

LIMITS & DERIVATIVES	
Class XI	
Q.1)	Differentiate using first principle method $f(x) = \cos(3x)$
Sol.1)	$\frac{dy}{dx} = \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\cos(3x+3h) - \cos(3x)}{h} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{-2 \sin\left(\frac{6x+3h}{2}\right) \cdot \sin\left(\frac{3h}{2}\right)}{h} \right) \quad \{\cos A - \cos B \text{ formula}\}$ $= \lim_{h \rightarrow 0} \left(\frac{-2 \sin\left(\frac{6x+3h}{2}\right) \cdot \sin\left(\frac{3h}{2}\right)}{\frac{3h}{2}} \times \frac{3}{2} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{3h}{2}\right) \cdot \sin\left(\frac{3h}{2}\right)}{\frac{3h}{2}} \right) \times \left[-3 \lim_{h \rightarrow 0} \left(\frac{6x+3h}{2} \right) \right]$ $= 1 \times (-3 \sin(3x)) \quad \left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$ $\frac{dy}{dx} = f'(x) = -3 \sin(3x) \text{ ans.}$
Q.2)	Differentiate using first formula $f(x) = \tan(2x)$
Sol.2)	$f'(x) = \frac{dy}{dx} = \lim_{h \rightarrow 0} \left(\frac{\tan(2x+2h) - \tan(2x)}{h} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\frac{\sin(2x+2h)}{\cos(2x+2h)} - \frac{\sin(2x)}{\cos(2x)}}{h} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\sin(2x+2h) \cdot \cos(2x) - \cos(2x+2h) \cdot \sin(2x)}{h \cdot \cos(2x+2h) \cos(2x)} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\sin(2x+2h-2x)}{h \cdot \cos(2x+2h) \cos(2x)} \right) \quad \{\sin(A - B) \text{ formula}\}$ $= \lim_{h \rightarrow 0} \left(\frac{\sin(2h)}{2h \cdot \cos(2x+2h) \cos(2x)} \times 2 \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\sin(2h)}{2h} \right) \times \lim_{h \rightarrow 0} \left(\frac{2}{\cos(2x+2h) \cdot \cos(2x)} \right)$ $= 1 \times \frac{2}{\cos(2x) \cdot \cos(2x)}$ $\therefore \frac{dy}{dx} = 2 \sec^2(2x) \text{ ans.}$
Q.3)	$f(x) = \sqrt{\tan x}$, find $f'(x)$ first principle method.
Sol.3)	$f(x) = \sqrt{\tan x}$

	$f'(x) = \lim_{h \rightarrow 0} \left[\frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h} \right]$ <p>Rationalize</p> $= \lim_{h \rightarrow 0} \left[\frac{\tan(x+h) - \tan x}{h\sqrt{\tan(x+h)} + \sqrt{\tan x}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\tan(x+h) - [1 + \tan(x+h) \cdot \tan x]}{h\sqrt{\tan(x+h)} + \sqrt{\tan x}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\tan h \cdot [1 + \tan(x+h) \cdot \tan x]}{h\sqrt{\tan(x+h)} + \sqrt{\tan x}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\tan h}{h} \right] \times \lim_{h \rightarrow 0} \left[\frac{1 + \tan(x+h) \cdot \tan x}{\sqrt{\tan(x+h)} + \sqrt{\tan x}} \right]$ $= 1 \times \frac{(1 + \tan(x+h) \cdot \tan x)}{\sqrt{\tan x} + \sqrt{\tan x}}$ $= \frac{1 + \tan^2 x}{2\sqrt{\tan x}}$ $\therefore f'(x) = \frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x \text{ ans.}$
Q.4)	$f(x) = \sec^2 x$ Using first principle method.
Sol.4)	$f'(x) = \lim_{h \rightarrow 0} \left[\frac{\sec^2(x+h) - \sec^2 x}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\frac{1}{\cos^2(x+h)} - \frac{1}{\cos^2 x}}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\cos^2 x - \cos^2(x+h)}{h \cdot \cos^2(x+h) \cdot \cos^2 x} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\{\cos x + \cos(x+h)\} \{\cos x - \cos(x+h)\}}{h \cdot \cos^2(x+h) \cdot \cos^2 x} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\{\cos x + \cos(x+h)\} \left\{-2 \sin\left(\frac{2x+h}{2}\right) \cdot \sin\left(-\frac{h}{2}\right)\right\}}{h \cdot \cos^2(x+h) \cdot \cos^2 x} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2 \sin\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right) \times \{\cos x + \cos(x+h)\}}{2 \times \frac{h}{2} \cdot \cos^2(x+h) \cdot \cos^2 x} \right]$ $= \lim_{h \rightarrow 0} \sin\left(\frac{2x+h}{2}\right) \times \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\right) \times \lim_{h \rightarrow 0} \left[\frac{\cos x + \cos(x+h)}{\cos^2(x+h) \cdot \cos^2 x} \right]$ $= \sin(x) \times 1 \times \frac{(\cos x + \cos x)}{\cos^2 x \cdot \cos^2 x}$ $= \frac{\sin x \cdot 2 \cos x}{\cos^2 x \cdot \cos^2 x}$ $= \frac{2 \sin x}{\cos x} \cdot \frac{1}{\cos^2 x}$ $f'(x) = 2 \tan x \cdot \sec^2 x \text{ ans.}$

