

<u><b>Class 11 Limits &amp; Derivatives</b></u> <b>Class 11<sup>th</sup></b>	
	<b>TYPE: 7</b>
	$\lim_{x \rightarrow 0} \left( \frac{e^x - 1}{x} \right)$ and $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}$
Q.1)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{2^x - 1}{\sqrt{1+x} - 1} \right)$
Sol.1)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{2^x - 1}{\sqrt{1+x} - 1} \right)</math></p> <p>Rationalize</p> $  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{2^x - 1}{\sqrt{1+x} - 1} \times \frac{(\sqrt{1+x} + 1)}{(\sqrt{1+x} + 1)} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{(2^x - 1)(\sqrt{1+x} + 1)}{1+x-1} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{(2^x - 1)}{1+x-1} \right) \times \lim_{x \rightarrow 0} (\sqrt{1+x} + 1) \\  &= \log 2 \times (1 + 1) \\  &= 2 \log 2 \text{ ans.} \qquad \qquad \qquad \left\{ \lim_{x \rightarrow 0} \left( \frac{a^x - 1}{x} \right) = \log a \right\}  \end{aligned}  $
Q.2)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{10^x - 2^x - 5^x + 1}{x \tan x} \right)$
Sol.2)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{10^x - 2^x - 5^x + 1}{x \tan x} \right)</math></p> $  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{2^x(5^x - 1) - 1(5^x - 1)}{x \tan x} \right) \qquad \qquad \qquad \{10^x = 2^x \cdot 5^x\} \\  &= \lim_{x \rightarrow 0} \left( \frac{(5^x - 1)(2^x - 1)}{x \tan x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\left( \frac{5^x - 1}{x} \right) \times x \cdot \left( \frac{2^x - 1}{x} \right) \times x}{x \left( \frac{\tan x}{x} \right) \times x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\left( \frac{5^x - 1}{x} \right) \cdot \left( \frac{2^x - 1}{x} \right)}{\left( \frac{\tan x}{x} \right)} \right) \\  &= \frac{\lim_{x \rightarrow 0} \left( \frac{5^x - 1}{x} \right) \times \lim_{x \rightarrow 0} \left( \frac{2^x - 1}{x} \right)}{\lim_{x \rightarrow 0} \left( \frac{\tan x}{x} \right)} \\  &= \frac{(\log 5)(\log 2)}{1} \qquad \qquad \qquad \left\{ \begin{array}{l} \lim_{x \rightarrow 0} \left( \frac{\tan x}{x} \right) = 1 \\ \left\{ \lim_{x \rightarrow 0} \left( \frac{a^x - 1}{x} \right) = \log a \right\} \end{array} \right\} \\  &= \log(5) \log(2) \text{ ans.}  \end{aligned}  $
Q.3)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{e^x + e^{-x} - 2}{x^2} \right)$



Sol.3)	We have $\lim_{x \rightarrow 0} \left( \frac{e^x + e^{-x} - 2}{x^2} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{\frac{e^x + 1}{e^x} - 2}{x^2} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{\frac{e^{2x} + 1 - 2e^x}{e^x \cdot x^2}}{e^x \cdot x^2} \right) \quad \{(e^x)^2 = e^{2x}\}$ $= \lim_{x \rightarrow 0} \left( \frac{(e^x - 1)^2}{e^x \cdot x^2} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{(e^x - 1)^2}{x} \right) \times \lim_{x \rightarrow 0} \left( \frac{1}{e^x} \right)$ $= (1)^2 \times \frac{1}{e^0} \quad \left\{ \lim_{x \rightarrow 0} \left( \frac{e^x - 1}{x} \right) = 1 \right\}$ $= 1 \text{ ans.} \quad \{e^0 = 1\}$
