

**LIMITS & DERIVATIVES****Class XI**

Q.1)	Evaluate: $\lim_{x \rightarrow 0} \left(\frac{\cosec x - \cot x}{x} \right)$
Sol.1)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{\cosec x - \cot x}{x} \right)$</p> $= \lim_{x \rightarrow 0} \left(\frac{\frac{1}{\sin x} - \frac{\cos x}{\sin x}}{x} \right)$ $= \lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x \sin x} \right)$ $= \lim_{x \rightarrow 0} \left(\frac{2 \sin^2 \left(\frac{x}{2} \right)}{x \sin x} \right)$ $= \lim_{x \rightarrow 0} \left(\frac{\frac{2 \sin^2 \left(\frac{x}{2} \right)}{x^2} \times \frac{x^2}{4}}{\frac{x \sin x}{x} \times x} \right)$ $= \frac{2}{4} \lim_{x \rightarrow 0} \left(\frac{\frac{\sin^2 \left(\frac{x}{2} \right)}{x^2}}{\frac{1}{4}} \right)$ $= \frac{1}{2} \times \frac{(1)}{(1)} \quad \left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$ $= \frac{1}{2} \text{ ans.}$
Q.2)	<p>TYPE: 6 TRIGO LIMITS</p> <p>When $\lim_{x \rightarrow a} f(x)$</p> <p>Evaluate: $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{1 + \cos(2x)}{(\pi - 2x)^2} \right)$</p> <p>We have : $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{1 + \cos(2x)}{(\pi - 2x)^2} \right)$</p> <p>Put $x = \frac{\pi}{2} + h$ and $h \rightarrow 0$</p> $\therefore \lim_{h \rightarrow 0} \left(\frac{1 + \cos \left(2 \left(\frac{\pi}{2} + h \right) \right)}{\left(\pi - 2 \left(\frac{\pi}{2} + h \right) \right)^2} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{1 + \cos(\pi + 2h)}{(\pi - \pi - 2h)^2} \right)$



	$ \begin{aligned} &= \lim_{h \rightarrow 0} \left(\frac{1 - \cos(2h)}{4h^2} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{2 \sin^2 h}{4h^2} \right) \\ &= \frac{1}{2} \lim_{h \rightarrow 0} \left(\frac{\sin^2 h}{h^2} \right) \\ &= \frac{1}{2} (1)^2 \\ &= \frac{1}{2} \text{ ans.} \end{aligned} $
Q.3)	Evaluate: $\lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{2 - \sqrt{3} \cos x - \sin x}{(6x - \pi)^2} \right)$
Sol.3)	<p>We have $\lim_{x \rightarrow \frac{\pi}{6}} \left(\frac{2 - \sqrt{3} \cos x - \sin x}{(6x - \pi)^2} \right)$</p> <p>Put $x = \frac{\pi}{6} + h$ and $h \rightarrow 0$</p> $ \begin{aligned} &= \lim_{h \rightarrow 0} \left(\frac{2 - \sqrt{3} \cos \left(\frac{\pi}{6} + h \right) - \sin \left(\frac{\pi}{6} + h \right)}{\left(6 \left(\frac{\pi}{6} + h \right) - \pi \right)^2} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{2 - \sqrt{3} \left(\cos \frac{\pi}{6} \cos h - \sin \frac{\pi}{6} \sin h \right) - \left(\sin \frac{\pi}{6} \sin h + \cos \frac{\pi}{6} \cos h \right)}{(\pi + 6h - \pi)^2} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{2 - \sqrt{3} \left(\frac{\sqrt{3}}{2} \cos h - \frac{1}{2} \sin h \right) - \left(\frac{1}{2} \cos h + \frac{\sqrt{3}}{2} \sin h \right)}{36h^2} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{2 - \frac{3}{2} \cos h + \frac{\sqrt{3}}{2} \sin h - \frac{1}{2} \cos h - \frac{\sqrt{3}}{2} \sin h}{36h^2} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{2 - 2 \cos h}{36h^2} \right) \\ &= \lim_{h \rightarrow 0} \left(\frac{2(1 - \cos h)}{36h^2} \right) \\ &= \frac{1}{18} \lim_{h \rightarrow 0} \left(\frac{2 \sin^2 \left(\frac{h}{2} \right)}{\frac{h^2}{4} \times 4} \right) \\ &= \frac{1}{36} \lim_{h \rightarrow 0} \left(\frac{\sin^2 \left(\frac{h}{2} \right)}{\frac{h^2}{4}} \right) \end{aligned} $ <p style="text-align: right;">$\left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$</p>



