LIMITS & DERIVATIVES

Class XI

Q.1) Evaluate: $\lim_{x\to 3}\left[\frac{x^3-7x^2+15x-9}{x^4-5x^3+27x-27}\right]$

$(x-3)$ is the factor of both polynomials.

Sol.1) We have $\lim_{x\to 3}\left[\frac{x^3-7x^2+15x-9}{x^4-5x^3+27x-27}\right]$

$(x-3)$ is the factor of both polynomials

$$\begin{array}{r|l} & x^2-4x+3 \\ \hline x-3 & x^3-7x^2+15x-9 \\ & -(x^3-3x^2) \\ \hline & -4x^2+15x-9 \\ & -(-4x^2+12x) \\ \hline & 3x-9 \\ & 3x-9 \\ \hline & \text{x} \\ \hline \end{array}$$

$$\begin{array}{r|l} & x^3-2x^2-6x+9 \\ \hline x-3 & x^4-5x^3+27x-27 \\ & -(x^4-3x^3) \\ \hline & -2x^3+27x-27 \\ & -(-2x^3+6x^2) \\ \hline & -6x^2+27x-27 \\ & -(-6x^2+18x) \\ \hline & 9x-27 \\ & 9x-27 \\ \hline & \text{x} \\ \hline \end{array}$$

$=\lim_{x\to 3}\left[\frac{(x-3)(x^2-4x+3)}{(x-3)(x^3-2x^2-6x+9)}\right]$

$=\lim_{x\to 3}\left[\frac{(x-1)(x-3)}{x^3-2x^2-6x+9}\right]$

Again $(x-3)$ is factor of D

$$=\lim_{x\to 3}\left[\frac{(x-1)(x-3)}{(x-3)(x^2+x-3)}\right]$$

$$=\lim_{x\to 3}\left[\frac{(x-1)}{x^2+x-3}\right]$$

$=\frac{3-1}{9+3-3}=\frac{2}{9}$ ans.

Q.2) Evaluate: $\lim_{x\to\sqrt{2}}\left[\frac{x^4-4}{x^2+3\sqrt{2}x-8}\right]$

Sol.2) We have $\lim_{x\to\sqrt{2}}\left[\frac{x^4-4}{x^2+3\sqrt{2}x-8}\right]$

$=\lim_{x\to\sqrt{2}}\left[\frac{(x^2)^2-(2)^2}{x^2+4\sqrt{2}x-\sqrt{2}x-8}\right]$

	$= \lim_{x\to\sqrt{2}}\left[\frac{(x^2+2)(x^2-2)}{x(x+4\sqrt{2})-\sqrt{2}(x+4\sqrt{2})}\right]$ $= \lim_{x\to\sqrt{2}}\left[\frac{(x^2+2)(x+\sqrt{2})(x-\sqrt{2})}{(x+4\sqrt{2})(x-\sqrt{2})}\right]$ $= \frac{\left((\sqrt{2})^2+2\right)(\sqrt{2}+\sqrt{2})}{(\sqrt{2}+4\sqrt{2})}$ $= \frac{(4)(2\sqrt{2})}{(5\sqrt{2})}$ $= \frac{8}{5}$ ans.
	TYPE: 3 RATIONALIZE **When rationalize:** $(\sqrt{\ } - \sqrt{\ });\ (\sqrt{\ } - \boldsymbol{function});\ (\boldsymbol{fnction} - \sqrt{\ })$
Q.3)	Evaluate: $\lim_{x\to 4}\left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right]$ Rationalize both N & D simultaniusly
Sol.3)	We have $\lim_{x\to 4}\left[\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right]$ $= \lim_{x\to 4}\left[\frac{(3-\sqrt{5+x})(1+\sqrt{5-x})(3+\sqrt{5+x})}{(1-\sqrt{5-x})(1+\sqrt{5-x})(3+\sqrt{5+x})}\right]$ $= \lim_{x\to 4}\left[\frac{(9-5-x)(1+\sqrt{5-x})}{(1-\sqrt{5+x})(3+\sqrt{5+x})}\right]$ $= \lim_{x\to 4}\left[\frac{(4-x)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}\right]$ $= \lim_{x\to 4}\left[\frac{-(x-4)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}\right]$ $= \frac{-(1+\sqrt{5-4})}{(3+\sqrt{5+x})}$ $= \frac{-(1+1)}{3+3} = -\frac{2}{6}$ $= -\frac{1}{3}$ ans.
Q.4)	Evaluate: $\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$
Sol.4)	We have, $\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$ Rationalize both N & D $= \lim_{x\to a}\left[\frac{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{3a+x}+2\sqrt{x})(\sqrt{a+2x}+\sqrt{3x})}{(\sqrt{3a+x}-2\sqrt{x})(\sqrt{3a+x}+2\sqrt{x})(\sqrt{a+2x}+\sqrt{3x})}\right]$ $= \lim_{x\to a}\left[\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}\right]$