Q.4)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{\log(1+x^3)}{\sin^3 x} \right)$
Sol.4)	We have $\lim_{x \rightarrow 0} \left( \frac{\log(1+x^3)}{\sin^3 x} \right)$ $= \lim_{x \rightarrow 0} \left[ \frac{\frac{\log(1+x^3)}{x^3} \times x^3}{\frac{\sin^3 x}{x^3} \times x^3} \right]$ $= \frac{\lim_{x \rightarrow 0} \left( \frac{\log(1+x^3)}{x^3} \right)}{\lim_{x \rightarrow 0} \left( \frac{\sin^3 x}{x^3} \right)} \quad \left\{ \lim_{x \rightarrow 0} \left( \frac{\log(1+x)}{x} \right) = 1 \right\}$ $= \frac{1}{1^3} = 1 \text{ ans.}$
Q.5)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{2^{3x} - 3^{2x}}{\sin(3x)} \right)$
Sol.5)	We have $\lim_{x \rightarrow 0} \left( \frac{2^{3x} - 3^{2x}}{\sin(3x)} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{2^{3x} - 3^{2x} - 1 + 1}{\sin(3x)} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{(2^{3x} - 1) - (3^{2x} - 1)}{\sin(3x)} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{\frac{(2^{3x} - 1) \times 3x - (3^{2x} - 1) \times 2x}{3x}}{\frac{\sin(3x)}{3x} \times 3x} \right)$ $= \frac{\lim_{x \rightarrow 0} \left[ \frac{2^{3x} - 1}{3x} \right] \times 3 - \lim_{x \rightarrow 0} \left[ \frac{3^{2x} - 1}{2x} \right] \times 2}{\lim_{x \rightarrow 0} \left[ \frac{\sin(3x)}{3x} \right] \times 3} \quad \left\{ \lim_{x \rightarrow 0} \left( \frac{a^x - 1}{x} \right) = \log a \right\}$ $= \frac{3(\log 2) - 2(\log 3)}{1 \times 3} \quad \{\log m^n = n \log m\}$ $= \frac{\log 2^3 - \log 3^2}{3}$ $= \frac{1}{3} \log \left( \frac{8}{9} \right) \text{ ans.} \quad \{\log A - \log B = \log \left( \frac{A}{B} \right)\}$

Q.6)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{x(e^x - 1)}{1 - \cos x} \right)$
Sol.6)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{x(e^x - 1)}{1 - \cos x} \right)</math></p> $= \lim_{x \rightarrow 0} \left( \frac{x(e^x - 1)}{2 \sin^2 \frac{x}{2}} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{\frac{x(e^x - 1) \times x}{x}}{\frac{2 \sin^2 \frac{x}{2} \times \frac{1}{2}}{\frac{x^2}{4}}} \right)$ $= \frac{2 \lim_{x \rightarrow 0} \left( \frac{(e^x - 1)}{x} \right)}{\lim_{x \rightarrow 0} \left( \frac{x^2}{4} \right)}$ $= \frac{2(1)}{1^2} = 2 \text{ ans.}$ <p style="text-align: right;"><math>\left\{ \lim_{x \rightarrow 0} \left( \frac{e^x - 1}{x} \right) = 1 \right\}</math></p>
Q.7)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x - 1}{x} \right)$
Sol.7)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x - 1}{x} \right)</math></p> $= \lim_{x \rightarrow 0} \left( \frac{a^x + b^x + c^x - 1 - 1 - 1}{x} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{(a^x - 1) + (b^x - 1) + (c^x - 1)}{x} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{a^x - 1}{x} + \frac{b^x - 1}{x} + \frac{c^x - 1}{x} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{a^x - 1}{x} \right) + \lim_{x \rightarrow 0} \left( \frac{b^x - 1}{x} \right) + \lim_{x \rightarrow 0} \left( \frac{c^x - 1}{x} \right)$ $= \log a + \log b + \log c = \log(abc) \text{ ans.}$ <p style="text-align: right;"><math>\{\log A + \log B = \log(AB)\}</math></p>
