

<u>Class 11 Limits & Derivatives</u> Class 11th	
	TYPE: 7
	$\lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x} \right)$ and $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}$
Q.1)	Evaluate $\lim_{x \rightarrow 0} \left(\frac{9^x - 6^x - 6^x + 4}{x} \right)$
Sol.1)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{9^x - 6^x - 6^x + 4}{x} \right)$</p> $ \begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{9^x - 6^x - 6^x + 4}{x^2} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{3^x(3^x - 2^x) - 2^x(3^x - 2^x)}{x^2} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{(3^x - 2^x)(3^x - 2^x)}{x^2} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{(3^x - 2^x)^2}{x^2} \right) \\ &= \lim_{x \rightarrow 0} \left[\left(\frac{3^x - 2^x}{x} \right)^2 \right] \\ &= \lim_{x \rightarrow 0} \left[\left(\frac{(3^x - 1) - (2^x - 1)}{x} \right)^2 \right] \\ &= \left\{ \lim_{x \rightarrow 0} \left[\frac{3^x - 1}{x} \right] - \lim_{x \rightarrow 0} \left[\frac{2^x - 1}{x} \right] \right\}^2 \\ &= (\log 3 - \log 2)^2 \quad \left\{ \lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x} \right) = \log a \right\} \\ &= \left(\log \left(\frac{3}{2} \right) \right)^2 \text{ ans.} \quad \{ \log A + \log B = \log(AB) \} \end{aligned} $
	Trigo formula used:
Q.2)	Differentiate using first principle method $f(x) = \cos(3x)$
Sol.2)	$ \begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{\cos(3x+3h) - \cos(3x)}{h} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{-2 \sin\left(\frac{6x+3h}{2}\right) \cdot \sin\left(\frac{3h}{2}\right)}{h} \right) \quad \{ \cos A - \cos B \text{ formula} \} \\ &= \lim_{h \rightarrow 0} \left(\frac{-2 \sin\left(\frac{6x+3h}{2}\right) \cdot \sin\left(\frac{3h}{2}\right)}{\frac{3h}{2}} \times \frac{3}{2} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{3h}{2}\right) \cdot \sin\left(\frac{3h}{2}\right)}{\frac{3h}{2}} \right) \times \left[-3 \lim_{h \rightarrow 0} \left(\frac{6x+3h}{2} \right) \right] \end{aligned} $

	$= 1 \times (-3 \sin(3x))$ $\left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$ $\frac{dy}{dx} = f'(x) = -3 \sin(3x)$ ans.
Q.3)	Differentiate using first formula $f(x) = \tan(2x)$
Sol.3)	$f'(x) = \frac{dy}{dx} = \lim_{h \rightarrow 0} \left(\frac{\tan(2x + 2h) - \tan(2x)}{h} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\frac{\sin(2x+2h)}{\cos(2x+2h)} - \frac{\sin(2x)}{\cos(2x)}}{h} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\sin(2x+2h).\cos(2x) - \cos(2x+2h).\sin(2x)}{h.\cos(2x+2h)\cos(2x)} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\sin(2x+2h-2x)}{h.\cos(2x+2h)\cos(2x)} \right) \quad \{ \sin(A - B) \text{ formula} \}$ $= \lim_{h \rightarrow 0} \left(\frac{\sin(2h)}{2h.\cos(2x+2h)\cos(2x)} \times 2 \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\sin(2h)}{2h} \right) \times \lim_{h \rightarrow 0} \left(\frac{2}{\cos(2x+2h).\cos(2x)} \right)$ $= 1 \times \frac{2}{\cos(2x).\cos(2x)}$ $\therefore \frac{dy}{dx} = 2 \sec^2(2x)$ ans.
Q.4)	$f(x) = \sqrt{\tan x}$, find $f'(x)$ first principle method.
Sol.4)	$f(x) = \sqrt{\tan x}$ $f'(x) = \lim_{h \rightarrow 0} \left[\frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h} \right]$ <p>Rationalize</p> $= \lim_{h \rightarrow 0} \left[\frac{\tan(x+h) - \tan x}{h\sqrt{\tan(x+h)} + \sqrt{\tan x}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\tan(x+h-x) - [1+\tan(x+h).\tan x]}{h\sqrt{\tan(x+h)} + \sqrt{\tan x}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\tan h.[1+\tan(x+h).\tan x]}{h\sqrt{\tan(x+h)} + \sqrt{\tan x}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\tan h}{h} \right] \times \lim_{h \rightarrow 0} \left[\frac{1+\tan(x+h).\tan x}{\sqrt{\tan(x+h)} + \sqrt{\tan x}} \right]$ $= 1 \times \frac{(1+\tan(x+h).\tan x)}{\sqrt{\tan x} + \sqrt{\tan x}}$ $= \frac{1+\tan^2 x}{2\sqrt{\tan x}}$ $\therefore f'(x) = \frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x$ ans.
Q.5)	$f(x) = \sec^2 x$ Using first principle method.

