CHAPTER - 11

CONIC SECTIONS

KEY POINTS

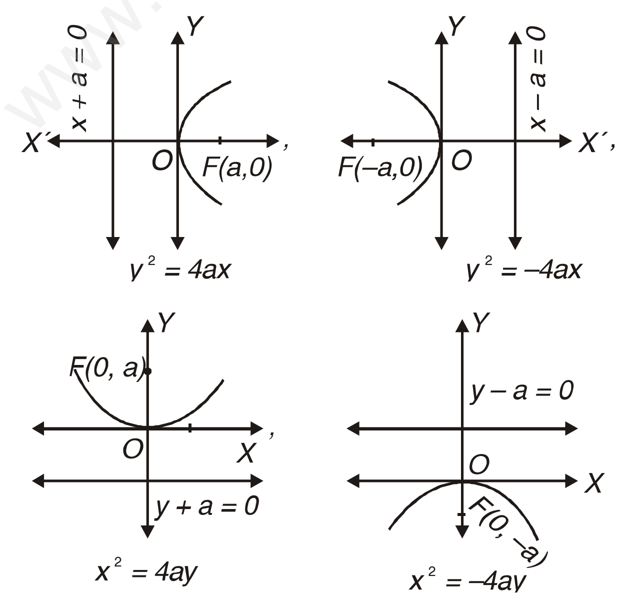
- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and cone in different positions

- **Circle :** It is the set of all points in a plane that are equidistant from a fixed point in that plane

- Equation of circle : $(x - h)^2 + (y - k)^2 = r^2$

  Centre (h, k), radius = r

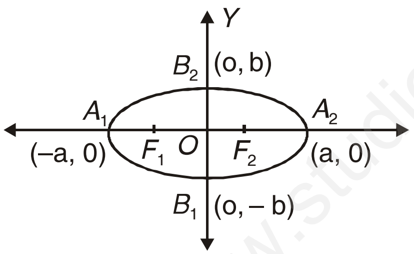
- **Parabola :** It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in the plane. Fixed point does not lie on the line.

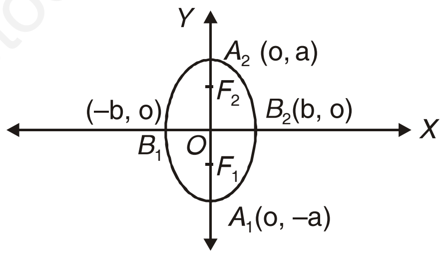


Main facts about the Parabola

Equation	$y^2 = 4ax$ (a > 0) Right hand	$y^2 = -4ax$ a > 0 Left hand	$x^2 = 4ay$ a > 0 Upwards	$x^2 = -4ay$ a > 0 Downwards
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
Focus	(a, 0)	(–a, 0)	(0, a)	(0, –a)
Length of latus–rectum	4a	4a	4a	4a
Equation of latus–rectum	$x - a = 0$	$x + a = 0$	$y - a = 0$	$y + a = 0$

- **Latus Rectum :** A chord through focus perpendicular to axis of parabola is called its latus rectum.
- **Ellipse :** It is the set of points in a plane the sum of whose distances from two fixed points in the plane is a constant and is always greater than the distances between the fixed points





$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad\qquad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$a > b > 0, \; a > b > 0$

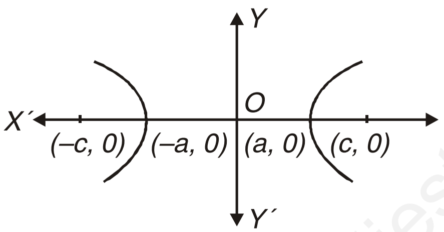
$$c = \sqrt{a^2 - b^2}$$

Main facts about the ellipse

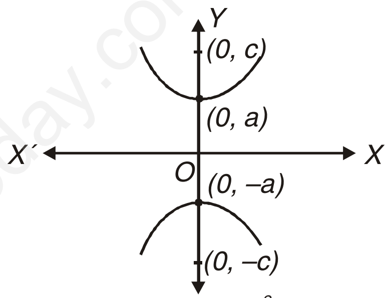
Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > 0, b > 0	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ a > 0, b > 0
Centre	(0,0)	(0,0)
Major axis lies along	x–axis	y–axis
Length of major axis	2a	2a
Length of minor axis	2b	2b

Foci	(–c, 0), (c, 0)	(0, –c),(0, c)
Vertices	(–a, 0), (a, 0)	(0, –a), (0, a)
Eccentricity e	$\frac{c}{a}$	$\frac{c}{a}$
Length of latus–rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

- **Latus rectum :** Chord through foci perpendicular to major axis called latus rectum.
- **Hyperbola :** It is the set of all points in a plane, the differences of whose distance from two fixed points in the plane is a constant.



