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|  | Class 11 Limits \& Derivatives Class $11^{\text {th }}$ |
| :---: | :---: |
|  | TYPE: 5 TRIGO LIMITS |
| Q.1) | Evaluate: $\lim _{x \rightarrow 0}\left(\frac{\operatorname{cosec} x-\cot x}{x}\right)$ |
| Sol.1) | We have $\lim _{x \rightarrow 0}\left(\frac{\operatorname{cosec} x-\cot x}{x}\right)$ $\begin{aligned} & =\lim _{x \rightarrow 0}\left(\frac{\frac{1}{\sin n}-\frac{\cos x}{\sin x}}{x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{1-\cos x}{x \sin x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{2 \sin ^{2}\left(\frac{x}{2}\right)}{x \sin x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{\frac{2 \sin ^{2}\left(\frac{x}{2}\right)}{\frac{x^{2}}{4}} \times \frac{x^{2}}{4}}{\frac{x \sin x}{x} \times x}\right) \\ & =\frac{2}{4} \frac{\lim _{x \rightarrow 0}\left(\frac{\sin ^{2}\left(\frac{x}{2}\right)}{\frac{x^{2}}{4}}\right)}{\lim _{x \rightarrow 0}\left(\frac{\sin x}{x}\right)} \\ & =\frac{1}{2} \times \frac{(1)}{(1)} \\ & =\frac{1}{2} \text { ans. } \end{aligned}$ |
|  | TYPE: 6 TRIGO LIMITS When $\lim _{x \rightarrow a} f(x)$ |
| Q.2) | Evaluate: $\lim _{x \rightarrow \frac{\pi}{2}}\left(\frac{1+\cos (2 x)}{(\pi-2 x)^{2}}\right)$ |
| Sol.2) | We have : $\lim _{x \rightarrow \frac{\pi}{2}}\left(\frac{1+\cos (2 x)}{(\pi-2 x)^{2}}\right)$ <br> Put $x=\frac{\pi}{2}+h$ and $h \rightarrow 0$ $\begin{aligned} & \therefore \lim _{h \rightarrow 0}\left(\frac{1+\cos \left(2\left(\frac{\pi}{2}+h\right)\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)\right)^{2}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{1+\cos (\pi+2 h)}{(\pi-\pi-2 h)^{2}}\right) \end{aligned}$ |

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|  | $\begin{aligned} & =\lim _{h \rightarrow 0}\left(\frac{1-\cos (2 h)}{4 h^{2}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{2 \sin ^{2} h}{4 h^{2}}\right) \\ & =\frac{1}{2} \lim _{h \rightarrow 0}\left(\frac{\sin ^{2} h}{h^{2}}\right) \\ & =\frac{1}{2}(1)^{2} \quad\left\{\lim _{x \rightarrow 0}\left(\frac{\sin x}{x}\right)=1\right\} \\ & =\frac{1}{2} \text { ans. } \end{aligned}$ |
| :---: | :---: |
| Q.3) | Evaluate: $\lim _{x \rightarrow \frac{\pi}{6}}\left(\frac{2-\sqrt{3} \cos x-\sin x}{(6 x-\pi)^{2}}\right)$ |
| Sol.3) | $\begin{aligned} & \text { We have } \lim _{x \rightarrow \frac{\pi}{6}}\left(\frac{2-\sqrt{3} \cos x-\sin x}{(6 x-\pi)^{2}}\right) \\ & \text { Put } x=\frac{\pi}{6}+h \text { and } h \rightarrow 0 \\ & =\lim _{h \rightarrow 0}\left(\frac{2-\sqrt{3} \cos \left(\frac{\pi}{6}+h\right)-\sin \left(\frac{\pi}{6}+h\right)}{\left(6\left(\frac{\pi}{6}+h\right)-\pi\right)^{2}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{2-\sqrt{3}\left(\cos \frac{\pi}{6} \cos h-\sin \frac{\pi}{6} \sin h\right)-\left(\sin \frac{\pi}{6} \sin h+\cos \frac{\pi}{6} \cos h\right)}{(\pi+6 