

<u>Class 11 Limits & Derivatives</u>	
Class 11th	
Q.1)	$f(x) = \frac{2x^2+1}{x-3}$ Using first principle method.
Sol.1)	$f'(x) = \lim_{h \rightarrow 0} \left[\frac{\left\{ \frac{2(x+h)^2 + 1}{x+h-3} \right\} - \left\{ \frac{2x^2 + 1}{x-3} \right\}}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\left\{ \frac{2(x+h)^2 + 1}{x+h-3} \right\} - \left\{ \frac{2x^2 + 1}{x-3} \right\}}{h} \right] \quad (\text{repeat})$ $= \lim_{h \rightarrow 0} \left[\frac{\left(\frac{(2x^2+h^2+2hx+1)(x-3)-(2x^2+1)(x+h-3)}{h(x+h-3)(x-3)} \right) - \frac{2x^2+1}{x-3}}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2x^3-6x^2+2h^2x-6h^2+2hx^2-6hx+x-3-2x^3-2x^2h+6x^2+x-h+3}{h(x+h-3)(x-3)} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2h^2x-6h^2-6hx-h}{h(x+h-3)(x-3)} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{h(2hx-6h-6x-1)}{h(x+h-3)(x-3)} \right]$ $\therefore f'(x) = \frac{-6x-1}{(x-3)(x-3)} = \frac{-6x-1}{(x-3)^2} \text{ ans.}$
Q.2)	$f(x) = \sqrt{2x+3}$ Using first principle method.
S.2)	$f'(x) = \lim_{h \rightarrow 0} \left[\frac{\sqrt{2x+2h+3} - \sqrt{2x+3}}{h} \right]$ <p>Rationalize</p> $= \lim_{h \rightarrow 0} \left[\frac{\sqrt{2x+2h+3} - \sqrt{2x+3}}{h} \times \frac{\sqrt{2x+2h+3} + \sqrt{2x+3}}{\sqrt{2x+2h+3} + \sqrt{2x+3}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{(2x+2h+3) - (2x+3)}{h(\sqrt{2x+2h+3} + \sqrt{2x+3})} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{(2x^2+h^2+2hx+1)(x-3) - (2x^2+1)(x+h-3)}{h(x+h-3)(x-3)} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2h}{h(\sqrt{2x+2h+3} + \sqrt{2x+3})} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2}{\sqrt{2x+3} + \sqrt{2x+3}} \right]$ $\therefore \frac{dy}{dx} = \frac{1}{\sqrt{2x+3}} \text{ ans.}$
Q.3)	$f(x) = x^2 \sin x$ Using first principle method.

Sol.3)	$ \begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \left[\frac{(x+h)^2 \cdot \sin(x+h) - x^2 \sin x}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{(x^2 + h^2 + 2hx) \sin(x+h) - x^2 \sin x}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x^2 \cdot \sin(x+h) + h^2 \sin(x+h) + 2hx \cdot \sin(x+h) - x^2 \sin x}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x^2 \{\sin(x+h) - \sin x\} + (h^2 + 2hx) \sin(x+h)}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{x^2 \cdot 2 \cos\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{h} + \frac{h(h+2x) \sin(x+h)}{h} \right] \\ &= \lim_{h \rightarrow 0} \left[\frac{2x^2 \cos\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{2 \times \frac{h}{2}} + (h+2x) \cdot \sin(x+h) \right] \\ &= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \right) \times \lim_{h \rightarrow 0} \left(x^2 \cos\left(\frac{2x+h}{2}\right) \right) + \lim_{h \rightarrow 0} ((h+2x) \cdot \sin(x+h)) \\ &= 1 \times x^2 \cos(x) + (2x) \sin x \\ \therefore f'(x) &= x^2 \cos(x) + (2x) \sin x \text{ ans.} \end{aligned} $
