	LIMITS & DERIVATIVES ***Class XI***
Q.1)	Evaluate $\lim_{x\to 0}\left(\frac{10^x-2^x-5^x+1}{x\tan x}\right)$
Sol.1)	We have $\lim_{x\to 0}\left(\frac{10^x-2^x-5^x+1}{x\tan x}\right)$ $=\lim_{x\to 0}\left(\frac{2^x(5^x-1)-1(5^x-1)}{x\tan x}\right)$ $\{10^x=2^x.5^x\}$ $=\lim_{x\to 0}\left(\frac{(5^x-1)(2^x-1)}{x\tan x}\right)$ $=\lim_{x\to 0}\left(\frac{\left(\frac{5^x-1}{x}\right)\times x.\left(\frac{2^x-1}{x}\right)\times x}{x\left(\frac{\tan x}{x}\right)\times x}\right)$ $=\lim_{x\to 0}\left(\frac{\left(\frac{5^x-1}{x}\right).\left(\frac{2^x-1}{x}\right)}{\left(\frac{\tan x}{x}\right)}\right)$ $=\frac{\lim_{x\to 0}\left(\frac{5^x-1}{x}\right)\times\lim_{x\to 0}\left(\frac{2^x-1}{x}\right)}{\lim_{x\to 0}\left(\frac{\tan x}{x}\right)}$ $=\frac{(\log 5)(\log 2)}{1}$ $\left\{\begin{matrix}\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1\\ \left\{\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)=\log a\right\}\end{matrix}\right\}$ $=\log(5)\log(2)$ ans.
Q.2)	Evaluate $\lim_{x\to 0}\left(\frac{e^x+e^{-x}-2}{x^2}\right)$
Sol.2)	We have $\lim_{x\to 0}\left(\frac{e^x+e^{-x}-2}{x^2}\right)$ $=\lim_{x\to 0}\left(\frac{e^x+\frac{1}{e^x}-2}{x^2}\right)$ $=\lim_{x\to 0}\left(\frac{e^{2x}+1-2e^x}{e^x.x^2}\right)$ $\{(e^x)^2=e^{2x}\}$ $=\lim_{x\to 0}\left(\frac{(e^x-1)^2}{e^x.x^2}\right)$ $=\lim_{x\to 0}\left(\frac{(e^x-1)^2}{x}\right)\times\lim_{x\to 0}\left(\frac{1}{e^x}\right)$ $=(1)^2\times\frac{1}{e^0}$ $\left\{\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=1\right\}$ $=1$ ans. $\{e^0=1\}$
Q.3)	Evaluate $\lim_{x\to 0}\left(\frac{\log(1+x^3)}{\sin^3 x}\right)$

Sol.3)	We have $\lim_{x\to 0}\left(\frac{\log(1+x^3)}{\sin^3 x}\right)$ $= \lim_{x\to 0}\left[\frac{\frac{\log(1+x^3)}{x^3}\times x^3}{\frac{\sin^3 x}{x^3}\times x^3}\right]$ $= \frac{\lim_{x\to 0}\left(\frac{\log(1+x^3)}{x^3}\right)}{\lim_{x\to 0}\left(\frac{\sin^3 x}{x^3}\right)}$ $\left\{\lim_{x\to 0}\left(\frac{\log(1+x)}{x}\right)=1\right\}$ $= \frac{1}{1^3} = 1$ ans.
