

<b>LIMITS &amp; DERIVATIVES</b>	
<b>Class XI</b>	
Q.1)	Evaluate: $\lim_{x \rightarrow 0} \left( \frac{5x+4 \sin(3x)}{4 \sin(2x)+7x} \right)$
Sol.1)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{5x+4 \sin(3x)}{4 \sin(2x)+7x} \right)</math></p> $  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{5x + \frac{4 \sin(3x)}{3x} \times 3x}{\frac{4 \sin 2x}{2x} \times 2x + 7x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{x \left( 5 + \frac{4 \sin(3x)}{3x} \times 3 \right)}{x \left( \frac{4 \sin 2x}{2x} \times 2x + 7 \right)} \right) \\  &= \frac{5+12 \lim_{x \rightarrow 0} \left( \frac{\sin(3x)}{3x} \right)}{8 \lim_{x \rightarrow 0} \left( \frac{\sin(2x)}{2x} \right) + 7} \\  &= \frac{5+12(1)}{8(1)+7} \\  &= \frac{17}{15} \text{ ans.} \quad \left\{ \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) = 1 \right\}  \end{aligned}  $
Q.2)	Evaluate: $\lim_{x \rightarrow 0} \left( \frac{1-\cos mx}{1-\cos nx} \right)$
Sol.2)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{1-\cos mx}{1-\cos nx} \right)</math></p> $  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{2 \sin^2 \frac{mx}{2}}{2 \sin^2 \frac{nx}{2}} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\frac{\sin^2 \frac{mx}{2}}{\frac{m^2 x^2}{4}} \times \frac{m^2 x^2}{4}}{\frac{\sin^2 \frac{nx}{2}}{\frac{n^2 x^2}{4}} \times \frac{n^2 x^2}{4}} \right) \\  &= \frac{m^2 \times \lim_{x \rightarrow 0} \left( \frac{\sin^2 \frac{mx}{2}}{\frac{m^2 x^2}{4}} \right)}{n^2 \times \left( \frac{\sin^2 \frac{nx}{2}}{\frac{n^2 x^2}{4}} \right)} \\  &= \frac{m^2(1)^2}{n^2(1)^2} \\  &= \frac{m^2}{n^2} \text{ ans.} \quad \left\{ \lim_{x \rightarrow 0} \left( \frac{\sin^2 x}{x^2} \right) = 1^2 \right\}  \end{aligned}  $
Q.3)	Evaluate: $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$
Sol.3)	We have $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$

	$  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{1}{\sin x} - \frac{\cos x}{\sin x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{1-\cos x}{\sin x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{2 \sin^2 \left( \frac{x}{2} \right)}{2 \sin \frac{x}{2} \cos \frac{x}{2}} \right) \\  &= \lim_{x \rightarrow 0} \left( \tan \frac{x}{2} \right) \\  &= \tan(0) = 0 \text{ ans.}  \end{aligned}  $
Q.4)	Evaluate: $\lim_{x \rightarrow 0} \left( \frac{x^3 \cot x}{1-\cos x} \right)$
Sol.4)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{x^3 \cot x}{1-\cos x} \right)</math></p> $  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( x^3 \cdot \frac{1}{\tan x} \cdot \frac{1}{1-\cos x} \right) \\  &= \lim_{x \rightarrow 0} \left( x^3 \cdot \frac{1}{\tan x} \cdot \frac{1}{2 \sin^2 \left( \frac{x}{2} \right)} \right) \\  &= \lim_{x \rightarrow 0} \left( x^3 \cdot \frac{1}{x} \cdot \frac{1}{2 \sin^2 \left( \frac{x}{2} \right)} \cdot \frac{x^2}{x^2} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{x^3}{x^3} \cdot \frac{1}{\frac{\tan x}{x}} \cdot \frac{1}{1/2} \cdot \frac{1}{\frac{2 \sin^2 \left( \frac{x}{2} \right)}{x^2}} \right) \\  &= \frac{1}{\lim_{x \rightarrow 0} \left( \frac{\tan x}{x} \right)} \cdot \frac{1}{1/2} \cdot \frac{1}{\lim_{x \rightarrow 0} \left( \frac{\sin^2 \left( \frac{x}{2} \right)}{x^2} \right)} \\  &= \frac{1}{1} \times \frac{1}{1/2} \times \frac{1}{1^2} \\  &= 2 \text{ ans.}  \end{aligned}  $ <p style="text-align: right;"><math>\left\{ \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1 \right\}</math></p>
Q.5)	Evaluate: $\lim_{x \rightarrow 0} \left( \frac{\sin(2x)+\sin(6x)}{\sin(5x)-\sin(3x)} \right)$
Sol.5)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{\sin(2x)+\sin(6x)}{\sin(5x)-\sin(3x)} \right)</math></p> $  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{\frac{\sin(2x)}{2x} \times 2x + \frac{\sin(6x)}{6x} \times 6x}{\frac{\sin(5x)}{5x} \times 5x - \frac{\sin(3x)}{3x} \times 3x} \right) \\  &= \lim_{x \rightarrow 0} \left[ \frac{x \left( \frac{\sin(2x)}{2x} \times 2 + \frac{\sin(6x)}{6x} \times 6 \right)}{x \left( \frac{\sin(5x)}{5x} \times 5 - \frac{\sin(3x)}{3x} \times 3 \right)} \right]  \end{aligned}  $