Q.5)	$f(x) = \sin(x^2)$ Using first principle method.
Sol.5)	$f'(x) = \lim_{h \rightarrow 0} \left[\frac{\sin(x+h)^2 - \sin x^2}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\sin(x^2+h^2+2hx) - \sin(x^2)}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2 \cos\left(\frac{2x^2+h^2+2hx}{2}\right) \cdot \sin\left(\frac{h^2+2hx}{2}\right)}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2 \cos\left(\frac{2x^2+h^2+2hx}{2}\right) \cdot \sin\left(\frac{h^2+2hx}{2}\right)}{h \cdot \left(\frac{h^2+2hx}{2}\right)} \times \left(\frac{h^2+2hx}{2}\right) \right]$ $= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{h^2+2hx}{2}\right)}{\frac{h^2+2hx}{2}} \right) \times \lim_{h \rightarrow 0} 2 \cos\left(\frac{2x^2+h^2+2hx}{2}\right) \times \lim_{h \rightarrow 0} \left(\frac{h^2+2hx}{h} \right)$ $= 1 \times (2 \cos(x^2)) \times \lim_{h \rightarrow 0} \left[\frac{h(h+2x)}{2h} \right]$ $= 2 \cos(x^2) \times \left(\frac{2x}{2}\right)$ $\therefore \frac{dy}{dx} = 2x \cdot \cos(x^2) \text{ ans.}$
Q.6)	$f(x) = \tan \sqrt{x}$ Using first principle method.
Sol.6)	$f'(x) = \lim_{h \rightarrow 0} \tan(\sqrt{x+h}) - \tan \sqrt{x}$ $= \lim_{h \rightarrow 0} \left[\frac{\frac{\sin(\sqrt{x+h})}{\cos(\sqrt{x+h})} - \frac{\sin \sqrt{x}}{\cos \sqrt{x}}}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h}) \cdot \cos \sqrt{x} \cdot \cos(\sqrt{x+h}) \cdot \sin \sqrt{x}}{h \cdot \cos(\sqrt{x+h}) \cdot \cos \sqrt{x}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h} - \sqrt{x})}{h \cdot \cos(\sqrt{x+h}) \cdot \cos \sqrt{x}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h} - \sqrt{x}) \times (\sqrt{x+h} - \sqrt{x})}{h(\sqrt{x+h} - \sqrt{x}) \cdot \cos(\sqrt{x+h}) \cdot \cos \sqrt{x}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})} \right] \times \lim_{h \rightarrow 0} \left[\frac{1}{\cos(\sqrt{x+h}) \cdot \cos \sqrt{x}} \right] \times \lim_{h \rightarrow 0} \left[\frac{(\sqrt{x+h} - \sqrt{x})}{h} \right]$ $= 1 \times \frac{1}{\cos \sqrt{x} \cdot \cos \sqrt{x}} \times \lim_{h \rightarrow 0} \left[\frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} \right]$ $= \sec^2 \sqrt{x} \times \frac{1}{(\sqrt{x} + \sqrt{x})}$ $\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \cdot \sec^2 \sqrt{x} \text{ ans.}$
Q.7)	$f(x) = x \cos x$ Using first principle method.