Q.8)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{a^x + b^x - c^x - d^x}{x} \right)$
Sol.8)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{a^x + b^x - c^x - d^x}{x} \right)</math></p> $= \lim_{x \rightarrow 0} \left( \frac{a^x + b^x - c^x - d^x - 1 - 1 + 1 + 1}{x} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{(a^x - 1) + (b^x - 1) - (c^x - 1) - (d^x - 1)}{x} \right)$ $= \lim_{x \rightarrow 0} \left[ \left( \frac{a^x - 1}{x} \right) + \left( \frac{b^x - 1}{x} \right) - \left( \frac{c^x - 1}{x} \right) - \left( \frac{d^x - 1}{x} \right) \right]$ $= \lim_{x \rightarrow 0} \left( \frac{a^x - 1}{x} \right) + \lim_{x \rightarrow 0} \left( \frac{b^x - 1}{x} \right) - \lim_{x \rightarrow 0} \left( \frac{c^x - 1}{x} \right) - \lim_{x \rightarrow 0} \left( \frac{d^x - 1}{x} \right)$ $= \log a + \log b - \log c - \log d$ <p style="text-align: right;"><math>\left\{ \lim_{x \rightarrow 0} \left( \frac{a^x - 1}{x} \right) = \log a \right\}</math></p> $= (\log a + \log b) - (\log c + \log d)$ $= \log(ab) - \log(cd)$ <p style="text-align: right;"><math>\{\log A + \log B = \log(AB)\}</math></p>



	$= \log\left(\frac{ab}{cd}\right)$ ans. $\{\log A - \log B = \log\left(\frac{A}{B}\right)\}$
Q.9)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{\log(5+x) - \log(5-x)}{x} \right)$
Sol.9)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{\log(5+x) - \log(5-x)}{x} \right)</math></p> $= \lim_{x \rightarrow 0} \left( \frac{\log\left(5\left(1+\frac{x}{5}\right)\right) - \log\left(5\left(1-\frac{x}{5}\right)\right)}{x} \right)$ $= \lim_{x \rightarrow 0} \left[ \frac{\{\log 5 + \log\left(1+\frac{x}{5}\right)\} - \{\log 5 + \log\left(1-\frac{x}{5}\right)\}}{x} \right] \quad \{\log A + \log B = \log(AB)\}$ $= \lim_{x \rightarrow 0} \frac{\log\left(1+\frac{x}{5}\right) - \log\left(1-\frac{x}{5}\right)}{x}$ $= \lim_{x \rightarrow 0} \frac{\frac{x}{5}}{\frac{x}{5} \times 5} - \lim_{x \rightarrow 0} \frac{\log\left(1+\frac{-x}{5}\right)}{x}$ $= \lim_{x \rightarrow 0} \frac{\log\left(1+\frac{x}{5}\right)}{\frac{x}{5} \times 5} + \lim_{x \rightarrow 0} \frac{\log\left(1+\frac{-x}{5}\right)}{-\frac{x}{5} \times 5}$ $= \frac{1}{5} + \frac{1}{5} \quad \left\{ \lim_{x \rightarrow 0} \left( \frac{\log(1+x)}{x} \right) = 1 \right\}$ $= \frac{2}{5}$ ans.
Q.10)	Evaluate $\lim_{x \rightarrow 0} \left( \frac{e^{3+x} - \sin x - e^3}{x} \right)$
Sol.10)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{e^{3+x} - \sin x - e^3}{x} \right)</math></p> $= \lim_{x \rightarrow 0} \left( \frac{e^{3+x} - e^3 - \sin x}{x} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{e^3 - (e^x - 1) - \sin x}{x} \right)$ $= \lim_{x \rightarrow 0} \left( \frac{e^3(e^x - 1)}{x} - \frac{\sin x}{x} \right)$ $= e^3 \lim_{x \rightarrow 0} \left[ \frac{e^x - 1}{x} \right] - \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)$ $= e^3 - (1)$ $= e^3 - 1$ ans.