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	Class 11 Limits & Derivatives **Class 11th**
Q.1)	$f(x)=\dfrac{\sin x-x\cos x}{x\sin x+\cos x}$
Sol.1)	We have $\frac{\sin x-x\cos x}{x\sin x+\cos x}$ Differentiate $w.r.t\ x$ (quotient rule) $f'(x)=\dfrac{(x\sin x+\cos x).\frac{d}{dx}(\sin x-x\cos x)-(\sin x-x\cos x).\frac{d}{dx}(x\sin x+\cos x)}{(x\sin x+\cos x)^2}$ $=$ $\dfrac{(x\sin x+\cos x).\left[\frac{d}{dx}(\sin x)-\left(x.\frac{d}{dx}(\cos x)\right)+\cos x\frac{d}{dx}(x)\right]-(\sin x-x\cos x)\left[x.\frac{d}{dx}(\sin x)+.\frac{d}{dx}(x)+\frac{d}{dx}\cos x\right]}{(x\sin x+\cos x)^2}$ $=\frac{(x\sin x+\cos x).[\cos x-(-x\sin x+\cos x)]-(\sin x-x\cos x)(x.\cos x+\sin x-\sin x)}{(x\sin x+\cos x)^2}$ $=\frac{(x\sin x+\cos x).(x\sin x)-(\sin x-x\cos x)(x\cos x)}{(x\sin x+\cos x)^2}$ $=\frac{x^2\sin^2 x+x\sin x\cos x-x\sin x\cos x+x^2\cos^2 x}{(x\sin x+\cos x)^2}$ $=\frac{x^2(\sin^2 x+\cos^2 x)}{(x\sin x+\cos x)^2}$ $f'(x)=\frac{x^2}{(x\sin x+\cos x)^2}$ ans.
Q.2)	Given $f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+\ldots\ldots\frac{x^2}{2}+x+1$ Show that $f'(1)=100f'(0)$
Sol.2)	We have $\frac{x^{100}}{100}+\frac{x^{99}}{99}+\ldots\ldots\frac{x^2}{2}+x+1$ Differentiate $w.r.t\ x$ $f'(x)=\frac{100x^{99}}{100}+\frac{99x^{98}}{99}+\ldots\ldots\frac{2x}{2}+1+0$ $=x^{99}+x^{98}+\ldots\ldots x+1$ $f'(1)=(1)^{99}+(1)^{99}+\ldots\ldots 1+1$ (put $x=1$) $=1+1+\ldots\ldots 1+1=100$ For $f'(0)$ put $x=0$ $f'(0)=0+0+\ldots\ldots 0+1=1$ L.H.S. $f'(1)=100$ R.H.S. $100\,f'(0)=100\times 1=100$ ∴ L.H.S = R.H.S. ans.