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$c = \sqrt{a^2 + b^2}$$

Main facts about the Hyperbola

Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $a > 0, b > 0$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ $a > 0, b > 0$
Centre	(0,0)	(0,0)
Transverse axis lies along	x–axis	y–axis
Length of transverse axis	2a	2a
Length of conjugate axis	2b	2b
Foci	(–c, 0), (c, 0)	(0, –c),(0, c)
Vertices	(–a, 0), (a, 0)	(0, –a), (0, a)
Eecentricity e	$\frac{c}{a}$	$\frac{c}{a}$
Length of latus–rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

- **Latus Rectum :** Chord through foci perpendicular to transverse axis is called latus rectum.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find the centre and radius of the circle

$$3x^2 + 3y^2 + 6x - 4y - 1 = 0$$

2. Does $2x^2 + 2y^2 + 3y + 10 = 0$ represent the equation of a circle? Justify.

3. Find equation of circle whose end points of one of its diameter are (–2, 3) and (0, –1).

4. Find the value(s) of p so that the equation $x^2 + y^2 - 2px + 4y - 12 = 0$ may represent a circle of radius 5 units.

5. If parabola $y^2 = px$ passes through point (2, –3), find the length of latus rectum.

6. Find the coordinates of focus, and length of latus rectum of parabola $3y^2 = 8x$.

7. Find the eccentricity of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

8. One end of diameter of a circle $x^2 + y^2 - 6x + 5y - 7 = 0$ is (7, –8). Find the coordinates of other end.

9. Find the equation of the ellipse coordinates of whose foci are (±2, 0) and length of latus rectum is $\frac{10}{3}$.

10. Find the equation of ellipse with eccentricity $\frac{3}{4}$, centre at origin, foci on y-axis and passing through point (6, 4).

11. Find the equation of hyperbola with centre at origin, transverse axis along x-axis, eccentricity $\sqrt{5}$ and sum of lengths of whose axes is 18.

12. Two diameters of a circle are along the lines $x - y - 9 = 0$ and $x - 2y - 7 = 0$ and area of circle is 154 square units, find its equation.

13. Find equation(s) of circle passing through points (1,1), (2,2) and whose radius is 1 unit.

14. Find equation of circle concentric with circle $4x^2 + 4y^2 - 12x - 16y - 21 = 0$ and of half its area.

15. Find the equation of a circle whose centre is at (4, –2) and $3x - 4y + 5 = 0$ is tangent to circle.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

16. Show that the four points (7,5), (6, –2) (–1,–1) and (0,6) are concyclic. [Concylic points : Four or more points which lie on a circle].

ANSWERS

1. $\left(-1, \frac{2}{3}\right), \quad \frac{4}{3}$

2. No

3. $x^2 + y^2 + 2x - 2y - 3 = 0$ or $(x + 1)^2 + (y - 1)^2 = 5$

4. –3, +3

5. $\frac{9}{2}$

6. $\left(\frac{2}{3}, 0\right), \frac{8}{3}$

7. $\frac{4}{5}$

8. (–1, 3)

9. $\frac{x^2}{9} + \frac{y^2}{5} = 1$

10. $16x^2 + 7y^2 = 688$

11. $4x^2 - y^2 = 36$

12. $x^2 + y^2 - 22x - 4y + 76 = 0$

    [**Hint :** Point of intersection of two diameters is the centre]

13. $x^2 + y^2 - 2x - 4y + 4 = 0$, $x^2 + y^2 - 4x - 2y + 4 = 0$

14. $2x^2 + 2y^2 - 6x + 8y + 1 = 0$

15. $x^2 + y^2 - 8x + 4y - 5 = 0$