Sol.5)	$ \begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left[\frac{\sec^2(x+h) - \sec^2 x}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\frac{1}{\cos^2(x+h)} - \frac{1}{\cos^2 x}}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\cos^2 x - \cos^2(x+h)}{h \cdot \cos^2(x+h) \cdot \cos^2 x} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\{\cos x + \cos(x+h)\}\{\cos x - \cos(x+h)\}}{h \cdot \cos^2(x+h) \cdot \cos^2 x} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\{\cos x + \cos(x+h)\}\{-2 \sin(\frac{2x+h}{2}) \cdot \sin(-\frac{h}{2})\}}{h \cdot \cos^2(x+h) \cdot \cos^2 x} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{2 \sin(\frac{2x+h}{2}) \cdot \sin(\frac{h}{2}) \times \{\cos x + \cos(x+h)\}}{2 \times \frac{h}{2} \cdot \cos^2(x+h) \cdot \cos^2 x} \right] \\ &= \lim_{h \rightarrow 0} \sin(\frac{2x+h}{2}) \times \lim_{h \rightarrow 0} \left(\frac{\sin(\frac{h}{2})}{\frac{h}{2}} \right) \times \lim_{h \rightarrow 0} \left[\frac{\cos x + \cos(x+h)}{\cos^2(x+h) \cdot \cos^2 x} \right] \\ &= \sin(x) \times 1 \times \frac{(\cos x + \cos x)}{\cos^2 x \cdot \cos^2 x} \\ &= \frac{\sin x \cdot 2 \cos x}{\cos^2 x \cdot \cos^2 x} \\ &= \frac{2 \sin x}{\cos x} \cdot \frac{1}{\cos^2 x} \\ f'(x) &= 2 \tan x \cdot \sec^2 x \text{ ans.} \end{aligned} $
Q.6)	$f(x) = \sin(x^2)$ Using first principle method.
Sol.6)	$ \begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left[\frac{\sin(x+h)^2 - \sin x^2}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\sin(x^2 + h^2 + 2hx) - \sin(x^2)}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{2 \cos(\frac{2x^2 + h^2 + 2hx}{2}) \cdot \sin(\frac{h^2 + 2hx}{2})}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{2 \cos(\frac{2x^2 + h^2 + 2hx}{2}) \cdot \sin(\frac{h^2 + 2hx}{2})}{h \cdot (\frac{h^2 + 2hx}{2})} \times (\frac{h^2 + 2hx}{2}) \right] \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin(\frac{h^2 + 2hx}{2})}{\frac{h^2 + 2hx}{2}} \right) \times \lim_{h \rightarrow 0} 2 \cos(\frac{2x^2 + h^2 + 2hx}{2}) \times \lim_{h \rightarrow 0} \left(\frac{h^2 + 2hx}{h} \right) \\ &= 1 \times (2 \cos(x^2)) \times \lim_{h \rightarrow 0} \left[\frac{h(h+2x)}{2h} \right] \\ &= 2 \cos(x^2) \times \left(\frac{2x}{2} \right) \\ \therefore \frac{dy}{dx} &= 2x \cdot \cos(x^2) \text{ ans.} \end{aligned} $
Q.7)	$f(x) = \tan \sqrt{x}$ Using first principle method.



