.7)	We have $\lim_{x\to\frac{\pi}{2}}\left(\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}\right)$ Rationalize $= \lim_{x\to\frac{\pi}{2}}\left[\frac{(\sqrt{2}-\sqrt{1+\sin x})(\sqrt{2}+\sqrt{1+\sin x})}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}\right]$ $= \lim_{x\to\frac{\pi}{2}}\left[\frac{2-1-\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}\right]$ $= \lim_{x\to\frac{\pi}{2}}\left[\frac{1-\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}\right]$ Put $x=\frac{\pi}{2}+h$ and $h\to 0$ $= \lim_{h\to 0}\left[\frac{1-\sin\left(\frac{\pi}{2}+h\right)}{\cos^2\left(\frac{\pi}{2}+h\right)}\right]\times\lim_{x\to\frac{\pi}{2}}\left[\frac{1}{\sqrt{2}+\sqrt{1+\sin x}}\right]$ $= \lim_{h\to 0}\left(\frac{1-\cos h}{\sin^2 h}\right)\times\frac{1}{(\sqrt{2}+\sqrt{2})}$ $= \lim_{h\to 0}\left(\frac{2\sin^2\left(\frac{h}{2}\right)}{\sin^2 h}\right)\times\frac{1}{2\sqrt{2}}$ $= \lim_{h\to 0}\left(\frac{\frac{2\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}}\times\frac{h^2}{4}}{\frac{\sin^2 h}{h^2}\times h^2}\right)\times\frac{1}{2\sqrt{2}}$

	$= \dfrac{\frac{2}{4}\lim\limits_{h\to 0}\left(\dfrac{\sin^2\left(\frac{h}{2}\right)}{\frac{h^2}{4}}\right)}{\lim\limits_{h\to 0}\left(\frac{\sin^2 h}{h^2}\right)} \times \dfrac{1}{2\sqrt{2}}$ $\left\{\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right) = 1\right\}$ $= \frac{1}{2} \times \frac{1}{1} \times \frac{1}{2\sqrt{2}} = \frac{1}{4\sqrt{2}}$ ans.
Q.8)	Evaluate: $\lim\limits_{x\to 0}\left(\frac{1-\cos(2x)}{\cos(2x)-\cos(8x)}\right)$
Sol.8)	We have $\lim\limits_{x\to 0}\left(\frac{1-\cos(2x)}{\cos(2x)-\cos(8x)}\right)$ $= \lim\limits_{x\to 0}\left[\frac{2\sin^2 x}{-2\sin(5x).\sin(-3x)}\right]$ $= \lim\limits_{x\to 0}\left[\dfrac{\frac{\sin^2 x}{x^2}\times x^2}{15\times\frac{\sin(5x)}{5x}.\frac{\sin(3x)}{3x}}\right]$ $= \dfrac{\lim\limits_{x\to 0}\frac{\sin^2 x}{x^2}}{15\times\lim\limits_{x\to 0}\frac{\sin(5x)}{5x}\times\lim\limits_{x\to 0}\frac{\sin(3x)}{3x}}$ $= \frac{1^2}{15(1)(1)} = \frac{1}{15}$ ans. $\left\{\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right) = 1\right\}$
Q.9)	Evaluate: $\lim\limits_{x\to a}\left(\frac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}\right)$
Sol.9)	We have $\lim\limits_{x\to a}\left(\frac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}\right)$ Rationalize $= \lim\limits_{x\to a}\left(\frac{(\sin x-\sin a)(\sqrt{x}-\sqrt{a})}{x-a}\right)$ Put $x = a + h$ and $h \to 0$ $= \lim\limits_{h\to 0}\left(\frac{(\sin(a+h)-\sin a)(\sqrt{a+h}-\sqrt{a})}{a+h-a}\right)$ $= \lim\limits_{h\to 0}\left(\frac{2\cos\left(\frac{2a+h}{2}\right).\sin\left(\frac{h}{2}\right).(\sqrt{a+h}-\sqrt{a})}{h}\right)$ $= \lim\limits_{h\to 0}\left(\dfrac{2\cos\left(\frac{2a+h}{2}\right).\sin\left(\frac{h}{2}\right).(\sqrt{a+h}-\sqrt{a})}{\frac{h}{2}\times 2}\right)$ $= \lim\limits_{h\to 0}\left(\dfrac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\right) \times \lim\limits_{h\to 0}\left(\cos\left(\frac{2a+h}{2}\right).\left(\sqrt{a+h}-\sqrt{a}\right)\right)$ $= 1 \times \cos(a)\left(\sqrt{a}+\sqrt{a}\right)$ $\left\{\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right) = 1\right\}$ $= 2\sqrt{a}\cos a$ ans.
	TYPE: 7 $\lim\limits_{x\to 0}\left(\frac{e^x-1}{x}\right)$ **and** $\lim\limits_{x\to 0}\frac{\log(1+x)}{x}$

Q.10)	Evaluate $\lim\limits_{x \to 0} \left(\frac{a^x - b^x}{x}\right)$
Sol.10)	We have $\lim\limits_{x \to 0} \left(\frac{a^x - b^x}{x}\right)$ $= \lim\limits_{x \to 0} \left(\frac{a^x - b^x - 1 + 1}{x}\right)$ $= \lim\limits_{x \to 0} \left(\frac{a^x - 1}{x} - \frac{b^x - 1}{x}\right)$ $= \lim\limits_{x \to 0} \left(\frac{a^x - 1}{x}\right) - \lim\limits_{x \to 0} \left(\frac{b^x - 1}{x}\right)$ $= \log a - \log b$ $= \log\left(\frac{a}{b}\right)$ ans. $\left\{\lim\limits_{x \to 0} \left(\frac{a^x - 1}{x}\right) = \log a\right\}$