

BAL BHARATI PUBLIC SCHOOL
Ganga Ram Hospital Marg, New Delhi-60

CLASS –XI
ASSIGNMENT- 8

SUBJECT – MATHEMATICS
TOPIC–LIMITS AND DERIVATIVES

Q1. Evaluate the following :-

(i) $\lim_{x \rightarrow 1} \frac{2}{1-x^2} + \frac{1}{x-1}$

(ii) $\lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{x^4 - 8x^2 + 16}$

(iii) $\lim_{x \rightarrow \sqrt{2}} \frac{x^4 - 4}{x^2 + 3x\sqrt{2} - 8}$

(iv) $\lim_{x \rightarrow 3} (x^2 - 9) \left(\frac{1}{x+3} + \frac{1}{x-3} \right)$

(v) $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$

(vi) $\lim_{x \rightarrow 0} \frac{\sqrt{a^2 + x^2} - \sqrt{a^2 - x^2}}{x^2}$

(vii) $\lim_{x \rightarrow 0} \frac{1 - \cos 2mx}{1 - \cos 2nx}$

(viii) $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$

(ix) $\lim_{x \rightarrow 0} \frac{\sin 2x + \sin 3x}{2x + \sin 3x}$

(x) $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$

(xi) $\lim_{x \rightarrow 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$

(xii) $\lim_{x \rightarrow a} \frac{(x+2)^{5/3} - (a+2)^{5/3}}{x-a}$

(xiii) $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^2 - 4}$

(xiv) $\lim_{x \rightarrow 1} \frac{x^{16} - 1}{x^5 - 1}$

(xv) $\lim_{x \rightarrow 0} \frac{e^{5x} - 1}{x}$

(xvi) $\lim_{x \rightarrow 0} \frac{e^{5x} - 1}{e^{8x} - 1}$

(xvii) $\lim_{x \rightarrow 0} \frac{\log(1+5x)}{x}$

(xviii) $\lim_{x \rightarrow 0} \frac{\log(1+5x)}{\log(1+10x)}$

(xix) $\lim_{x \rightarrow 3} \frac{e^x - 3}{x - 3}$

(xx) $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{\sin^2 x}$

Q2. If $\lim_{x \rightarrow a} \frac{x^3 - a^3}{x - a} = \lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1}$, then find the value of a.

Q3. If $\lim_{x \rightarrow 2} \frac{x^n - a^n}{x - 2} = 80$ and if n is a +ve integer, find n.

Q4. If $f(x) = \begin{cases} 5x - 4 & \text{if } x \leq 1 \\ 4x^3 - 3x & \text{if } x > 1 \end{cases}$ Find (i) $\lim_{x \rightarrow 1} f(x)$ (ii) $\lim_{x \rightarrow 4} f(x)$ (iii) $\lim_{x \rightarrow 3} f(x)$ (iv) $\lim_{x \rightarrow 2} f(x)$

Q5. If $f(x) = \begin{cases} 4 & \text{if } x \geq 3 \\ Kx + 1 & \text{if } x < 3 \end{cases}$ and $\lim_{x \rightarrow 3} f(x)$ exists, find K.

Q6. If $f(x) = \begin{cases} \cos x & \text{if } x \geq 0 \\ x + k & \text{if } x < 0 \end{cases}$ and $\lim_{x \rightarrow 0} f(x)$ exists, find K.

Q7. If $f(x) = \begin{cases} (x - 1)K & \text{if } x \geq 3 \\ x + 1 & \text{if } x < 3 \end{cases}$ and $\lim_{x \rightarrow 3} f(x)$ exists, then find the value of K.

Q8. Find the derivative of the following functions from first principle / delta Method / ab-initio method.

(i) $\frac{\sin x}{x}$ (ii) \sqrt{x} (iii) $\sin \sqrt{x+a}$ (iv) $\sqrt{\cos x}$

(v) $\tan(2x+1)$ (vi) $\frac{x+1}{x-1}$ (vii) $\sin(ax+b)$ (ix) $(ax+b)^2$

Q9. If $y = x^5$ show that $\frac{xdy}{dx} 5y = 0$

Q10. If $y = \left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$

Q11. Differentiate w.r. to x

(i) $\frac{2x+3}{x^2-5}$ (ii) $\frac{1+\tan x}{1-\tan x}$ (iii) $\frac{\sec x + \tan x}{\sec x - \tan x}$

(iv) $\frac{x+\cos x}{\tan x}$ (v) $x^4 - 2\sin x + 3 \cos x$ (vi) $\left(x + \frac{1}{x}\right) \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$

(viii) $(x + \cos x)(x - \tan x)$ (vii) $\frac{x}{1+x}$ (ix) $\frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}$

(x) $\frac{x^2(1+\cos x)}{\sin x}$ (xi) $(x^2 - 3x + 2)(x + 2)$

Q12. If $y = \frac{x}{x+4}$, prove that $\frac{xdy}{dx} = y(1-y)$

Q13. If $Y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ Show that $\frac{dy}{dx} = y$

Q14. If $y = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$, find $\frac{dy}{dx}$

Q15. Differentiate $\frac{4x^2 - 1}{(5-2x)^3}$ and then find its value at $x = 2$.

Q16. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + C = 0$, find $\frac{dy}{dx}$

Q17. Find the derivative of $f(x) = (1-x)(1+x)(1+x^2)(1+x^4)(1+x^8)$ & hence find $f'(1)$