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|  | LIMITS \& DERIVATIVES Class XI |
| :---: | :---: |
| Q.1) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{10^{x}-2^{x}-5^{x}+1}{x \tan x}\right)$ |
| Sol.1) | We have $\lim _{x \rightarrow 0}\left(\frac{10^{x}-2^{x}-5^{x}+1}{x \tan x}\right)$ $\begin{aligned} & =\lim _{x \rightarrow 0}\left(\frac{2^{x}\left(5^{x}-1\right)-1\left(5^{x}-1\right)}{x \tan x}\right) \quad\left\{10^{x}=2^{x} \cdot 5^{x}\right\} \\ & =\lim _{x \rightarrow 0}\left(\frac{\left(5^{x}-1\right)\left(2^{x}-1\right)}{x \tan x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{\left(\frac{5^{x}-1}{x}\right) \times x \cdot\left(\frac{2^{x}-1}{x}\right) \times x}{x\left(\frac{\tan x}{x}\right) \times x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{\left(\frac{5^{x}-1}{x}\right) \cdot\left(\frac{2^{x}-1}{x}\right)}{\left(\frac{\tan x}{x}\right)}\right) \\ & =\frac{\lim _{x \rightarrow 0}\left(\frac{5^{x}-1}{x}\right) \times \lim _{x \rightarrow 0}\left(\frac{2^{x}-1}{x}\right)}{\lim _{x \rightarrow 0}\left(\frac{\tan x}{x}\right)} \\ & =\frac{(\log 5)(\log 2)}{1} \\ & =\log (5) \log (2) \text { ans. } \end{aligned} \quad\left\{\begin{array}{c} \lim _{x \rightarrow 0}\left(\frac{\tan x}{x}\right)=1 \\ \left\{\lim _{x \rightarrow 0}\left(\frac{a^{x}-1}{x}\right)=\log a\right\} \end{array}\right\}$ |
| Q.2) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{e^{x}+e^{-x}-2}{x^{2}}\right)$ |
| Sol.2) | We have $\lim _{x \rightarrow 0}\left(\frac{e^{x}+e^{-x}-2}{x^{2}}\right)$ $\begin{array}{ll} =\lim _{x \rightarrow 0}\left(\frac{e^{x}+\frac{1}{e^{x}}-2}{x^{2}}\right) & \\ =\lim _{x \rightarrow 0}\left(\frac{e^{2 x}+1-2 e^{x}}{e^{x} \cdot x^{2}}\right) & \left\{\left(e^{x}\right)^{2}=e^{2 x}\right\} \\ =\lim _{x \rightarrow 0}\left(\frac{\left(e^{x}-1\right)^{2}}{e^{x} \cdot x^{2}}\right) & \\ =\lim _{x \rightarrow 0}\left(\frac{\left(e^{x}-1\right)^{2}}{x}\right) \times \lim _{x \rightarrow 0}\left(\frac{1}{e^{x}}\right) & \\ =(1)^{2} \times \frac{1}{e^{0}} & \left\{\lim _{x \rightarrow 0}\left(\frac{e^{x}-1}{x}\right)=1\right\} \\ =1 \text { ans. } & \left\{e^{0}=1\right\} \end{array}$ |
| Q.3) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{\log \left(1+x^{3}\right)}{\sin ^{3} x}\right)$ |