h-\pi)^{2}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{2-\sqrt{3}\left(\frac{\sqrt{3}}{2} \cos h-\frac{1}{2} \sin h\right)-\left(\frac{1}{2} \cos h+\frac{\sqrt{3}}{2} \sin h\right)}{36 h^{2}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{2-\frac{3}{2} \cos h+\frac{\sqrt{3}}{2} \sin h-\frac{1}{2} \cos h-\frac{\sqrt{3}}{2} \sin h}{36 h^{2}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{2-2 \cos h}{36 h^{2}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{2(1-\cos h)}{36 h^{2}}\right) \\ & =\frac{1}{18} \lim _{h \rightarrow 0}\left(\frac{2 \sin ^{2}\left(\frac{h}{2}\right)}{\frac{h^{2}}{4} \times 4}\right) \\ & =\frac{1}{36} \lim _{h \rightarrow 0}\left(\frac{\sin ^{2}\left(\frac{h}{2}\right)}{\frac{h^{2}}{4}}\right) \\ & =\frac{1}{36} \times(1)^{2}=\frac{1}{36} \text { ans. } \end{aligned}$ |

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| Q.5) | Evaluate: $\lim _{x \rightarrow \pi}\left(\frac{\sin (3 x)-3 \sin x}{(\pi-x)^{3}}\right)$ |
| :---: | :---: |
| Sol.5) | We have $\lim _{x \rightarrow \pi}\left(\frac{\sin (3 x)-3 \sin x}{(\pi-x)^{3}}\right)$ $\begin{aligned} & =\lim _{x \rightarrow \pi}\left(\frac{3 \sin x-4 \sin ^{3} x-3 \sin x}{(\pi-x)^{3}}\right) \\ & =\lim _{x \rightarrow \pi}\left(\frac{-4 \sin ^{3} x}{(\pi-x)^{3}}\right) \end{aligned}$ <br> Put $x=\pi+h$ and $h \rightarrow 0$ $\begin{aligned} & =\lim _{h \rightarrow 0}\left(\frac{-4 \sin ^{3}(\pi+h)-3 \sin (\pi+h)}{(\pi-\pi+h)^{3}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{+4 \sin ^{3} h}{-h^{3}}\right) \\ & =-4 \lim _{h \rightarrow 0}\left(\frac{\sin ^{3} h}{h^{3}}\right) \\ & =-4(1)^{3}=-4 \text { ans. } \end{aligned}$ $\{\sin (\pi+\theta)=-\sin \theta\}$ $\left\{\lim _{x \rightarrow 0}\left(\frac{\sin x}{x}\right)=1\right\}$ |
| Q.6) | Evaluate: $\lim _{x \rightarrow \frac{\pi}{2}}\left(\frac{\cot x-\cos x}{(\pi-2 x)^{3}}\right)$ |
| Sol.6) | We have $\lim _{x \rightarrow \frac{\pi}{2}}\left(\frac{\cot x-\cos x}{(\pi-2 x)^{3}}\right)$ <br> Put $x=\frac{\pi}{2}+h$ and $h \rightarrow 0$ $\begin{aligned} & =\lim _{h \rightarrow 0}\left(\frac{\cot \left(\frac{\pi}{2}+h\right)-\cos \left(\frac{\pi}{2}+h\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)\right)^{3}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{-\tan h+\sin h}{(\pi-\pi+2 h)^{3}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{-\tan h+\sin h}{-8 h^{3}}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{\tan h-\sin h}{8 h^{3}}\right) \\ & =\frac{1}{8} \lim _{h \rightarrow 0}\left(\frac{\frac{\sin h}{\cos h}-\sin h}{h^{3}}\right) \\ & =\frac{1}{8} \lim _{h \rightarrow 0}\left(\frac{\sin h-\sin h \cdot \cos h}{h^{3} \cdot \cos h}\right) \\ & =\frac{1}{8} \lim _{h \rightarrow 0}\left(\frac{\sin h(1-\cos h)}{h^{3} \cdot \cos h}\right) \\ & =\frac{1}{8} \lim _{h \rightarrow 0}\left(\frac{\sin h \cdot 2 \sin 2\left(\frac{h}{2}\right)}{h^{3} \cdot \cos h}\right) \end{aligned}$ |