Q.4)	Differentiate w.r.t x (product rule) $f(x) = (ax + b)^m(cx + d)^n$
Sol.4)	We have $f(x) = (ax + b)^m(cx + d)^n$ Differentiate both sides w.r.t x (product rule) $ \begin{aligned} f'(x) &= (ax + b)^m \cdot \frac{d}{dx}(cx + d)^n + (cx + d)^n \cdot \frac{d}{dx}(ax + b)^m \\ &= (ax + b)^m \cdot n(cx + d)^{n-1} \cdot \frac{d}{dx}(cx + d) + (cx + d)^n \cdot m(ax + b)^{m-1} \cdot \frac{d}{dx}(ax + b) \\ &= (ax + b)^m \cdot n(cx + d)^{n-1} \cdot (c) + (cx + d)^n \cdot m(ax + b)^{m-1} \cdot (a) \\ &= (ax + b)^{m-1} \cdot (cx + d)^{n-1} [(ax + b)nc + (cx + d)ma] \\ &= (ax + b)^{m-1} \cdot (cx + d)^{n-1} [ancx + bnc + cmax + dma] \\ f'(x) &= (ax + b)^{m-1} \cdot (cx + d)^{n-1} [ax(nc + mc) + (bnc + dma)] \text{ ans.} \end{aligned} $
Q.5)	Differentiate w.r.t x (product rule) $f(x) = (x + \sec x)(x - \tan x)$
Sol.5)	We have $f(x) = (x + \sec x)(x - \tan x)$ Differentiate both sides w.r.t x (product rule) $ \begin{aligned} f'(x) &= (x + \sec x) \cdot \frac{d}{dx}(x - \tan x) + (x - \tan x) \cdot \frac{d}{dx}(x + \sec x) \\ &= (x + \sec x)(1 - \sec^2 x) + (x - \tan x)(1 + \sec x \tan x) \text{ ans.} \end{aligned} $
Q.6)	Differentiate w.r.t x (product rule) $f(x) = (x \sin x + \cos x) + (x \cos x - \sin x)$



Sol.6)	<p>We have $f(x) = (x \sin x + \cos x) + (x \cos x - \sin x)$</p> <p>Differentiate both sides w.r.t x (product rule)</p> $ \begin{aligned} f'(x) &= (x \sin x + \cos x) \cdot \frac{d}{dx}(x \cos x - \sin x) + (x \cos x - \sin x) \cdot \frac{d}{dx}(x \sin x + \cos x) \\ &= (x \sin x + \cos x) \left[x \cdot \frac{d}{dx}(\cos x) + \cos x \cdot \frac{d}{dx}(x) - \frac{d}{dx}(\sin x) \right] + [x \cos x - \sin x] \left[x \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x) + \frac{d}{dx}(\cos x) \right] \\ &= (x \sin x + \cos x)(-x \sin x + \cos x) + (x \cos x - \sin x)(x \cos x + \sin x - \sin x) \\ &= (x \sin x + \cos x)(-x \sin x) + (x \cos x - \sin x)(x \cos x) \\ &= -x^2 \sin^2 x - x \sin x \cdot \cos x + x^2 \cos^2 x - x \sin x \cdot \cos x \\ &= x^2(\cos^2 x - \sin^2 x) - 2x \sin x \cos x \\ f'(x) &= x^2 \cdot \cos(2x) - x \sin(2x) \text{ ans.} \end{aligned} $
Q.7)	<p>Differentiate w.r.t x (product rule)</p> $f(x) = x^{-4}(3 - 4x^{-5})$