Q.4)	Evaluate $\lim_{x\to 0}\left(\frac{2^{3x}-3^{2x}}{\sin(3x)}\right)$
Sol.4)	We have $\lim_{x\to 0}\left(\frac{2^{3x}-3^{2x}}{\sin(3x)}\right)$ $= \lim_{x\to 0}\left(\frac{2^{3x}-3^{2x}-1+1}{\sin(3x)}\right)$ $= \lim_{x\to 0}\left(\frac{(2^{3x}-1)-(3^{2x}-1)}{\sin(3x)}\right)$ $= \lim_{x\to 0}\left(\frac{\frac{(2^{3x}-1)}{3x}\times 3x-\frac{(3^{2x}-1)}{2x}\times 2x}{\frac{\sin(3x)}{3x}\times 3x}\right)$ $= \frac{\lim_{x\to 0}\left[\frac{2^{3x}-1}{3x}\right]\times 3-\lim_{x\to 0}\left[\frac{3^{2x}-1}{2x}\right]\times 2}{\lim_{x\to 0}\left[\frac{\sin(3x)}{3x}\right]\times 3}$ $= \frac{3(\log 2)-2(\log 3)}{1\times 3}$ $\left\{\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)=\log a\right\}$ $= \frac{\log 2^3-\log 3^2}{3}$ $\{\log m^n = n\log m\}$ $= \frac{1}{3}\log\left(\frac{8}{9}\right)$ ans. $\left\{\log A-\log B=\log\left(\frac{A}{B}\right)\right\}$
Q.5)	Evaluate $\lim_{x\to 0}\left(\frac{x(e^x-1)}{1-\cos x}\right)$
Sol.5)	We have $\lim_{x\to 0}\left(\frac{x(e^x-1)}{1-\cos x}\right)$ $= \lim_{x\to 0}\left(\frac{x(e^x-1)}{2\sin^2\frac{x}{2}}\right)$ $= \lim_{x\to 0}\left(\frac{\frac{x(e^x-1)}{x}\times x}{\frac{2\sin^2\frac{x}{2}}{\frac{x^2}{4}}\times\frac{1}{2}}\right)$ $= \frac{2\lim_{x\to 0}\left(\frac{(e^x-1)}{x}\right)}{\lim_{x\to 0}\left(\frac{2\sin^2\frac{x}{2}}{\frac{x^2}{4}}\right)}$ $\left\{\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)=1\right\}$

	$= \frac{2(1)}{1^2} = 2$ ans.
Q.6)	Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x-1}{x}\right)$
Sol.6)	We have $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x-1}{x}\right)$ $= \lim_{x\to 0}\left(\frac{a^x+b^x+c^x-1-1-1}{x}\right)$ $= \lim_{x\to 0}\left(\frac{(a^x-1)+(b^x-1)+(c^x-1)}{x}\right)$ $= \lim_{x\to 0}\left(\frac{a^x-1}{x}+\frac{b^x-1}{x}+\frac{c^x-1}{x}\right)$ $= \lim_{x\to 0}\left(\frac{a^x-1}{x}\right)+\lim_{x\to 0}\left(\frac{b^x-1}{x}\right)+\lim_{x\to 0}\left(\frac{c^x-1}{x}\right)$ $= \log a + \log b + \log c = \log(abc)$ ans. $\{\log A + \log B = \log(AB)\}$
Q.7)	Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x-c^x-d^x}{x}\right)$
Sol.7)	We have $\lim_{x\to 0}\left(\frac{a^x+b^x-c^x-d^x}{x}\right)$ $= \lim_{x\to 0}\left(\frac{a^x+b^x-c^x-d^x-1-1+1+1}{x}\right)$ $= \lim_{x\to 0}\left(\frac{(a^x-1)+(b^x-1)-(c^x-1)-(d^x-1)}{x}\right)$ $= \lim_{x\to 0}\left[\left(\frac{a^x-1}{x}\right)+\left(\frac{b^x-1}{x}\right)-\left(\frac{c^x-1}{x}\right)-\left(\frac{d^x-1}{x}\right)\right]$ $= \lim_{x\to 0}\left(\frac{a^x-1}{x}\right)+\lim_{x\to 0}\left(\frac{b^x-1}{x}\right)-\lim_{x\to 0}\left(\frac{c^x-1}{x}\right)-\lim_{x\to 0}\left(\frac{d^x-1}{x}\right)$ $= \log a + \log h - \log c - \log d$ $\left\{\lim_{x\to 0}\left(\frac{a^x-1}{x}\right) = \log a\right\}$ $= (\log a + \log b) - (\log c + \log d)$ $= \log(ab) - \log(cd)$ $\{\log A + \log B = \log(AB)\}$ $= \log\left(\frac{ab}{cd}\right)$ ans. $\left\{\log A - \log B = \log\left(\frac{A}{B}\right)\right\}$