	$  \begin{aligned}  &= \frac{2 \times \lim_{x \rightarrow 0} \left( \frac{\sin(2x)}{2x} \right) + 6 \times \lim_{x \rightarrow 0} \left( \frac{\sin(6x)}{6x} \right)}{5 \times 2 \times \lim_{x \rightarrow 0} \left( \frac{\sin(5x)}{5x} \right) - 3 \times \lim_{x \rightarrow 0} \left( \frac{\sin(3x)}{3x} \right)} \\  &= \frac{2(1) + 6(1)}{5(1) - 3(3)} \\  &= \frac{8}{2} = 4 \text{ ans.}  \end{aligned}  $
Q.6)	Evaluate: $\lim_{x \rightarrow 0} \left( \frac{\tan x - \sin x}{x^3} \right)$
Sol.6)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{\tan x - \sin x}{x^3} \right)</math></p> $  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{\frac{\sin x}{\cos x} - \sin x}{x^3} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\sin x - \sin x \cdot \cos x}{x^3 \cdot \cos x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\sin x(1 - \cos x)}{x^3 \cdot \cos x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\sin x \cdot 2 \sin^2 \left( \frac{x}{2} \right)}{x^3 \cdot \cos x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \cdot \frac{2 \sin^2 \left( \frac{x}{2} \right)}{x^2} \cdot \frac{1}{\cos x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \cdot \frac{2 \sin^2 \left( \frac{x}{2} \right)}{\frac{x^2}{4} \times 4} \cdot \frac{1}{\cos x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) \times \frac{1}{2} \lim_{x \rightarrow 0} \left( \frac{\sin^2 \left( \frac{x}{2} \right)}{\frac{x^2}{4}} \right) \lim_{x \rightarrow 0} \left( \frac{1}{\cos x} \right) \\  &= 1 \times \frac{1}{2} \times (1)^2 \times \frac{1}{1} \\  &= \frac{1}{2} \text{ ans.}  \end{aligned}  $
Q.7)	Evaluate: $\lim_{x \rightarrow 0} \left( \frac{\tan x - \sin x}{\sin^3 x} \right)$
Sol.7)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{\tan x - \sin x}{\sin^3 x} \right)</math></p> $  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\sin x - \sin x \cdot \cos x}{\sin^3 x \cdot \cos x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\sin x(1 - \cos x)}{\sin^3 x \cdot \cos x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\sin x \cdot 2 \sin^2 \left( \frac{x}{2} \right)}{\sin^3 x \cdot \cos x} \right)  \end{aligned}  $

	$  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{2 \sin^2\left(\frac{x}{2}\right)}{\sin^2 x \cdot \cos x} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\frac{2 \sin^2\left(\frac{x}{2}\right)}{\frac{x^2}{4}} \times \frac{x^2}{4}}{\frac{\sin^2 x}{x^2} \times x^2 \cdot \cos x} \right) \\  &= \frac{\frac{1}{2} \lim_{x \rightarrow 0} \left( \frac{\sin^2\left(\frac{x}{2}\right)}{\frac{x^2}{4}} \right)}{\lim_{x \rightarrow 0} \left( \frac{\sin^2 x}{x^2} \right) \lim_{x \rightarrow 0} (\cos x)} \\  &= \frac{\frac{1}{2}(1)^2}{(1)(1)} \quad \left\{ \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) = 1 \right\} \\  &= \frac{1}{2} \text{ ans.}  \end{aligned}  $
Q.8)	Evaluate: $\lim_{x \rightarrow 0} \left( \frac{\sec(4x) - \sec(2x)}{\sec(3x) - \sec x} \right)$
Sol.8)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{\sec(4x) - \sec(2x)}{\sec(3x) - \sec x} \right)</math></p> $  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{\frac{1}{\cos(4x)} - \frac{1}{\cos 2x}}{\frac{1}{\cos 3x} - \frac{1}{x}} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\frac{\cos(2x) - \cos(4x)}{\cos(4x)\cos(2x)}}{\frac{\cos x - \cos(3x)}{\cos(3x)\cos x}} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\cos(2x) - \cos(4x)}{\cos x - \cos(3x)} \cdot \frac{\cos x \cdot \cos(3x)}{\cos(4x) \cdot \cos(2x)} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{-2 \sin(3x) \cdot \sin(-x)}{-2 \sin(2x) \cdot \sin(-x)} \cdot \frac{\cos(3x) \cdot \cos x}{\cos(4x) \cdot \cos(2x)} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{\frac{\sin(3x)}{3x} \times 3x}{\frac{\sin(2x)}{2x} \times 2x} \cdot \frac{\cos(3x) \cdot \cos x}{\cos(4x) \cdot \cos(2x)} \right) \\  &= \frac{3 \lim_{x \rightarrow 0} \left( \frac{\sin(3x)}{3x} \right)}{2 \lim_{x \rightarrow 0} \left( \frac{\sin(2x)}{2x} \right)} \cdot \lim_{x \rightarrow 0} \left( \frac{\cos(3x) \cdot \cos x}{\cos(4x) \cdot \cos(2x)} \right) \\  &= \frac{3 \times 1}{2 \times 1} \cdot \left( \frac{(1)(1)}{(1)(1)} \right) \quad \left\{ \begin{array}{l} \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) = 1 \\ \text{and} \\ \cos \theta = 1 \end{array} \right\} \\  &= \frac{3}{2} \text{ ans.}  \end{aligned}  $
Q.9)	Evaluate: $\lim_{y \rightarrow 0} \left( \frac{(x+y) \sec(x+y) - x \sec x}{y} \right)$
Sol.9)	<p>We have <math>\lim_{y \rightarrow 0} \left( \frac{(x+y) \sec(x+y) - x \sec x}{y} \right)</math></p> <p>Here <math>y</math> is the variable not <math>x</math></p>