	$= \lim_{x\to a}\left[\frac{(a-x)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{(3a+3x)\left(\sqrt{a+2x}+\sqrt{3x}\right)}\right]$ $= \lim_{x\to a}\left[\frac{(a-x)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{3(a-x)\left(\sqrt{a+2x}+\sqrt{3x}\right)}\right]$ $= \frac{\left(\sqrt{3a+a}+2\sqrt{a}\right)}{3\left(\sqrt{a+2a}+\sqrt{3a}\right)}$ $= \frac{2\sqrt{a}+2\sqrt{a}}{3\left(\sqrt{3a}+\sqrt{3a}\right)}$ $= \frac{4\sqrt{a}}{3\left(2\sqrt{3a}\right)}$ $= \frac{4\sqrt{a}}{6\sqrt{3}\sqrt{a}} = \frac{2}{3\sqrt{3}}$ ans.
Q.5)	Evaluate: $\lim_{x\to 1}\left[\frac{(2x-3)-(\sqrt{x}-1)}{2x^2+x-3}\right]$
Sol.5)	We have, $\lim_{x\to 1}\left[\frac{(2x-3)-(\sqrt{x}-1)}{2x^2+x-3}\right]$ Rationalize both N & D $= \lim_{x\to 1}\left[\frac{(2x-3)\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}{(2x^2+3x-2x-3)\left(\sqrt{x}+1\right)}\right]$ $= \lim_{x\to 1}\left[\frac{(2x-3)(x-1)}{(2x+3)(x-1)\left(\sqrt{x}+1\right)}\right]$ $= \lim_{x\to 1}\left[\frac{(2x-3)}{(2x+3)\left(\sqrt{x}+1\right)}\right]$ $= \frac{(2x-3)}{(2x+3)(1+1)}$ $= \frac{-1}{(5)(2)}$ $= \frac{-1}{10}$ ans.
	TYPE: 4 $\boldsymbol{\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = na^{n-1}}$
Q.6)	Evaluate: $\lim_{x\to 2}\left(\frac{x^{10}-1024}{x^5-32}\right)$
Sol.6)	We have $\lim_{x\to 2}\left(\frac{x^{10}-1024}{x^5-32}\right)$ $= \lim_{x\to 2}\left[\frac{x^{10}-2^{10}}{x^5-2^5}\right]$