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| Sol.3) | We have $\lim _{x \rightarrow 0}\left(\frac{\log \left(1+x^{3}\right)}{\sin ^{3} x}\right)$ $\begin{aligned} & =\lim _{x \rightarrow 0}\left[\frac{\frac{\log \left(1+x^{3}\right)}{x^{3}} \times x^{3}}{\frac{\sin ^{3} x}{x^{3}} \times x^{3}}\right] \\ & =\frac{\lim _{x \rightarrow 0}\left(\frac{\log \left(1+x^{3}\right)}{x^{3}}\right)}{\lim _{x \rightarrow 0}\left(\frac{\sin ^{3} x}{x^{3}}\right)} \\ & =\frac{1}{1^{3}}=1 \text { ans. } \end{aligned} \quad\left\{\lim _{x \rightarrow 0}\left(\frac{\log (1+x)}{x}\right)=1\right\}$ |
| :---: | :---: |
| Q.4) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{2^{3 x}-3^{2 x}}{\sin (3 x)}\right)$ |
| Sol.4) | We have $\lim _{x \rightarrow 0}\left(\frac{2^{3 x}-3^{2 x}}{\sin (3 x)}\right)$ $\left.\begin{array}{l} =\lim _{x \rightarrow 0}\left(\frac{2^{3 x}-3^{2 x}-1+1}{\sin (3 x)}\right) \\ =\lim _{x \rightarrow 0}\left(\frac{\left(2^{3 x}-1\right)-\left(3^{2 x}-1\right)}{\sin (3 x)}\right) \\ =\lim _{x \rightarrow 0}\left(\frac{\left(2^{3 x}-1\right)}{3 x} \times 3 x-\frac{\left(3^{2 x}-1\right)}{2 x} \times 2 x\right. \\ \frac{\sin (3 x)}{3 x} \times 3 x \end{array}\right) .$ $\left\{\lim _{x \rightarrow 0}\left(\frac{a^{x}-1}{x}\right)=\log a\right\}$ $\left\{\log m^{n}=n \log m\right\}$ $\left\{\log A-\log B=\log \left(\frac{A}{B}\right)\right\}$ |
| Q.5) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{x\left(e^{x}-1\right)}{1-\cos x}\right)$ |
| Sol.5) | We have $\lim _{x \rightarrow 0}\left(\frac{x\left(e^{x}-1\right)}{1-\cos x}\right)$ $\begin{aligned} & =\lim _{x \rightarrow 0}\left(\frac{x\left(e^{x}-1\right)}{2 \sin ^{2} \frac{x}{2}}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{\frac{x\left(e^{x}-1\right)}{x} x x}{\frac{2 \sin ^{2} \frac{x}{2}}{\frac{1}{2}} \times \frac{1}{2}}\right) \\ & =\frac{2 \lim _{x \rightarrow 0}\left(\frac{\left(e^{x}-1\right)}{x}\right)}{\lim _{x \rightarrow 0}\left(\frac{2 \sin ^{2} \frac{x}{2}}{\frac{x^{2}}{4}}\right)} \end{aligned}$ $\left\{\lim _{x \rightarrow 0}\left(\frac{e^{x}-1}{x}\right)=1\right\}$ |

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|  | $=\frac{2(1)}{1^{2}}=2 \mathrm{ans} .$ |
| :---: | :---: |
| Q.6) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{a^{x}+b^{x}+c^{x}-1}{x}\right)$ |
| Sol.6) | We have $\begin{aligned} & \mathrm{e} \lim _{x \rightarrow 0}\left(\frac{a^{x}+b^{x}+c^{x}-1}{x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{a^{x}+b^{x}+c^{x}-1-1-1}{x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{\left(a^{x}-1\right)+\left(b^{x}-1\right)+\left(c^{x}-1\right)}{x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{a^{x}-1}{x}+\frac{b^{x}-1}{x}+\frac{c^{x}-1}{x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{a^{x}-1}{x}\right)+\lim _{x \rightarrow 0}\left(\frac{b^{x}-1}{x}\right)+\lim _{x \rightarrow 0}\left(\frac{c^{x}-1}{x}\right) \\ & =\log a+\log b+\log c=\log (a b c) \text { ans. } \quad\{\log A+\log B=\log (A B)\} \end{aligned}$ |
| Q.7) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{a^{x}+b^{x}-c^{x}-d^{x}}{x}\right)$ |
| Sol.7) | We have $\lim _{x \rightarrow 0}\left(\frac{a^{x}+b^{x}-c^{x}-d^{x}}{x}\right)$ $\begin{array}{ll} =\lim _{x \rightarrow 0}\left(\frac{a^{x}+b^{x}-c^{x}-d^{x}-1-1+1+1}{x}\right) & \\ =\lim _{x \rightarrow 0}\left(\frac{\left(a^{x}-1\right)+\left(b^{x}-1\right)-\left(c^{x}-1\right)-\left(d^{x}-1\right)}{x}\right) & \\ =\lim _{x \rightarrow 0}\left[\left(\frac{a^{x}-1}{x}\right)+\left(\frac{b^{x}-1}{x}\right)-\left(\frac{c^{x}-1}{x}\right)-\left(\frac{d^{x}-1}{x}\right)\right] \\ =\lim _{x \rightarrow 0}\left(\frac{a^{x}-1}{x}\right)+\lim _{x \rightarrow 0}\left(\frac{b^{x}-1}{x}\right)-\lim _{x \rightarrow 0}\left(\frac{c^{x}-1}{x}\right)-\lim _{x \rightarrow 0}\left(\frac{d^{x}-1}{x}\right) \\ =\log a+\log h-\log c-\log d & \left\{\lim _{x \rightarrow 0}\left(\frac{a^{x}-1}{x}\right)=\log a\right\} \\ =(\log a+\log b)-(\log c+\log d) & \\ =\log (a b)-\log (c d) & \{\log A+\log B=\log (A B)\} \\ =\log \left(\frac{a b}{c d}\right) \text { ans. } & \left\{\log A-\log B=\log \left(\frac{A}{B}\right)\right\} \end{array}$ |
| Q.8) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{\log (5+x)-\log (5-x)}{x}\right)$ |
| Sol.8) | We have $\lim _{x \rightarrow 0}\left(\frac{\log (5+x)-\log (5-x)}{x}\right)$ $\begin{aligned} & =\lim _{x \rightarrow 0}\left(\frac{\log \left(5\left(1+\frac{x}{5}\right)\right)-\log \left(5\left(1-\frac{x}{5}\right)\right)}{x}\right) \\ & =\lim _{x \rightarrow 0}\left[\frac{\left[\log 5+\log \left(1+\frac{x}{5}\right)\right\}-\left\{\log 5+\log \left(1-\frac{x}{5}\right)\right\}}{x}\right] \\ & =\lim _{x \rightarrow 0} \frac{\log \left(1+\frac{x}{5}\right)-\log \left(1-\frac{x}{5}\right)}{x} \end{aligned}$ $\{\log A+\log B=\log (A B)\}$ |

