

	LIMITS & DERIVATIVES
	Class XI
Q.1)	Evaluate $\lim_{x \to 0} \left(\frac{10^x - 2^x - 5^x + 1}{x \tan x} \right)$
Sol.1)	We have $\lim_{x \to 0} \left(\frac{10^x - 2^x - 5^x + 1}{x \tan x} \right)$
	$= \lim_{x \to 0} \left(\frac{2^{x} (5^{x} - 1) - 1(5^{x} - 1)}{x \tan x} \right) $ {10 ^x = 2 ^x .5 ^x }
	$= \lim_{x \to 0} \left(\frac{(5^{x} - 1)(2^{x} - 1)}{x \tan x} \right)$
	$= \lim_{x \to 0} \left(\frac{\left(\frac{5^{x}-1}{x}\right) \times x. \left(\frac{2^{x}-1}{x}\right) \times x}{x \left(\frac{\tan x}{x}\right) \times x} \right)$
	$= \lim_{x \to 0} \left(\frac{\left(\frac{5^{x}-1}{x}\right) \cdot \left(\frac{2^{x}-1}{x}\right)}{\left(\frac{\tan x}{x}\right)} \right)$
	$=\frac{\lim_{x\to 0}\left(\frac{5^{x}-1}{x}\right)\times\lim_{x\to 0}\left(\frac{2^{x}-1}{x}\right)}{\lim_{x\to 0}\left(\frac{\tan x}{x}\right)}$
	$= \frac{(\log 5)(\log 2)}{1} \qquad \qquad$
	$= \log(5)\log(2)$ ans.
Q.2)	Evaluate $\lim_{x \to 0} \left(\frac{e^x + e^{-x} - 2}{x^2} \right)$
Sol.2)	We have $\lim_{x \to 0} \left(\frac{e^{x} + e^{-x} - 2}{x^2} \right)$ $= \lim_{x \to 0} \left(\frac{e^{x} + \frac{1}{e^{x}} - 2}{x^2} \right)$
	$= \lim_{x \to 0} \left(\frac{1}{x^2} \right)$
	$= \lim_{x \to 0} \left(\frac{e^{2x} + 1 - 2e^x}{e^x \cdot x^2} \right) \qquad \{ (e^x)^2 = e^{2x} \}$
	$= \lim_{x \to 0} \left(\frac{(e^x - 1)^2}{e^x \cdot x^2} \right)$
	$= \lim_{x \to 0} \left(\frac{(e^x - 1)^2}{x} \right) \times \lim_{x \to 0} \left(\frac{1}{e^x} \right)$
	$= (1)^{2} \times \frac{1}{e^{0}} \qquad \left\{ \lim_{x \to 0} \left(\frac{e^{x} - 1}{x} \right) = 1 \right\}$
Q.3)	$= 1 \text{ ans.} \qquad \{e^0 = 1\}$
~~~/	Evaluate $\lim_{x \to 0} \left( \frac{\log(1+x^3)}{\sin^3 x} \right)$

Copyright © www.studiestoday.com All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission.

	Studies Today.com	
Sol.3)	We have $\lim_{x \to 0} \left( \frac{\log(1+x^3)}{\sin^3 x} \right)$	
	$= \lim_{x \to 0} \left[ \frac{\frac{\log(1+x^3)}{x^3} \times x^3}{\frac{\sin^3 x}{x^3} \times x^3} \right]$	
	$=\frac{\lim_{x\to 0}\left(\frac{\log(1+x^3)}{x^3}\right)}{\lim_{x\to 0}\left(\frac{\sin^3 x}{x^3}\right)}$	$\left\{\lim_{x\to 0} \left(\frac{\log(1+x)}{x}\right) = 1\right\}$
	$=\frac{1}{1^3}=1$ ans.	