	Divide N & D by $(x-2)$ $= \lim_{x\to 2}\left[\dfrac{\frac{x^{10}-2^{10}}{x-2}}{\frac{x^5-2^5}{x-2}}\right]$ $= \dfrac{\lim_{x\to 2}\left[\frac{x^{10}-2^{10}}{x-2}\right]}{\lim_{x\to 2}\left[\frac{x^5-2^5}{x-2}\right]}$ $= \frac{10(2)^{10-1}}{5(2)^{5-1}}$ $\quad\quad \left\{\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = na^{n-1}\right\}$ $= 2(2)^5$ $= 2\times 32 = 64$ ans.
Q.7)	Evaluate: $\lim_{x\to a}\left(\frac{(x+2)^{5/3}-(a+2)^{5/3}}{x-a}\right)$
Sol.7)	We have $\lim_{x\to a}\left(\frac{(x+2)^{5/3}-(a+2)^{5/3}}{x-a}\right)$ Substitution : put $x+2=y$ Limits change: when $x\to a$ then $y\to a+2$ $\therefore \lim_{y\to a+2}\left[\frac{y^{5/3}-(a+2)^{5/3}}{y-2-a}\right]$ $= \lim_{y\to a+2}\left[\frac{y^{5/3}-(a+2)^{5/3}}{y-(a+2)}\right]$ $= \frac{5}{3}(a+2)^{5/3-1}$ $\quad\quad \left\{\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = na^{n-1}\right\}$ $= \frac{5}{3}(a+2)^{2/3}$ ans.
Q.8)	Evaluate: $\lim_{x\to 0}\left(\frac{(1+x)^6-1}{(1+x)^2-1}\right)$
Sol.8)	We have $\lim_{x\to 0}\left(\frac{(1+x)^6-1}{(1+x)^2-1}\right)$ put $1+x=y$, when $x\to 0$ then $y\to 1$ $\therefore \lim_{y\to 1}\left[\frac{y^6-1}{y^2-1}\right]$ Divide N & D by $(y-1)$ $= \lim_{y\to 1}\left[\dfrac{\frac{y^6-1}{y-1}}{\frac{y^2-1}{y-1}}\right]$

	$= \dfrac{\lim_{y\to 1}\left[\dfrac{y^6-1}{y-1}\right]}{\lim_{y\to 1}\left[\dfrac{y^2-1}{y-1}\right]}$ $= \frac{6(1)^5}{5(1)^1}$ $= \frac{6}{2} = 3$ ans.
Q.9)	Evaluate : $\lim_{x\to 1}\left\{\frac{(x+x^2+x^3 \ldots\ldots x^n)-n}{x-1}\right\}$
Sol.9)	We have $\lim_{x\to 1}\left\{\frac{(x+x^2+x^3 \ldots\ldots x^n)-n}{x-1}\right\}$ $= \lim_{x\to 1}\left\{\frac{(x+x^2+x^3 \ldots\ldots x^n)-(1+1+1\ldots n\ terms)}{x-1}\right\}$ $= \lim_{x\to 1}\left\{\frac{(x-1)+(x^2-1)+(x^3-1)\ldots\ldots(x^n-1)}{x-1}\right\}$ $= \lim_{x\to 1}\left[\frac{x-1}{x-1}\right] + \lim_{x\to 1}\left(\frac{x^2-1^2}{x-1}\right) + \lim_{x\to 1}\left(\frac{x^3-1^3}{x-1}\right) + \ldots\ldots \lim_{x\to 1}\left(\frac{x^n-1^n}{x-1}\right)$ $= 1 + 2(1)^1 + 3(1)^2 + \ldots\ldots n(1)^{n-1}$ $= 1 + 2 + 3 + \ldots\ldots n$ $= \frac{n(n+1)}{2}$ ans.
Q.10)	If $\lim_{x\to -a}\left(\frac{x^9+a^9}{x+a}\right) = 9$. Find the value of a.
Sol.10)	We have $\lim_{x\to -a}\left(\frac{x^9-(-a^9)}{x-(-a)}\right) = 9$ $\Rightarrow 9(-a)^{9-1} = 9$ $\quad\quad$ $\left\{\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right) = na^{n-1}\right\}$ $\Rightarrow 9(-9)^8 = 9$ $\Rightarrow (-a)^8 = 1$ $\Rightarrow a^8 = 1$ $\Rightarrow a = \pm 1$ ans.