Sol.7)	$f'(x) = \lim_{h \rightarrow 0} \left[\frac{(x+h) \cdot \cos(x+h) - x \cos x}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{x \cos(x+h) + h \cos(x+h) - x \cos x}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\sin(\sqrt{x+h}) \cdot \cos \sqrt{x} \cdot \cos(\sqrt{x+h}) \cdot \sin \sqrt{x}}{h \cdot \cos(\sqrt{x+h}) \cdot \cos \sqrt{x}} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{x \{ \cos(x+h) \} + h \cos(x+h) - x \cos x}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{x \left\{ -2 \sin\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right) \right\} + h \cdot \cos(x+h)}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{-2x \cdot \sin\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{2 \times \frac{h}{2}} + \frac{h \cdot \cos(x+h)}{h} \right]$ $= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \right) \times \left[x \lim_{h \rightarrow 0} \left(-\sin\left(\frac{2x+h}{2}\right) \right) \right] + \lim_{h \rightarrow 0} (\cos(x+h))$ $= (1)(-x \sin x) + \cos x$ $\therefore f'(x) = -x \sin x + \cos x \text{ ans.}$
Q.8)	$f(x) = \frac{\sin x}{x} \text{ Using first principle method.}$
Sol.8)	$f'(x) = \lim_{h \rightarrow 0} \left[\frac{\frac{\sin(x+h)}{x+h} - \frac{\sin x}{x}}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{x \sin(x+h) - (x+h) \sin x}{h(x+h)x} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{x \sin(x+h) - x \sin x - h \sin x}{h(x+h)x} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{x \{ \sin(x+h) - \sin x \} - h \sin x}{h(x+h)x} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{x \{ \sin(x+h) - \sin x \} - h \sin x}{h(x+h)x} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{x \{ \sin(x+h) - \sin x \}}{h(x+h)x} - \frac{h \sin x}{h(x+h)x} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{x \cdot 2 \cos\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{2 \times \frac{h}{2} (x+h)x} - \frac{\sin x}{(x+h)x} \right]$ $= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \right) \times \lim_{h \rightarrow 0} \left(\frac{x \cdot \cos\left(\frac{2x+h}{2}\right)}{(x+h)x} \right) - \lim_{h \rightarrow 0} \left(\frac{\sin x}{(x+h)x} \right)$ $= 1 \times \left(\frac{x \cos x}{x^2} \right) - \frac{\sin x}{x^2}$ $\therefore \frac{dy}{dx} = \frac{x \cos x - \sin x}{x^2} \text{ ans.}$



Q.9)	$f(x) = \sin x - \cos x$ Using first principle method.
Sol.9)	$f'(x) = \lim_{h \rightarrow 0} \left[\frac{\{\sin(x+h) - \cos(x+h)\} - \{\sin x - \cos x\}}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\{\sin(x+h) - \sin x\} - \{\cos(x+h) - \cos x\}}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2 \cos\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right) + 2 \sin\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right)}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2 \sin\left(\frac{h}{2}\right) \left\{ \cos\left(\frac{2x+h}{2}\right) + \sin\left(\frac{2x+h}{2}\right) \right\}}{2 \times \frac{h}{2}} \right]$ $= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \right) \times \lim_{h \rightarrow 0} \left(\cos\left(\frac{2x+h}{2}\right) + \sin\left(\frac{2x+h}{2}\right) \right)$ $= 1 \times (\cos x + \sin x)$ <p>$\therefore f'(x) = \cos x + \sin x$ ans.</p>
Q.10)	$f(x) = \frac{2x^2+1}{x-3}$ Using first principle method.
Sol.10)	$f'(x) = \lim_{h \rightarrow 0} \left[\frac{\left\{ \frac{2(x+h)^2+1}{x+h-3} \right\} - \left\{ \frac{2x^2+1}{x-3} \right\}}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{\left\{ \frac{2(x+h)^2+1}{x+h-3} \right\} - \left\{ \frac{2x^2+1}{x-3} \right\}}{h} \right] \quad (\text{repeat})$ $= \lim_{h \rightarrow 0} \left[\frac{\left(\frac{2x^2+h^2+2hx+1}{x+h-3} \right) - \frac{2x^2+1}{x-3}}{h} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{(2x^2+h^2+2hx+1)(x-3) - (2x^2+1)(x+h-3)}{h(x+h-3)(x-3)} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2x^3 - 6x^2 + 2h^2x - 6h^2 + 2hx^2 - 6hx + x - 3 - 2x^3 - 2x^2h + 6x^2 + x - h + 3}{h(x+h-3)(x-3)} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{2h^2x - 6h^2 - 6hx - h}{h(x+h-3)(x-3)} \right]$ $= \lim_{h \rightarrow 0} \left[\frac{h(2hx - 6h - 6x - 1)}{h(x+h-3)(x-3)} \right]$ <p>$\therefore f'(x) = \frac{-6x-1}{(x-3)(x-3)} = \frac{-6x-1}{(x-3)^2}$ ans.</p>