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|  | $\begin{aligned} & =\frac{1}{8} \lim _{h \rightarrow 0}\left(\frac{\sin h}{h} \cdot \frac{2 \sin ^{2}\left(\frac{h}{2}\right)}{h^{2}} \cdot \frac{1}{\cos h}\right) \\ & =\frac{1}{8} \lim _{h \rightarrow 0}\left(\frac{\sin h}{h} \cdot \frac{2 \sin ^{2}\left(\frac{h}{2}\right)}{\frac{h^{2}}{4} \times 4} \cdot \frac{1}{\cos h}\right) \\ & =\frac{1}{8} \lim _{h \rightarrow 0}\left(\frac{\sin h}{h}\right) \times \frac{2}{4} \cdot \lim _{h \rightarrow 0}\left(\frac{\sin ^{2}\left(\frac{h}{2}\right)}{\frac{h^{2}}{4}}\right) \times \lim _{h \rightarrow 0}\left(\frac{1}{\cos h}\right) \\ & =\frac{1}{8} \times 1 \times \frac{1}{2} \times 1^{2} \times 1 \quad \quad\left\{\lim _{x \rightarrow 0}\left(\frac{\sin x}{x}\right)=1\right\} \\ & =\frac{1}{16} \text { ans. } \end{aligned}$ |
| :---: | :---: |
| Q.7) | Evaluate: $\lim _{x \rightarrow \frac{\pi}{2}}\left(\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos ^{2} x}\right)$ |
| Sol.7) | We have $\lim _{x \rightarrow \frac{\pi}{2}}\left(\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos ^{2} x}\right)$ <br> Rationalize $\begin{aligned} & =\lim _{x \rightarrow \frac{\pi}{2}}\left[\frac{(\sqrt{2}-\sqrt{1+\sin x})(\sqrt{2}+\sqrt{1+\sin x})}{\cos ^{2} x(\sqrt{2}+\sqrt{1+\sin x})}\right] \\ & =\lim _{x \rightarrow \frac{\pi}{2}}\left[\frac{2-1-\sin x}{\cos ^{2} x(\sqrt{2}+\sqrt{1+\sin x})}\right] \\ & =\lim _{x \rightarrow \frac{\pi}{2}}\left[\frac{1-\sin x}{\cos ^{2} x(\sqrt{2}+\sqrt{1+\sin x})}\right] \end{aligned}$ <br> Put $x=\frac{\pi}{2}+h$ and $h \rightarrow 0$ $\begin{aligned} & =\lim _{h \rightarrow 0}\left[\frac{1-\sin \left(\frac{\pi}{2}+h\right)}{\cos ^{2}\left(\frac{\pi}{2}+h\right)}\right] \times \lim _{x \rightarrow \frac{\pi}{2}}\left[\frac{1}{\sqrt{2}+\sqrt{1+\sin x}}\right] \\ & =\lim _{h \rightarrow 0}\left(\frac{1-\cos h}{\sin ^{2} h}\right) \times \frac{1}{(\sqrt{2}+\sqrt{2})} \\ & =\lim _{h \rightarrow 0}\left(\frac{2 \sin ^{2}\left(\frac{h}{2}\right)}{\sin ^{2} h}\right) \times \frac{1}{2 \sqrt{2}} \\ & =\lim _{h \rightarrow 0}\left(\frac{\frac{2 \sin ^{2}\left(\frac{h}{2}\right)}{\frac{h^{2}}{4}} \times \frac{h^{2}}{4}}{\frac{\sin ^{2} h}{h^{2}} \times h^{2}}\right) \times \frac{1}{2 \sqrt{2}} \end{aligned}$ |

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|  | $\begin{aligned} & =\frac{\frac{2}{4} \lim _{h \rightarrow 0}\left(\frac{\sin ^{2}\left(\frac{h}{2}\right)}{\frac{h^{2}}{4}}\right)}{\lim _{h \rightarrow 0}\left(\frac{\sin ^{2} h}{h^{2}}\right)} \times \frac{1}{2 \sqrt{2}} \\ & =\frac{1}{2} \times \frac{1}{1} \times \frac{1}{2 \sqrt{2}}=\frac{1}{4 \sqrt{2}} \text { ans. } \end{aligned} \quad\left\{\lim _{x \rightarrow 0}\left(\frac{\sin x}{x}\right)=1\right\}$ |
| :---: | :---: |