Sol.7)	$ \begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \tan(\sqrt{x+h}) - \tan \sqrt{x} \\ &= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h})}{\cos(\sqrt{x+h})} - \frac{\sin \sqrt{x}}{\cos \sqrt{x}} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h}).\cos \sqrt{x}.\cos(\sqrt{x+h}).\sin \sqrt{x}}{h.\cos(\sqrt{x+h}).\cos \sqrt{x}} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h}-\sqrt{x})}{h.\cos(\sqrt{x+h}).\cos \sqrt{x}} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h}-\sqrt{x}) \times (\sqrt{x+h}-\sqrt{x})}{h(\sqrt{x+h}-\sqrt{x}).\cos(\sqrt{x+h}).\cos \sqrt{x}} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h}+\sqrt{x})}{(\sqrt{x+h}+\sqrt{x})} \right] \times \lim_{h \rightarrow 0} \left[\frac{1}{\cos(\sqrt{x+h}).\cos \sqrt{x}} \right] \times \lim_{h \rightarrow 0} \left[\frac{(\sqrt{x+h}-\sqrt{x})}{h} \right] \\ &= 1 \times \frac{1}{\cos \sqrt{x}.\cos \sqrt{x}} \times \lim_{h \rightarrow 0} \left[\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} \right] \\ &= \sec^2 \sqrt{x} \times \frac{1}{(\sqrt{x}+\sqrt{x})} \\ \therefore \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \cdot \sec^2 \sqrt{x} \text{ ans.} \end{aligned} $
Q.8)	$f(x) = x \cos x$ Using first principle method.
Sol.8)	$ \begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left[\frac{(x+h).\cos(x+h) - x \cos x}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x \cos(x+h) + h \cos(x+h) - x \cos x}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h}).\cos \sqrt{x}.\cos(\sqrt{x+h}).\sin \sqrt{x}}{h.\cos(\sqrt{x+h}).\cos \sqrt{x}} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x\{\cos(x+h)\} + h \cos(x+h) - x \cos x}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x\{-2 \sin\left(\frac{2x+h}{2}\right).\sin\left(\frac{h}{2}\right)\} + h \cos(x+h)}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{-2x \sin\left(\frac{2x+h}{2}\right) \sin\left(\frac{h}{2}\right)}{2 \times \frac{h}{2}} + \frac{h \cos(x+h)}{h} \right] \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \right) \times \left[x \lim_{h \rightarrow 0} \left(-\sin\left(\frac{2x+h}{2}\right) \right) \right] + \lim_{h \rightarrow 0} (\cos(x+h)) \\ &= (1)(-x \sin x) + \cos x \\ \therefore f'(x) &= -x \sin x + \cos x \text{ ans.} \end{aligned} $
Q.9)	$f(x) = \frac{\sin x}{x}$ Using first principle method.
Sol.9)	$ f'(x) = \lim_{h \rightarrow 0} \left[\frac{\sin(x+h) - \sin x}{x+h - x} \right] $

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	$ \begin{aligned} &= \lim_{h \rightarrow 0} \left[\frac{x \sin(x+h) - (x+h) \sin x}{h.(x+h)x} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x \sin(x+h) - x \sin x - h \sin x}{h(x+h)x} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x\{\sin(x+h) - \sin x\} - h \sin x}{h(x+h)x} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x\{\sin(x+h) - \sin x\} - h \sin x}{h(x+h)x} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x\{\sin(x+h) - \sin x\}}{h(x+h)x} - \frac{h \sin x}{h(x+h)x} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x.2 \cos\left(\frac{2x+h}{2}\right). \sin\left(\frac{h}{2}\right)}{2 \times \frac{h}{2}(x+h)x} - \frac{\sin x}{(x+h)x} \right] \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{h}{2}\right). \sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \right) \times \lim_{h \rightarrow 0} \left(\frac{x \cos\left(\frac{2x+h}{2}\right)}{(x+h)x} \right) - \lim_{h \rightarrow 0} \left(\frac{\sin x}{(x+h)x} \right) \\ &= 1 \times \left(\frac{x \cos x}{x^2} \right) - \frac{\sin x}{x^2} \\ \therefore \frac{dy}{dx} &= \frac{x \cos x - \sin x}{x^2} \text{ ans.} \end{aligned} $
Q.10)	$f(x) = \sin x - \cos x$ Using first principle method.
Sol.10)	$ \begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left[\frac{\{\sin(x+h) - \cos(x+h)\} - \{\sin x - \cos x\}}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{\{\sin(x+h) - \sin x\} - \{\cos(x+h) - \cos x\}}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{2 \cos\left(\frac{2x+h}{2}\right). \sin\left(\frac{h}{2}\right) + 2 \sin\left(\frac{2x+h}{2}\right). \sin\left(\frac{h}{2}\right)}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{2 \sin\left(\frac{h}{2}\right) \{ \cos\left(\frac{2x+h}{2}\right) + \sin\left(\frac{2x+h}{2}\right) \}}{2 \times \frac{h}{2}} \right] \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \right) \times \lim_{h \rightarrow 0} \left(\cos\left(\frac{2x+h}{2}\right) + \sin\left(\frac{2x+h}{2}\right) \right) \\ &= 1 \times (\cos x + \sin x) \\ \therefore f'(x) &= \cos x + \sin x \text{ ans.} \end{aligned} $