	$= \frac{1}{36} \times (1)^2 = \frac{1}{36}$ ans.
Q.4)	Evaluate: $\lim_{x \rightarrow \pi} \left(\frac{\sin(3x) - 3 \sin x}{(\pi - x)^3} \right)$
Sol.4)	<p>We have $\lim_{x \rightarrow \pi} \left(\frac{\sin(3x) - 3 \sin x}{(\pi - x)^3} \right)$</p> $= \lim_{x \rightarrow \pi} \left(\frac{3 \sin x - 4 \sin^3 x - 3 \sin x}{(\pi - x)^3} \right)$ $= \lim_{x \rightarrow \pi} \left(\frac{-4 \sin^3 x}{(\pi - x)^3} \right)$ <p>Put $x = \pi + h$ and $h \rightarrow 0$</p> $= \lim_{h \rightarrow 0} \left(\frac{-4 \sin^3(\pi + h) - 3 \sin(\pi + h)}{(\pi - \pi + h)^3} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{+4 \sin^3 h}{-h^3} \right) \quad \{ \sin(\pi + \theta) = -\sin \theta \}$ $= -4 \lim_{h \rightarrow 0} \left(\frac{\sin^3 h}{h^3} \right)$ $= -4(1)^3 = -4$ ans. $\left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$
Q.5)	Evaluate: $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\cot x - \cos x}{(\pi - 2x)^3} \right)$
Sol.5)	<p>We have $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\cot x - \cos x}{(\pi - 2x)^3} \right)$</p> <p>Put $x = \frac{\pi}{2} + h$ and $h \rightarrow 0$</p> $= \lim_{h \rightarrow 0} \left(\frac{\cot \left(\frac{\pi}{2} + h \right) - \cos \left(\frac{\pi}{2} + h \right)}{\left(\pi - 2 \left(\frac{\pi}{2} + h \right) \right)^3} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{-\tan h + \sin h}{(\pi - \pi + 2h)^3} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{-\tan h + \sin h}{-8h^3} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{\tan h - \sin h}{8h^3} \right)$ $= \frac{1}{8} \lim_{h \rightarrow 0} \left(\frac{\frac{\sin h}{\cos h} - \sin h}{h^3} \right)$ $= \frac{1}{8} \lim_{h \rightarrow 0} \left(\frac{\sin h - \sin h \cdot \cos h}{h^3 \cdot \cos h} \right)$ $= \frac{1}{8} \lim_{h \rightarrow 0} \left(\frac{\sin h (1 - \cos h)}{h^3 \cdot \cos h} \right)$



	$ \begin{aligned} &= \frac{1}{8} \lim_{h \rightarrow 0} \left(\frac{\sin h \cdot 2 \sin^2 \left(\frac{h}{2} \right)}{h^3 \cdot \cos h} \right) \\ &= \frac{1}{8} \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \cdot \frac{2 \sin^2 \left(\frac{h}{2} \right)}{h^2} \cdot \frac{1}{\cos h} \right) \\ &= \frac{1}{8} \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \cdot \frac{2 \sin^2 \left(\frac{h}{2} \right)}{\frac{h^2}{4} \times 4} \cdot \frac{1}{\cos h} \right) \\ &= \frac{1}{8} \lim_{h \rightarrow 0} \left(\frac{\sin h}{h} \right) \times \frac{2}{4} \cdot \lim_{h \rightarrow 0} \left(\frac{\sin^2 \left(\frac{h}{2} \right)}{\frac{h^2}{4}} \right) \times \lim_{h \rightarrow 0} \left(\frac{1}{\cos h} \right) \\ &= \frac{1}{8} \times 1 \times \frac{1}{2} \times 1^2 \times 1 \quad \left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\} \\ &= \frac{1}{16} \text{ ans.} \end{aligned} $
Q.6)	Evaluate: $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x} \right)$
Sol.6)	<p>We have $\lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x} \right)$</p> <p>Rationalize</p> $ \begin{aligned} &= \lim_{x \rightarrow \frac{\pi}{2}} \left[\frac{(\sqrt{2} - \sqrt{1 + \sin x})(\sqrt{2} + \sqrt{1 + \sin x})}{\cos^2 x (\sqrt{2} + \sqrt{1 + \sin x})} \right] \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \left[\frac{2 - 1 - \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1 + \sin x})} \right] \\ &= \lim_{x \rightarrow \frac{\pi}{2}} \left[\frac{1 - \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1 + \sin x})} \right] \end{aligned} $ <p>Put $x = \frac{\pi}{2} + h$ and $h \rightarrow 0$</p> $ \begin{aligned} &= \lim_{h \rightarrow 0} \left[\frac{1 - \sin \left(\frac{\pi}{2} + h \right)}{\cos^2 \left(\frac{\pi}{2} + h \right)} \right] \times \lim_{x \rightarrow \frac{\pi}{2}} \left[\frac{1}{\sqrt{2} + \sqrt{1 + \sin x}} \right] \\ &= \lim_{h \rightarrow 0} \left(\frac{1 - \cos h}{\sin^2 h} \right) \times \frac{1}{(\sqrt{2} + \sqrt{2})} \\ &= \lim_{h \rightarrow 0} \left(\frac{2 \sin^2 \left(\frac{h}{2} \right)}{\sin^2 h} \right) \times \frac{1}{2\sqrt{2}} \end{aligned} $