| Q.8) | Evaluate: $\lim _{x \rightarrow 0}\left(\frac{1-\cos (2 x)}{\cos (2 x)-\cos (8 x)}\right)$ |
| Sol.8) | We have $\lim _{x \rightarrow 0}\left(\frac{1-\cos (2 x)}{\cos (2 x)-\cos (8 x)}\right)$ $\begin{aligned} & =\lim _{x \rightarrow 0}\left[\frac{2 \sin ^{2} x}{-2 \sin (5 x) \cdot \sin (-3 x)}\right] \\ & =\lim _{x \rightarrow 0}\left[\frac{\frac{\sin ^{2} x}{x^{2}} \times x^{2}}{\left.15 \times \frac{\sin (5 x)}{5 x} \cdot \frac{\sin (3 x)}{3 x}\right]}\right. \\ & =\frac{\lim _{x \rightarrow 0} \frac{\sin ^{2} x}{x^{2}}}{15 \times \lim _{x \rightarrow 0}^{\sin (5 x)}} 5 \times \lim _{5 x}^{\sin (3 x)} \\ & 3 x \rightarrow 0 \\ & \end{aligned} \frac{1^{2}}{15(1)(1)}=\frac{1}{15} \text { ans. } \quad\left\{\lim _{x \rightarrow 0}\left(\frac{\sin x}{x}\right)=1\right\},$ |
| Q.9) | Evaluate: $\lim _{x \rightarrow a}\left(\frac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}\right)$ |
| Sol.9) | We have $\lim _{x \rightarrow a}\left(\frac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}\right)$ <br> Rationalize $=\lim _{x \rightarrow a}\left(\frac{(\sin x-\sin a)(\sqrt{x}-\sqrt{a})}{x-a}\right)$ <br> Put $x=a+h$ and $h \rightarrow 0$ $\begin{aligned} & =\lim _{h \rightarrow 0}\left(\frac{(\sin (a+h)-\sin a)(\sqrt{a+h}-\sqrt{a})}{a+h-a}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{2 \cos \left(\frac{2 a+h}{2}\right) \cdot \sin \left(\frac{h}{2}\right) \cdot(\sqrt{a+h}-\sqrt{a})}{h}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{2 \cos \left(\frac{2 a+h}{2}\right) \cdot \sin \left(\frac{h}{2}\right) \cdot(\sqrt{a+h}-\sqrt{a})}{\frac{h}{2} \times 2}\right) \\ & =\lim _{h \rightarrow 0}\left(\frac{\sin \left(\frac{h}{2}\right)}{\frac{h}{2}}\right) \times \lim _{h \rightarrow 0}\left(\cos \left(\frac{2 a+h}{2}\right) \cdot(\sqrt{a+h}-\sqrt{a})\right) \\ & =1 \times \cos (a)(\sqrt{a}+\sqrt{a}) \quad\left\{\lim _{x \rightarrow 0}\left(\frac{\sin x}{x}\right)=1\right\} \\ & =2 \sqrt{a} \cos a \text { ans. } \end{aligned}$ |
|  | TYPE: 7 $\lim _{x \rightarrow 0}\left(\frac{e^{x}-1}{x}\right) \text { and } \lim _{x \rightarrow 0} \frac{\log (1+x)}{x}$ |

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| Q.10) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{a^{x}-b^{x}}{x}\right)$ |  |
| :--- | :--- | :--- |
| Sol.10) | We have $\lim _{x \rightarrow 0}\left(\frac{a^{x}-b^{x}}{x}\right)$  <br>  $=\lim _{x \rightarrow 0}\left(\frac{a^{x}-b^{x}-1+1}{x}\right)$ <br>  $=\lim _{x \rightarrow 0}\left(\frac{a^{x}-1}{x}-\frac{b^{x}-1}{x}\right)$ <br>  $=\lim _{x \rightarrow 0}\left(\frac{a^{x}-1}{x}\right)-\lim _{x \rightarrow 0}\left(\frac{b^{x}-1}{x}\right)$ <br>  $=\log a-\log b$ <br>  $=\log \left(\frac{a}{b}\right)$ ans. |  |
|  |  | $\left\{\begin{array}{l}\left.\lim _{x \rightarrow 0}\left(\frac{a^{x}-1}{x}\right)=\log a\right\}\end{array}\right.$ |

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