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|  | $\begin{aligned} & =\lim _{x \rightarrow 0} \frac{\log \left(1+\frac{x}{5}\right)}{\frac{x}{5} \times 5}-\lim _{x \rightarrow 0} \frac{\log \left(1+\frac{-x}{5}\right)}{x} \\ & =\lim _{x \rightarrow 0} \frac{\log \left(1+\frac{x}{5}\right)}{\frac{x}{5} \times 5}+\lim _{x \rightarrow 0} \frac{\log \left(1+\frac{-x}{5}\right)}{-\frac{x}{5} \times 5} \\ & =\frac{1}{5}+\frac{1}{5} \\ & =\frac{2}{5} \text { ans. } \end{aligned}$ |
| :---: | :---: |
| Q.9) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{e^{3+x}-\sin x-e^{3}}{x}\right)$ |
| Sol.9) | We have $\lim _{x \rightarrow 0}\left(\frac{e^{3+x}-\sin x-e^{3}}{x}\right)$ $\begin{aligned} & =\lim _{x \rightarrow 0}\left(\frac{e^{3+x}-e^{3}-\sin x}{x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{e^{3}-\left(e^{x}-1\right)-\sin x}{x}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{e^{3}\left(e^{x}-1\right)}{x}-\frac{\sin x}{x}\right) \\ & =e^{3} \lim _{x \rightarrow 0}\left[\frac{e^{x}-1}{x}\right]-\lim _{x \rightarrow 0}\left(\frac{\sin x}{x}\right) \\ & =e^{3}-(1) \\ & =e^{3}-1 \text { ans. } \end{aligned}$ |
| Q.10) | Evaluate $\lim _{x \rightarrow 0}\left(\frac{9^{x}-6^{x}-6^{x}+4}{x}\right)$ |
| Sol.10) | We have $\lim _{x \rightarrow 0}\left(\frac{9^{x}-6^{x}-6^{x}+4}{x}\right)$ $\begin{aligned} & =\lim _{x \rightarrow 0}\left(\frac{9^{x}-6^{x}-6^{x}+4}{x^{2}}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{3^{x}\left(3^{x}-2^{x}\right)-2^{x}\left(3^{x}-2^{x}\right)}{x^{2}}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{\left(3^{x}-2^{x}\right) \cdot\left(3^{x}-2^{x}\right)}{x^{2}}\right) \\ & =\lim _{x \rightarrow 0}\left(\frac{\left(3^{x}-2^{x}\right)^{2}}{x^{2}}\right) \\ & =\lim _{x \rightarrow 0}\left[\left(\frac{3^{x}-2^{x}}{x}\right)^{2}\right] \\ & =\lim _{x \rightarrow 0}\left[\left[\frac{\left(3^{x}-1\right)-\left(2^{x}-1\right)}{x}\right]^{2}\right] \\ & =\left\{\lim _{x \rightarrow 0}\left[\frac{3^{x}-1}{x}\right]-\lim _{x \rightarrow 0}\left[\frac{2^{x}-1}{x}\right]\right\}^{2} \\ & =(\log 3-\log 2)^{2} \quad\left\{\lim _{x \rightarrow 0}\left(\frac{a^{x}-1}{x}\right)=\log a\right\} \end{aligned}$ |

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|  | $=\left(\log \left(\frac{3}{2}\right)\right)^{2}$ ans. |
| :--- | :--- | \(\begin{cases}\{\log A+\log B=\log (A B)\} <br>

\hline\end{cases}\)

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