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Q.3)	$y = \sqrt{\frac{1-\cos x}{1+\cos x}}$, find $\frac{dy}{dx}$.
Sol.3)	We have $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$ $\Rightarrow y = \sqrt{\frac{2\sin^2\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)}}$ $\Rightarrow y = \sqrt{\tan^2\left(\frac{x}{2}\right)}$ $\Rightarrow y = \tan\frac{x}{2}$ Differentiate $w.r.t\ x$ $\frac{dy}{dx} = \sec^2\left(\frac{x}{2}\right).\frac{d}{dx}.\left(\frac{x}{2}\right)$ $\frac{dy}{dx} = \sec^2\left(\frac{x}{2}\right) \times \frac{1}{2} = \frac{1}{2}\sec^2\left(\frac{x}{2}\right)$ ans.
Q.4)	$f(x) = \frac{\sin(x+a)}{\cos x}$, find $f^1(x)$
Sol.4)	We have $\frac{\sin(x+a)}{\cos x}$ Differentiate $w.r.t\ x$ (quotient rule) $f'(x) = \frac{\cos x.\frac{d}{dx}(\sin(x+a)) - \sin(x+a).\frac{d}{dx}(\cos x)}{\cos^2 x}$ $= \frac{\cos x.\cos(x+a).\frac{d}{dx}(x+a) - \sin(x+a)(-\sin x)}{\cos^2 x}$ $= \frac{\cos x.\cos(x+a)(1) + \sin(x+a)\sin x}{\cos^2 x}$ $= \frac{\cos(x+a).\cos x + \sin x.\sin(x+a)}{\cos^2 x}$ $= \frac{\cos x + a - x}{\cos^2 x}$ $= \frac{\cos a}{\cos^2 x}$ $f'(x) = \cos a \sec^2 x$ ans.
	TYPE: 2 $\lim_{x\to\infty} f(x)$
Q.5)	$\lim_{x\to\infty}\left(\frac{\sqrt{3x^2+1}+\sqrt{2x^2-1}}{4x+3}\right)$ Divide N & D by x
Sol.5)	$= \lim_{x\to\infty}\left(\frac{\frac{\sqrt{3x^2+1}}{x}+\frac{\sqrt{2x^2-1}}{x}}{\frac{4x+3}{x}}\right)$

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	$= \lim_{x\to\infty}\left(\frac{\sqrt{3+\frac{1}{x^2}}+\sqrt{2-\frac{1}{x^2}}}{4+\frac{3}{x}}\right)$ $= \frac{\sqrt{3}+\sqrt{2}}{4}$ ans.
Q.6)	Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$
Sol.6)	First make function in fraction by rationalize $= \lim_{x\to\infty}\frac{\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)}{\left(\sqrt{x^2+x+1}+\sqrt{x^2+1}\right)}\left(\sqrt{x^2+x+1}+\sqrt{x^2+1}\right)$ $= \lim_{x\to\infty}\left(\frac{x^2+x+1-x^2+1}{\sqrt{x^2+x+1}+\sqrt{x^2+1}}\right)$ $= \lim_{x\to\infty}\left(\frac{x}{\sqrt{x^2+x+1}+\sqrt{x^2+1}}\right)$ Divide N & D by x $= \lim_{x\to\infty}\left(\frac{1}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1+\frac{1}{x^2}}}\right)$ $= \frac{1}{\sqrt{1}+\sqrt{1}} = \frac{1}{1+1} = \frac{1}{2}$ ans.
Q.7)	Evaluate $\lim_{n\to\infty}\left(\frac{1+2+3\ldots\ldots n}{n^2}\right)$
Sol.7)	We have $\lim_{n\to\infty}\left(\frac{1+2+3\ldots\ldots n}{n^2}\right)$ $= \lim_{n\to\infty}\left(\frac{\frac{n(n+1)}{2}}{n^2}\right)$ $= \frac{1}{2}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$ $= \frac{1}{2}(1+0) = \frac{1}{2}$ ans.
Q.8)	Evaluate $\lim_{x\to-\infty}\left(\sqrt{x^2-x+1}+x\right)$
Sol.8)	Let $x = -y$ When $x \to -\infty$ (limits change) Then $y \to \infty$ $\Rightarrow \lim_{y\to\infty}\left(\sqrt{y^2+y+1}-y\right)$ Rationalize & proceed yourself