Q.4)	Evaluate $\lim_{x \to 0} \left( \frac{2^{3x} - 3^{2x}}{\sin(3x)} \right)$	
Sol.4)	We have $\lim_{x \to 0} \left( \frac{2^{3x} - 3^{2x}}{\sin(3x)} \right)$	
	$= \lim_{x \to 0} \left( \frac{2^{3x} - 3^{2x} - 1 + 1}{\sin(3x)} \right)$	
	$= \lim_{x \to 0} \left( \frac{(2^{3x} - 1) - (3^{2x} - 1)}{\sin(3x)} \right)$	
	$= \lim_{x \to 0} \left( \frac{\frac{(2^{3x} - 1)}{3x} \times 3x - \frac{(3^{2x} - 1)}{2x} \times 2x}{\frac{\sin(3x)}{3x} \times 3x} \right)$	600
	$= \frac{\lim_{x \to 0} \left( \frac{\sin(3x)}{3x} \times 3x \right)}{\lim_{x \to 0} \left[ \frac{2^{3x} - 1}{3x} \right] \times 3 - \lim_{x \to 0} \left[ \frac{3^{2x} - 1}{2x} \right] \times 2}{\lim_{x \to 0} \left[ \frac{\sin(3x)}{3x} \right] \times 3}$ $= \frac{3(\log 2) - 2(\log 3)}{2}$	5
	$=\frac{3(\log 2)-2(\log 3)}{1\times 3}$	$\left\{\lim_{x \to 0} \left(\frac{a^{x} - 1}{x}\right) = \log a\right\}$
	$=\frac{\log 2^3 - \log 3^2}{3}$	$\{\log m^n = n \log m\}$
	$=\frac{1}{3}\log\left(\frac{8}{9}\right)$ ans.	$\left\{\log A - \log B = \log\left(\frac{A}{B}\right)\right\}$
Q.5)	Evaluate $\lim_{x \to 0} \left( \frac{x(e^x - 1)}{1 - \cos x} \right)$	
Sol.5)	We have $\lim_{x \to 0} \left( \frac{x(e^x - 1)}{1 - \cos x} \right)$	
	$= \lim_{x \to 0} \left( \frac{x(e^x - 1)}{2 \sin^2 \frac{x}{2}} \right)$	
	$= \lim_{x \to 0} \left( \frac{\frac{x(e^{x}-1)}{x} \times x}{\frac{2 \sin^2 x}{\frac{x^2}{4}} \times \frac{1}{2}} \right)$	
	$=\frac{2\lim_{x\to 0}\left(\frac{(e^{x}-1)}{x}\right)}{\lim_{x\to 0}\left(\frac{2\sin^{2}\frac{x}{2}}{\frac{x^{2}}{4}}\right)}$	$\left\{\lim_{x\to 0} \left(\frac{e^{x}-1}{x}\right) = 1\right\}$

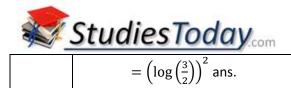
Copyright © www.studiestoday.com All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission.

Studies Today.com			
	$=\frac{2(1)}{1^2}=2$ ans.		
Q.6)	Evaluate $\lim_{x \to 0} \left( \frac{a^x + b^x + c^x - 1}{x} \right)$		
Sol.6)	We have $\lim_{x \to 0} \left( \frac{a^x + b^x + c^x - 1}{x} \right)$		
	$= \lim_{x \to 0} \left( \frac{a^{x} + b^{x} + c^{x} - 1 - 1 - 1}{x} \right)$		
	$= \lim_{x \to 0} \left( \frac{(a^{x}-1) + (b^{x}-1) + (c^{x}-1)}{x} \right)$		
	$= \lim_{x \to 0} \left( \frac{a^{x} - 1}{x} + \frac{b^{x} - 1}{x} + \frac{c^{x} - 1}{x} \right)$		
	$= \lim_{x \to 0} \left( \frac{a^{x} - 1}{x} \right) + \lim_{x \to 0} \left( \frac{b^{x} - 1}{x} \right) + \lim_{x \to 0} \left( \frac{c^{x} - 1}{x} \right)$		
	$= \log a + \log b + \log c = \log(abc) \text{ ans.}$	$\{\log A + \log B = \log(AB)\}\$	
Q.7)	Evaluate $\lim_{x \to 0} \left( \frac{a^x + b^x - c^x - d^x}{x} \right)$	G	
Sol.7)	We have $\lim_{x \to 0} \left( \frac{a^x + b^x - c^x - d^x}{x} \right)$	2	
	$= \lim_{x \to 0} \left( \frac{a^{x} + b^{x} - c^{x} - d^{x} - 1 - 1 + 1 + 1}{x} \right)$	30	
	$= \lim_{x \to 0} \left( \frac{(a^x - 1) + (b^x - 1) - (c^x - 1) - (d^x - 1)}{x} \right)$		
	$= \lim_{x \to 0} \left[ \left( \frac{a^{x} - 1}{x} \right) + \left( \frac{b^{x} - 1}{x} \right) - \left( \frac{c^{x} - 1}{x} \right) - \left( \frac{d^{x} - 1}{x} \right) \right]$	-)]	
	$= \lim_{x \to 0} \left( \frac{a^{x} - 1}{x} \right) + \lim_{x \to 0} \left( \frac{b^{x} - 1}{x} \right) - \lim_{x \to 0} \left( \frac{c^{x} - 1}{x} \right) -$	$-\lim_{x\to 0}\left(\frac{d^{x-1}}{x}\right)$	
	$= \log a + \log h - \log c - \log d$	$\left\{\lim_{x\to 0} \left(\frac{a^x-1}{x}\right) = \log a\right\}$	
	$= (\log a + \log b) - (\log c + \log d)$		
	$= \log(ab) - \log(cd)$	$\{\log A + \log B = \log(AB)\}\$	
	$=\log\left(\frac{ab}{cd}\right)$ ans.	$\left\{\log A - \log B = \log\left(\frac{A}{B}\right)\right\}$	
Q.8)	Evaluate $\lim_{x \to 0} \left( \frac{\log(5+x) - \log(5-x)}{x} \right)$		
Sol.8)	We have $\lim_{x \to 0} \left( \frac{\log(5+x) - \log(5-x)}{x} \right)$		
	$= \lim_{x \to 0} \left( \frac{\log\left(5\left(1 + \frac{x}{5}\right)\right) - \log\left(5\left(1 - \frac{x}{5}\right)\right)}{x} \right)$		
	$= \lim_{x \to 0} \left[ \frac{\{\log 5 + \log(1 + \frac{x}{5})\} - \{\log 5 + \log(1 - \frac{x}{5})\}}{x} \right]$	$\{\log A + \log B = \log(AB)\}\$	
	$= \lim_{x \to 0} \frac{\log\left(1 + \frac{x}{5}\right) - \log\left(1 - \frac{x}{5}\right)}{x}$		

Copyright © www.studiestoday.com All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission.

