StudiesToday.com

	LIMITS & DERIVATIVES ***Class XI***
	Limits:-
Q.1)	If $f(x) = \begin{cases} 5x-4\,; 0<x<1 \\ 4x^3-3x\,; 1<x<2 \end{cases}$ Evaluate $\lim_{x\to a} f(x)$
Sol.1)	For L.H.L. : $f(x) = 5x-4$ For R.H.L : $f(x) = 4x^3-3x$ L.H.L. $= \lim_{x\to 1^-}(5x-4)$ Put $x = 1-h$ & $h \to 0$ $\therefore$ L.H.L. $= \lim_{h\to 0}(5(1-h)-4)$ $\Rightarrow$ L.H.L $= 5-4 = 1$ $\therefore$ L.H.L.$= 1$ Now, R.H.L.$= \lim_{x\to 1^+}(4x^3-3x)$ Put $x = 1+h$ & $h \to 0$ $\therefore$ R.H.L. $= \lim_{h\to 0}[4(1+h)^3-3(1+h)]$ $\Rightarrow$ R.H.L $= 4(1)^3-3(1) = 1$ $\therefore$ R.H.L.$= 1$ Since, L.H.L. = R.H.L. = 1 $\therefore$ $\lim_{x\to 1} f(x)$ Exists & $\lim_{x\to 1} f(x) = 1$ ans.
Q.2)	If $f(x) = \begin{cases} \frac{x-\|x\|}{x}\,; x \neq 0 \\ 2\,; x = 0 \end{cases}$ Show that $\lim_{x\to 0} f(x)$ does not exists.
Sol.2)	Here, for L.H.L. & R.H.L. $f(x) = \begin{cases} \frac{x-\|x\|}{x} \end{cases}$ L.H.L. $= \lim_{x\to 0^-}\left[\frac{x-\|x\|}{x}\right]$ Put $x = 0-h = -h$ & $h \to 0$

	$\therefore$ L.H.L. $= \lim\limits_{h\to 0}\left(\frac{-h-\|-h\|}{-h}\right)$ $\Rightarrow$ L.H.L. $= \lim\limits_{h\to 0}\left(\frac{-h-h}{-h}\right) = \lim\limits_{h\to 0}\left(\frac{-2h}{-h}\right)$ $\Rightarrow \lim\limits_{h\to 0}(2)$ $\therefore$ L.H.L.$= 2$ Now, R.H.L. $= \lim\limits_{x\to 0^+}\left[\frac{x-\|x\|}{x}\right]$ Put $x = 0 + h = h$ & $h \to 0$ $\therefore$ R.H.L. $= \lim\limits_{h\to 0}\left(\frac{h-\|-h\|}{-h}\right) = \lim\limits_{h\to 0}\left(\frac{h-h}{h}\right)$ $\Rightarrow$ R.H.L. $= \lim\limits_{h\to 0}\left(\frac{0}{h}\right) = \lim\limits_{h\to 0}(0) = 0$ $\therefore$ R.H.L.$= 0$ Clearly L.H.L. $\neq$ R.H.L. $\therefore \lim\limits_{x\to 0} f(x)$ does not exists ans.
Q.3)	$f(x) = \begin{cases} \frac{4x-5}{x-4} ; x \le 2 \\ x - \pi ; x > 2 \end{cases}$ Find value of π if $\lim\limits_{x\to 2} f(x)$ exists.
Sol.3)	For L.H.L. $f(x) = 4x - 5$ For R.H.L. $f(x) = x - \pi$ L.H.L. $= \lim\limits_{x\to 2^+}(4x-5)$ Put $x = 2 - h$ & $h \to 0$ $\Rightarrow$ L.H.L. $= \lim\limits_{h\to 0}(4(2-h)-5) = \lim\limits_{h\to 0}(8-5)$ $\Rightarrow$ L.H.L. $= \lim\limits_{h\to 0}(3)$ $\Rightarrow$ L.H.L. $= 3$ Now, R.H.L. $= \lim\limits_{x\to 2^+}(x-\pi)$ Put $x = 2 + h$ and $h \to 0$ $\Rightarrow$ R.H.L. $= \lim\limits_{h\to 0}(2+h-\pi) = \lim\limits_{h\to 0}(2-\pi)$ $\therefore$ R.H.L.$= 2 - \pi$ Since, L.H.L. = R.H.L. $= 1$ $\therefore \lim\limits_{x\to 2} f(x)$ exists (given)