Sol.7)	<p>Differentiate w.r.t x (product rule)</p> $ \begin{aligned} f'(x) &= x^{-4} \cdot \frac{d}{dx}(3 - 4x^{-5}) + (3 - 4x^{-5}) \cdot \frac{d}{dx}(x^{-4}) \\ &= x^{-4}(0 + 20x^{-6}) + (3 - 4x^{-5})(-4x^{-5}) \\ &= x^{-4}(20x^{-6}) - (3 - 4x^{-5})(4x^{-5}) \\ &= 20x^{-10} - 12x^{-5} + 16x^{-10} \\ &= 36x^{-10} - 10x^{-5} \text{ ans.} \end{aligned} $
Q.8)	<p>Differentiate w.r.t x (product rule)</p> $f(x) = \frac{1 - \tan x}{1 + \tan x}$
Sol.8)	<p>We have $f(x) = \frac{1 - \tan x}{1 + \tan x}$</p> $ \begin{aligned} f'(x) &= \frac{1 - \tan x}{1 + \tan x} \\ &= \frac{\frac{\sin x}{\cos x}}{1 + \frac{\sin x}{\cos x}} \\ &= \frac{\cos x - \sin x}{\cos x + \sin x} \end{aligned} $ <p>Differentiate w.r.t x (quotient rule)</p> $ \begin{aligned} f'(x) &= \frac{(\cos x + \sin x) \cdot \frac{d}{dx}(\cos x - \sin x) - (\cos x - \sin x) \frac{d}{dx}(\cos x + \sin x)}{(\cos x + \sin x)^2} \\ &= \frac{(\cos x + \sin x)(-\sin x - \cos x) - (\cos x - \sin x)(-\sin x + \cos x)}{(\cos x + \sin x)^2} \\ &= \frac{-(\cos x + \sin x)(\cos x + \sin x) - (\cos x - \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2} \end{aligned} $



	$= \frac{-(\cos^2 x + \sin^2 x + 2 \sin x \cos x) - (\cos^2 x - \sin^2 x - 2 \sin x \cos x)}{(\cos x + \sin x)^2}$ $= \frac{-(1 + \sin(2x)) - (1 - 2 \sin(2x))}{(\cos x + \sin x)^2}$ $f'(x) = \frac{-2}{(\cos x + \sin x)^2} \text{ ans.}$
Q.9)	<p>Differentiate w.r.t x (product rule)</p> $f(x) = \frac{x}{\sin^n x}$
Sol.9)	<p>We have $f(x) = \frac{x}{\sin^n x}$</p> <p>Differentiate w.r.t x (quotient rule)</p> $\frac{dy}{dx} = \frac{\sin^n x \cdot \frac{d}{dx}(x) - x \cdot \frac{d}{dx}(\sin^n x)}{(\sin^n x)^2}$ $= \frac{\sin^n x \cdot (1) - x \cdot n \sin^{n-1} x \cdot \frac{d}{dx}(\sin x)}{(\sin^{2n} x)^2}$ $= \frac{\sin^n x - nx \cdot \sin^{n-1} x \cdot \cos x}{\sin^{2n} x}$ $= \frac{\sin^{n-1} x (\sin x - nx \cos x)}{\sin^{2n} x}$ $= \frac{\sin x - nx \cos x}{\sin^{2n-n+1}(x)}$ $f'(x) = \frac{\sin x - nx \cos x}{\sin^{n+1} x} \text{ ans.}$
Q.10)	$f(x) = \frac{x^2 \cos\left(\frac{\pi}{4}\right)}{\sin x}$
Sol.10)	<p>We have $\frac{x^2 \cos\left(\frac{\pi}{4}\right)}{\sin x}$</p> $= f(x) = \frac{1}{\sqrt{2}} \cdot \frac{x^2}{\sin x}$ <p>Differentiate w.r.t x (quotient rule)</p> $f'(x) = \frac{1}{\sqrt{2}} \left[\frac{\sin x \cdot \frac{d}{dx}(x^2) - (x^2) \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2} \right]$ $= \frac{1}{\sqrt{2}} \left[\frac{\sin x \cdot (2x) - x^2 \cdot \cos x}{\sin^2 x} \right]$ $f'(x) = \frac{1}{\sqrt{2}} \left[\frac{2x \cdot \sin x - x^2 \cdot \cos x}{\sin^2 x} \right] \text{ ans.}$