	$  \begin{aligned}  &= \lim_{y \rightarrow 0} \left( \frac{x \sec(x+y) + y \sec(x+y) - x \sec x}{y} \right) \\  &= \lim_{y \rightarrow 0} \left( \frac{x\{\sec(x+y)-\sec x\} + y \sec(x+y)}{y} \right) \\  &= \lim_{y \rightarrow 0} \left( \frac{x\left\{\frac{1}{\cos(x+y)} - \frac{1}{\cos x}\right\}}{y} + \frac{y \sec(x+y)}{y} \right) \quad \left\{ \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \right\} \\  &= \lim_{y \rightarrow 0} \left( \frac{x\{\cos(x) - \cos(x+y)\}}{y \cdot \cos(x+y) \cdot \cos x} + \sec(x+y) \right) \\  &= \lim_{y \rightarrow 0} \left( \frac{x\{-2 \sin\left(\frac{2x+y}{2}\right) - \sin\left(\frac{-y}{2}\right)\}}{2 \times \frac{y}{2} \cdot \cos(x+y) \cdot \cos x} + \sec(x+y) \right) \\  &= \lim_{y \rightarrow 0} \left( \frac{\sin\frac{y}{2}}{\frac{y}{2}} \right) \cdot \lim_{y \rightarrow 0} \left( \frac{x \cdot \sin\left(\frac{2x+y}{2}\right)}{\cos(x+y) \cdot \cos x} \right) + \lim_{y \rightarrow 0} \sec(x+y) \\  &= 1 \times x \frac{\sin x}{\cos x \cdot \cos x} + \sec x \quad \left\{ \lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right) = 1 \right\} \\  &= x \tan x \cdot \sec x + \sec x \text{ ans.}  \end{aligned}  $
Q.10)	Evaluate: $\lim_{x \rightarrow 0} \left( \frac{1-\cos x \sqrt{\cos 2x}}{x^2} \right)$
Sol.10)	<p>We have <math>\lim_{x \rightarrow 0} \left( \frac{1-\cos x \sqrt{\cos 2x}}{x^2} \right)</math></p> <p>Rationalize</p> $  \begin{aligned}  &= \lim_{x \rightarrow 0} \left( \frac{1-\cos x \sqrt{\cos 2x}(1+\cos x \sqrt{\cos 2x})}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{1-\cos^2 x (\cos 2x)}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{1-\cos^2 x (2 \cos^2 x - 1)}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{1-2 \cos^4 x + \cos^2 x}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\  &= -\lim_{x \rightarrow 0} \left( \frac{2 \cos^4 x - \cos^2 x - 1}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\  &= -\lim_{x \rightarrow 0} \left( \frac{2 \cos^4 x - 2 \cos^2 x + \cos^2 x - 1}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\  &= -\lim_{x \rightarrow 0} \left( \frac{2 \cos^4 x - (\cos^2 x - 1) + (\cos^2 x - 1)}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\  &= -\lim_{x \rightarrow 0} \left( \frac{(2 \cos^2 x + 1)(\cos^2 x - 1)}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{(2 \cos^2 x + 1)(1-\cos^2 x)}{x^2(1+\cos x \sqrt{\cos 2x})} \right) \\  &= \lim_{x \rightarrow 0} \left( \frac{(2 \cos^2 x + 1) \sin^2 x}{x^2(1+\cos x \sqrt{\cos 2x})} \right)  \end{aligned}  $

$$\begin{aligned}&= \lim_{x \rightarrow 0} \left( \frac{\sin^2 x}{x^2} \right) \times \lim_{x \rightarrow 0} \left( \frac{(2 \cos^2 x + 1)}{1 + \cos x \sqrt{\cos 2x}} \right) \\&= (1)^2 \times \left( \frac{2(1) + 1}{1 + (1)(1)} \right) \\&= \frac{3}{2} \text{ ans.}\end{aligned}$$