	$\therefore$ L.H.L. = R.H.L. $\Rightarrow\ 3-2\pi \Rightarrow \pi = -1$ ans.
Q.4)	Show that $\lim\limits_{x\to 0}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ does not exists.
Sol.4)	For L.H.L. & R.H.: $f(x) = \left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ L.H.L. $= \lim\limits_{x\to 0^-}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ Put $x = 0-h = -h$ & $h \to 0$ $\therefore$ L.H.L. $= \lim\limits_{h\to 0}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ Put directly $h = 0$ $\Rightarrow$ L.H.L $= \frac{e^{-\infty}-1}{e^{-\infty}+1} = \frac{0-1}{0+1}$ $\qquad e^{-\infty} = 0$ $\Rightarrow$ L.H.L.$= -1$ Now, R.H.L. $= \lim\limits_{x\to 0^+}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ Put $x = 0+h = h$ & $h \to 0$ $\therefore$ R.H.L. $= \lim\limits_{h\to 0}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ (don't put directly $h = 0$) $\qquad \frac{\infty}{\infty}$ form $\Rightarrow$ R.H.L. $= \lim\limits_{h\to 0}\left(\frac{1-\frac{1}{e^{1/h}}}{1+\frac{1}{e^{1/h}}}\right)$ $\qquad$ divide by $e^{1/h}$ $\Rightarrow$R.H.L. $= \lim\limits_{h\to 0}\left(\frac{1-e^{-1/h}}{1+e^{-1/h}}\right)$ Put $h = 0$ $\Rightarrow$ R.H.L. $= \lim\limits_{h\to 0}\left(\frac{1-e^{-\infty}}{1+e^{-\infty}}\right) = \frac{1-0}{1+0}$ $\quad e^{-\infty} = 0$ $\Rightarrow$ R.H.L. $= 1$ Since, L.H.L. $\neq$ R.H.L. $= 1$ $\therefore\ \lim\limits_{x\to 0} f(x)$ does not Exists ans.
Q.5)	$f(x) = \begin{cases} a+bx\,; x<1 \\ 4\,; x=1 \\ b-ax\,; x>1 \end{cases}$ and if $\lim\limits_{x\to 1} f(x) = f(1)$, what are possible values of a & b ?

<table>
<tr><td>Sol.5)</td><td>For L.H.L. $f(x) = a + bx$
For R.H.L. $f(x) = b - ax$
and $f(1) = 4$ (when $x = 1; f(x) = 4$)
given, $\lim_{x \to 1} f(x) = f(1)$
$\Rightarrow \lim_{x \to 1} f(x) = 4$
$\Rightarrow$ L.H.L. = R.H.L. $= 4$
$\Rightarrow \lim_{x \to 1^-}(a + bx) = \lim_{x \to 1^+}(b - ax) = 4$
Put $x = 1 - h$ Put $x = 1 + h$
$\& \; h \to 0$ $\& \; h \to 0$
$\Rightarrow \lim_{h \to 0}\left(a + b(1-h)\right) = \lim_{h \to 0}\left(b - a(1+h)\right) = 4$
$\Rightarrow a + b = b - a = 4$
$\Rightarrow a + b = 4 \; \& \; b - a = 4$
Solving we get $a = 0 \; \& \; b = 4$ ans.</td></tr>
<tr><td>Q.6)</td><td>$f(x) = \begin{cases} mx^2 + n; x < 0 \\ nx + m\,; 0 \le x \le 1 \\ nx^3 + m\,; x > 1 \end{cases}$
For what integers m and n does the $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$ exists.</td></tr>
<tr><td>Sol.6)</td><td>Given that $\lim_{x \to 0} f(x)$ exists
For L.H.L. $f(x) = nx + m$
L.H.L=R.H.L.
$\Rightarrow \lim_{x \to 0^-}(mx^2 + n) = \lim_{x \to 0^+}(nx + m)$
Put $x = 0 - h = -h$ Put $x = 0 + h = h$
$\& \; h \to 0$ $\& \; h \to 0$
$\therefore \lim_{h \to 0}(m(-h)^2 + n) = \lim_{h \to 0}(n(h) + m)$
$\Rightarrow 0 + n = 0 + m$
$\Rightarrow = m = n$ .... (i)
Given that $\lim_{x \to 0} f(x)$ exists
For L.H.L. $f(x) = nx + m$
For R.H.L. $f(x) = nx^3 + m$
L.H.L. = R.H.L.</td></tr>
</table>