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	$\frac{1}{2}$ ans.
Q.9)	Find the derivative of $\sqrt[3]{\sin x}$ using first principle method.
Sol.9)	Here $f(x) = \sqrt[3]{\sin x} = (\sin x)^{\frac{1}{3}}$ $f'(x) = \lim_{h\to 0}\left[\frac{\sin(x+h)^{\frac{1}{3}} - (\sin x)^{\frac{1}{3}}}{h}\right]$ $= \lim_{h\to 0}\left[\frac{\sin(x+h)^{\frac{1}{3}} - (\sin x)^{\frac{1}{3}}}{\sin(x+h) - \sin x} \times \frac{\sin(x+h) - \sin x}{h}\right]$ When $h \to 0$ then $\sin(x+h) \to \sin x$ $= \lim_{\sin(x+h)\to \sin x}\left[\frac{\sin(x+h)^{\frac{1}{3}} - (\sin x)^{\frac{1}{3}}}{\sin(x+h) - \sin x}\right] \times \lim_{h\to 0}\left[\frac{\sin(x+h) - \sin x}{h}\right]$ $= \frac{1}{3}(\sin x)^{\frac{1}{3}-1} \times \lim_{h\to 0}\left[\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right] \times \lim_{h\to 0}\left(\cos\left(\frac{2x+h}{2}\right)\right)$ $= \frac{1}{3}\sin^{-\frac{2}{3}} x \lim_{h\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right) \times \lim_{h\to 0}\left(\cos\left(\frac{2x+h}{2}\right)\right)$ $= \frac{1}{3}\sin^{-\frac{2}{3}} x \times 1 \times \cos x$ $\therefore f'(x) = \frac{1}{3}\sin^{-\frac{2}{3}} x . \cos x$ ans.
Q.10)	Find derivative of $e^{\sqrt{\tan x}}$ using first principle method.
Sol.10)	$f(x) = e^{\sqrt{\tan x}}$ $f'(x) = \lim_{h\to 0}\left(\frac{e^{\sqrt{\tan(x+h)}} - e^{\sqrt{\tan x}}}{h}\right)$ $\Rightarrow f'(x) = e^{\sqrt{\tan x}} \lim_{h\to 0}\left(\frac{e^{\sqrt{\tan(x+h)}-\sqrt{\tan x}} - 1}{h}\right)$ $= e^{\sqrt{\tan x}} \lim_{h\to 0}\left(\frac{e^{\sqrt{\tan(x+h)}-\sqrt{\tan x}} - 1}{\sqrt{\tan(x+h)} - \sqrt{\tan x}} \times \frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h}\right)$ When $h \to 0$; $\sqrt{\tan(x+h)} \to \sqrt{\tan x}$ $f'(x) = e^{\sqrt{\tan x}} \lim_{\sqrt{\tan(x+h)}\to\sqrt{\tan x}}\left[\frac{e^{\sqrt{\tan(x+h)}-\sqrt{\tan x}} - 1}{\sqrt{\tan(x+h)} - \sqrt{\tan x}}\right] \times \lim_{h\to 0}\left[\frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h}\right]$ $= e^{\sqrt{\tan x}} \times 1 \lim_{h\to 0}\left[\frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h} \times \frac{\sqrt{\tan(x+h)} + \sqrt{\tan x}}{\sqrt{\tan(x+h)} + \sqrt{\tan x}}\right]$

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	$= e^{\sqrt{\tan x}} \times 1 \lim_{h\to 0}\left[\frac{\tan(x+h)-\tan x}{h} \times \frac{1}{\sqrt{\tan(x+h)}+\sqrt{\tan x}}\right]$ $= e^{\sqrt{\tan x}} \times \lim_{h\to 0}\left[\frac{\tan(h)\{1+\tan(x+h)\tan x\}}{h} \times \frac{1}{\sqrt{\tan(x+h)}+\sqrt{\tan x}}\right]$ $= e^{\sqrt{\tan x}} \times \lim_{h\to 0}\left(\frac{\tan h}{h}\right) \times \lim_{h\to 0}\left[\frac{1+\tan(x+h)\tan x}{\sqrt{\tan(x+h)}+\sqrt{\tan x}}\right]$ $= e^{\sqrt{\tan x}} \times 1\left(\frac{1+\tan^2 x}{\sqrt{\tan x}+\sqrt{\tan x}}\right)$ $f'(x) = \frac{e^{\sqrt{\tan x}}.\sec^2 x}{2\sqrt{\tan x}}$ ans.

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