	$= \lim_{h \rightarrow 0} \left(\frac{\frac{2 \sin^2(\frac{h}{2})}{\frac{h^2}{4}} \times \frac{h^2}{4}}{\frac{\sin^2 h}{h^2} \times h^2} \right) \times \frac{1}{2\sqrt{2}}$ $= \frac{\frac{2}{4} \lim_{h \rightarrow 0} \left(\frac{\sin^2(\frac{h}{2})}{\frac{h^2}{4}} \right)}{\lim_{h \rightarrow 0} \left(\frac{\sin^2 h}{h^2} \right)} \times \frac{1}{2\sqrt{2}} \quad \left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$ $= \frac{1}{2} \times \frac{1}{1} \times \frac{1}{2\sqrt{2}} = \frac{1}{4\sqrt{2}} \text{ ans.}$
Q.7)	Evaluate: $\lim_{x \rightarrow 0} \left(\frac{1-\cos(2x)}{\cos(2x)-\cos(8x)} \right)$
Sol.7)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{1-\cos(2x)}{\cos(2x)-\cos(8x)} \right)$</p> $= \lim_{x \rightarrow 0} \left[\frac{2 \sin^2 x}{-2 \sin(5x) \cdot \sin(-3x)} \right]$ $= \lim_{x \rightarrow 0} \left[\frac{\frac{\sin^2 x}{x^2} \times x^2}{15 \times \frac{\sin(5x)}{5x} \times \frac{\sin(3x)}{3x}} \right]$ $= \frac{\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2}}{15 \times \lim_{x \rightarrow 0} \frac{\sin(5x)}{5x} \times \lim_{x \rightarrow 0} \frac{\sin(3x)}{3x}}$ $= \frac{1^2}{15(1)(1)} = \frac{1}{15} \text{ ans.} \quad \left\{ \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) = 1 \right\}$
Q.8)	Evaluate: $\lim_{x \rightarrow a} \left(\frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} \right)$
Sol.8)	<p>We have $\lim_{x \rightarrow a} \left(\frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}} \right)$</p> <p>Rationalize</p> $= \lim_{x \rightarrow a} \left(\frac{(\sin x - \sin a)(\sqrt{x} - \sqrt{a})}{x - a} \right)$ <p>Put $x = a + h$ and $h \rightarrow 0$</p> $= \lim_{h \rightarrow 0} \left(\frac{(\sin(a+h) - \sin a)(\sqrt{a+h} - \sqrt{a})}{a+h-a} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{2 \cos\left(\frac{2a+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right) \cdot (\sqrt{a+h} - \sqrt{a})}{h} \right)$ $= \lim_{h \rightarrow 0} \left(\frac{2 \cos\left(\frac{2a+h}{2}\right) \cdot \sin\left(\frac{h}{2}\right) \cdot (\sqrt{a+h} - \sqrt{a})}{\frac{h}{2} \times 2} \right)$



	$ \begin{aligned} &= \lim_{h \rightarrow 0} \left(\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \right) \times \lim_{h \rightarrow 0} \left(\cos\left(\frac{2a+h}{2}\right) \cdot (\sqrt{a+h} - \sqrt{a}) \right) \\ &= 1 \times \cos(a)(\sqrt{a} + \sqrt{a}) \\ &= 2\sqrt{a} \cos a \text{ ans.} \end{aligned} $
Q.9)	Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x - b^x}{x} \right)$
Sol.9)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{a^x - b^x}{x} \right)$</p> $ \begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{a^x - b^{x-1+1}}{x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x} - \frac{b^{x-1}}{x} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x} \right) - \lim_{x \rightarrow 0} \left(\frac{b^{x-1}}{x} \right) \\ &= \log a - \log b \\ &= \log \left(\frac{a}{b} \right) \text{ ans.} \end{aligned} $ <p style="text-align: right;">$\left\{ \lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x} \right) = \log a \right\}$</p>
Q.10)	Evaluate $\lim_{x \rightarrow 0} \left(\frac{2^x - 1}{\sqrt{1+x-1}} \right)$
Sol.10)	<p>We have $\lim_{x \rightarrow 0} \left(\frac{2^x - 1}{\sqrt{1+x-1}} \right)$</p> <p>Rationalize</p> $ \begin{aligned} &= \lim_{x \rightarrow 0} \left(\frac{2^x - 1}{\sqrt{1+x-1}} \times \frac{(\sqrt{1+x}+1)}{(\sqrt{1+x}+1)} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{(2^x - 1)(\sqrt{1+x}+1)}{1+x-1} \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{(2^x - 1)}{1+x-1} \right) \times \lim_{x \rightarrow 0} (\sqrt{1+x} + 1) \\ &= \log 2 \times (1 + 1) \\ &= 2 \log 2 \text{ ans.} \end{aligned} $ <p style="text-align: right;">$\left\{ \lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x} \right) = \log a \right\}$</p>