	$\Rightarrow \lim_{x\to 1^-}(nx+m) = \lim_{x\to 1^+}(nx^3+m)$ Put $x = 1-h$ Put $x = 1+h$ $\& h \to 0$ $\& h \to 0$ $\Rightarrow \lim_{h\to 0}(n(1-h)+m) = \lim_{h\to +}(n(1+h)^3+m)$ (put directly $h=0$) $\Rightarrow n+m = n+m$ ....(ii) From (i) & (ii) m and n can be any integers such that $m=n$ ans.
Q.7)	$f(x) = \begin{cases} \lvert x\rvert + 1\,; x<0 \\ 0\,; x=0 \\ \lvert x\rvert - 1\,; x>0 \end{cases}$ For what value(s) of a does the $\lim_{x\to 0} f(x)$ exists.
Sol.7)	For L.H.L. $f(x) = \lvert x\rvert + 1$ For R.H.L. $f(x) = \lvert x\rvert - 1$ L.H.L. $= \lim_{x\to 0^-}(\lvert x\rvert + 1)$ Put $x = 0-h = -h \,\&\, h\to 0$ $\therefore$ L.H.L. $= \lim_{h\to 0}(\lvert -h\rvert + 1) = \lim_{h\to 0}(h+1) = 0+1$ $\Rightarrow$ L.H.L. $=1$ Now, R.H.L. $= \lim_{x\to 0^+}(\lvert x\rvert - 1)$ Put $x = 0+h = h \,\&\, h\to 0$ $\therefore$ R.H.L. $\lim_{h\to 0}(\lvert -h\rvert - 1) = \lim_{h\to 0}(h-1) = 0-1$ $\Rightarrow$ R.H.L. $=-1$ Since L.H.L $\neq$ R.H.L. $\therefore \lim_{x\to 0} f(x)$ Does not exist. .... (i) But we are given, $\lim_{x\to a} f(x)$ exists. .... (ii) From (i) & (ii) We conclude that a can be any real no. except $a=0$ $\therefore a \in R - \{0\}$ ans.
Q.8)	$f(x)\begin{cases} 2x+3\,; x\le 0 \\ 3(x+1); x>0 \end{cases}$

	Evaluate $\lim\limits_{x\to 1} f(x)$
Sol.8)	For L.H.L. $f(x) = 3(x+1)$ Also for R.H.L. $f(x) = 3(x+1)$ L.H.L. $= \lim\limits_{x\to 1^-} 3(x+1)$ Put $x = 1-h$ & $h \to 0$ $\therefore$ L.H.L. $= \lim\limits_{h\to 0}(3(1-h+1)) = 3(2) = 6$ $\Rightarrow$ L.H.L. $= 6$ Now, R.H.L. $= \lim\limits_{x\to 1^+}(3(x+1))$ Put $x = 1+h = h$ & $h \to 0$ $\therefore$ R.H.L. $\lim\limits_{h\to 0}(3(1+h+1) = 3(2) = 6$ $\Rightarrow$ R.H.L. $= 6$ Since L.H.L = R.H.L. $\therefore \lim\limits_{x\to 0} f(x)$ Exist and $\lim\limits_{x\to 1} f(x) = 6$ ans.
Q.9)	a_1, a_2, a_3..........a_n are any real numbers $f(x) = (x-a_1)(x-a_2)(x-a_3)$........$(x-a_n)$. What is $\lim\limits_{x\to a} f(x)$? Also compute $\lim\limits_{x\to a} f(x)$.
Sol.9)	$\lim\limits_{x\to a_1} f(x) = \lim\limits_{x\to a_1} [(x-a_1)(x-a_2)(x-a_3) \dots \dots \dots . (x-a_n)]$ $= (a_1-a_1)(a_1-a_2)(a_1-a_3) \dots \dots \dots . (a_1-a_n)$ $= 0(a_1-a_2)(a_1-a_3)$..........$a_1 - a_n$ $= 0$ ans. $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a}[(x-a_1)(x-a_2) \dots \dots \dots . (x-a_n)]$ $= (a-a_1)(a-a_2)$.........$(a-a_n)$ ans.
	TYPE: 2 FACTORIZE **Formula: $a^2-b^2, a^3-b^3, a^4-b^4$, quadratic equation, cubic (hit & trial) L.C.M.**
Q.10)	Evaluate : $\lim\limits_{x\to 1}\left[\frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x}\right]$
Sol.10)	We have $\lim\limits_{x\to 1}\left[\frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x}\right]$ $= \lim\limits_{x\to 1}\left[\frac{x-2}{x(x-1)} - \frac{1}{x(x^2+3x+2)}\right]$ $= \lim\limits_{x\to 1}\left[\frac{x-2}{x(x-1)} - \frac{1}{x(x-1)(x-2)}\right]$

	$= \lim_{x \to 1} \left[\frac{(x-2)^2-1}{x(x-1)(x-2)}\right]$ $= \lim_{x \to 1} \left[\frac{x^2-4x+4-1}{x(x-1)(x-2)}\right]$ $= \lim_{x \to 1} \left[\frac{(x-3)(x-1)}{x(x-1)(x-2)}\right]$ $= \lim_{x \to 1} \left[\frac{(x-3)}{x(x-2)}\right]$ $= \frac{(1-3)}{(1)(1-2)} = \frac{-2}{-1} = 2$ ans.