Q.8)	Evaluate $\lim_{x\to 0}\left(\frac{\log(5+x)-\log(5-x)}{x}\right)$
Sol.8)	We have $\lim_{x\to 0}\left(\frac{\log(5+x)-\log(5-x)}{x}\right)$ $= \lim_{x\to 0}\left(\frac{\log\left(5\left(1+\frac{x}{5}\right)\right)-\log\left(5\left(1-\frac{x}{5}\right)\right)}{x}\right)$ $= \lim_{x\to 0}\left[\frac{\left\{\log 5+\log\left(1+\frac{x}{5}\right)\right\}-\left\{\log 5+\log\left(1-\frac{x}{5}\right)\right\}}{x}\right]$ $\{\log A + \log B = \log(AB)\}$ $= \lim_{x\to 0}\frac{\log\left(1+\frac{x}{5}\right)-\log\left(1-\frac{x}{5}\right)}{x}$

	$= \lim_{x\to 0} \frac{\log\left(1+\frac{x}{5}\right)}{\frac{x}{5}\times 5} - \lim_{x\to 0} \frac{\log\left(1+\frac{-x}{5}\right)}{x}$ $= \lim_{x\to 0} \frac{\log\left(1+\frac{x}{5}\right)}{\frac{x}{5}\times 5} + \lim_{x\to 0} \frac{\log\left(1+\frac{-x}{5}\right)}{-\frac{x}{5}\times 5}$ $= \frac{1}{5} + \frac{1}{5}$ $\left\{\lim_{x\to 0}\left(\frac{log(1+x)}{x}\right) = 1\right\}$ $= \frac{2}{5}$ ans.
Q.9)	Evaluate $\lim_{x\to 0}\left(\frac{e^{3+x}-\sin x-e^3}{x}\right)$
Sol.9)	We have $\lim_{x\to 0}\left(\frac{e^{3+x}-\sin x-e^3}{x}\right)$ $= \lim_{x\to 0}\left(\frac{e^{3+x}-e^3-\sin x}{x}\right)$ $= \lim_{x\to 0}\left(\frac{e^3-(e^x-1)-\sin x}{x}\right)$ $= \lim_{x\to 0}\left(\frac{e^3(e^x-1)}{x} - \frac{\sin x}{x}\right)$ $= e^3 \lim_{x\to 0}\left[\frac{e^x-1}{x}\right] - \lim_{x\to 0}\left(\frac{\sin x}{x}\right)$ $= e^3 - (1)$ $= e^3 - 1$ ans.
Q.10)	Evaluate $\lim_{x\to 0}\left(\frac{9^x-6^x-6^x+4}{x}\right)$
Sol.10)	We have $\lim_{x\to 0}\left(\frac{9^x-6^x-6^x+4}{x}\right)$ $= \lim_{x\to 0}\left(\frac{9^x-6^x-6^x+4}{x^2}\right)$ $= \lim_{x\to 0}\left(\frac{3^x(3^x-2^x)-2^x(3^x-2^x)}{x^2}\right)$ $= \lim_{x\to 0}\left(\frac{(3^x-2^x).(3^x-2^x)}{x^2}\right)$ $= \lim_{x\to 0}\left(\frac{(3^x-2^x)^2}{x^2}\right)$ $= \lim_{x\to 0}\left[\left(\frac{3^x-2^x}{x}\right)^2\right]$ $= \lim_{x\to 0}\left[\left[\frac{(3^x-1)-(2^x-1)}{x}\right]^2\right]$ $= \left\{\lim_{x\to 0}\left[\frac{3^x-1}{x}\right] - \lim_{x\to 0}\left[\frac{2^x-1}{x}\right]\right\}^2$ $= (\log 3 - \log 2)^2$ $\left\{\lim_{x\to 0}\left(\frac{a^x-1}{x}\right) = \log a\right\}$

	$= \left(\log\left(\frac{3}{2}\right)\right)^2$ ans.	$\{\log A + \log B = \log(AB)\}$