StudiesToday.com			
	$= \lim_{x \to 0} \frac{\log(1 + \frac{x}{5})}{\frac{x}{5} \times 5} - \lim_{x \to 0} \frac{\log(1 + \frac{-x}{5})}{x}$		
	$= \lim_{x \to 0} \frac{\log(1 + \frac{x}{5})}{\frac{x}{5} \times 5} + \lim_{x \to 0} \frac{\log(1 + \frac{-x}{5})}{\frac{-x}{5} \times 5}$		
	$= \frac{1}{5} + \frac{1}{5} \qquad \left\{ \lim_{x \to 0} \left( \frac{\log(1+x)}{x} \right) = 1 \right\}$		
	$=\frac{2}{5}$ ans.		
Q.9)	Evaluate $\lim_{x \to 0} \left( \frac{e^{3+x} - \sin x - e^3}{x} \right)$		
Sol.9)	We have $\lim_{x \to 0} \left( \frac{e^{3+x} - \sin x - e^3}{x} \right)$		
	$=\lim_{x\to 0}\left(\frac{e^{3+x}-e^3-\sin x}{x}\right)$		
	$= \lim_{x \to 0} \left( \frac{e^3 - (e^x - 1) - \sin x}{x} \right)$ = $\lim_{x \to 0} \left( \frac{e^3 (e^x - 1)}{x} - \frac{\sin x}{x} \right)$		
	$= \lim_{x \to 0} \left( \frac{e^{3}(e^{x}-1)}{x} - \frac{\sin x}{x} \right)$		
	$= e^{3} \lim_{x \to 0} \left[ \frac{e^{x} - 1}{x} \right] - \lim_{x \to 0} \left( \frac{\sin x}{x} \right)$		
	$= e^{3} - (1)$ = $e^{3} - 1$ ans.		
Q.10)	$= e^{x} - 1 \text{ ans.}$ Evaluate $\lim_{x \to 0} \left( \frac{9^{x} - 6^{x} - 6^{x} + 4}{x} \right)$		
Sol.10)	We have $\lim_{x \to 0} \left( \frac{9^x - 6^x - 6^x + 4}{x} \right)$		
	$= \lim_{x \to 0} \left( \frac{9^x - 6^x - 6^x + 4}{x^2} \right)$		
	$= \lim_{x \to 0} \left( \frac{3^{x}(3^{x} - 2^{x}) - 2^{x}(3^{x} - 2^{x})}{x^{2}} \right)$		
	$= \lim_{x \to 0} \left( \frac{(3^{x} - 2^{x}) \cdot (3^{x} - 2^{x})}{x^{2}} \right)$		
	$= \lim_{x \to 0} \left( \frac{(3^x - 2^x)^2}{x^2} \right)$		
	$= \lim_{x \to 0} \left[ \left( \frac{3^x - 2^x}{x} \right)^2 \right]$		
	$= \lim_{x \to 0} \left[ \left[ \frac{(3^{x} - 1) - (2^{x} - 1)}{x} \right]^{2} \right]$		
	$= \left\{ \lim_{x \to 0} \left[ \frac{3^{x} - 1}{x} \right] - \lim_{x \to 0} \left[ \frac{2^{x} - 1}{x} \right] \right\}^{2}$		
	$= (\log 3 - \log 2)^2 \qquad \left\{ \lim_{x \to 0} \left( \frac{a^x - 1}{x} \right) = \log a \right\}$		

Copyright © www.studiestoday.com All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission.



 $\{\log A + \log B = \log(AB)\}\$ 

www.studiestoday.com

Copyright © www.studiestoday.com All rights reserved. No part of this publication may be reproduced, distributed, or transmitted in any form or by any means, including photocopying, recording, or other electronic or mechanical methods, without the prior written permission.