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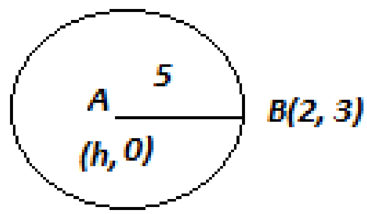
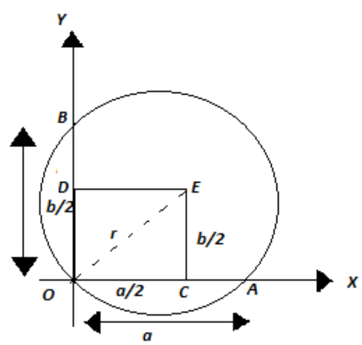
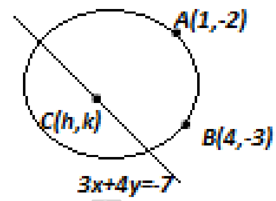
	Class 11 Conic Sections
Q.1)	Find the equation of a circle of radius 5 whose centre lies on $X - axis$ and passes through the point $(2,3)$.
Sol.1)	Let the entre of circle is $A(h, 0)$ Clearly, $AB = radius$ $\Rightarrow AB = 5$ $\Rightarrow \sqrt{(h-2)^2 + 9} = 5$ $\Rightarrow (h-2)^2 + 9 = 25$ $\Rightarrow h^2 - 4h + 4 + 9 = 25$ $\Rightarrow h^2 - 4h - 12 = 0$ $\Rightarrow (h-6)(h+2) = 0$ $\Rightarrow h = 6$ and $h = -2$ $\therefore$ coordinates of centre are $(6, 0)$ and $(-2, 0)$ $\therefore$ equations of the required circle are $(x-6)^2 + (y-0)^2 = 25$ $\Rightarrow x^2 + +y^2 - 12x + 36 = 25$ $\Rightarrow x^2 + y^2 - 12x + 11 = 0$ ans. 
Q.2)	Find the equation of the circle which passes through the origin & cuts off intercepts $'a'$ and $'b'$ on the coordinate axis.
Sol.2)	$OA = a$ and $OB = b$ (given) $\Rightarrow OC = \frac{a}{2}$ and $OD = \frac{b}{2}$ $\therefore h = \frac{a}{2}$ and $k = \frac{b}{2}$ $\therefore$ centre of circle is $\left(\frac{a}{2}, \frac{b}{2}\right)$ Now, $OE = r(radius)$ $\Rightarrow \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} = r$ .... (By Pythagoras) $\Rightarrow r^2 = \frac{a^2}{4} + \frac{b^2}{4}$ Now, equation of circle is given by $\Rightarrow (x-h)^2 + (y-k)^2 = r^2$ $\Rightarrow \left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = r^2$ $\Rightarrow x^2 + \frac{a^2}{4} - ax + y^2 + \frac{b^2}{4} - by = \frac{a^2}{4} + \frac{b^2}{4}$ $\Rightarrow x^2 + y^2 - ax - by = 0$ is the required equation of circle. ans. 
Q.3)	Find the equation of the circle which passes through the points $(1, -2)$ and $(4, -3)$ and has its centre on the line $3x + 4y = 7$.
Sol.3)	Let the equation of circle is $(x-h)^2 + (y-k)^2 = r^2$ $A(1, -2)$ lies on it $\therefore (1-h)^2 - (-2-k)^2 = r^2$ $\Rightarrow 1 + h^2 - 2h + 4 + k^2 + 4k = r^2$ $\Rightarrow h^2 + k^2 - 2h + 4k + 5 = r^2$ .... (i) Now, $B(4, -3)$ lies on circle

$\therefore (4-h)^2 + (-3-k)^2 = r^2$
$\Rightarrow h^2 + 16 - 8h + 9 + k^2 + 6k = r^2$
$\Rightarrow h^2 + k^2 - 8h + 6k + 25 - r^2$ .... (ii)
Also $C(h,k)$ lies on line $3x + 4y = 7$
$\Rightarrow 3h + 4k = 7$ .... (iii)
Equation (i) and (ii)
$h^2 + k^2 - 2h + 4k + 5 = h^2 + k^2 - 8h + 6k + 25$
$\Rightarrow 6h - 2k = 20$
Or $3h - k = 10$ .... (iv)



Now solving (iii) & (iv)
We get, $h = \frac{47}{15}$ and $k = \frac{-3}{5}$
$\therefore$ centre is $\left(\frac{47}{15}, \frac{-3}{5}\right)$
Put in equation (i)
We get $r^2 = \frac{1465}{225}$
$\therefore$ equation of circle is
$\left(x - \frac{47}{15}\right)^2 + \left(y + \frac{3}{k}\right)^2$ ans.

Q.4) Find the equation of the circle which passes through the point $(3,7)$, $(5,5)$ and has its centre on the line $x - 4y = 1$.

Sol.4) Let us consider the equation of circle in general form
$x^2 + y^2 + 2gx + 2fy + c = 0$.......(1)
Where $(-g, -f)$ are the center of circle.
So put $(x, y) = (-g, -f)$ in equation of line we will get $4f - g = 1$.
Then put(3,7) & (5,5) in (1) equation ......
We will get
$58 + 6g + 14f + c = 0$ .....(a)
$50 + 10g + 10f + c = 0$ .....(b)
Apply (a)- (b)
We get, $4f - 4g = -8$ & already we have $4f - g = 1$
On solving both we will get $g = 3$, $f = 1$ & $c = -90$
Hence equation of circle is
$x^2 + y^2 + 6x + 2y - 90 = 0$ ans.

Q.5) Find the centre and radius of the following equation
i) $x^2 + y^2 - 4x + 6y = 12$ ii) $2x^2 + 2y^2 - 6y = 2$

Sol.5) i) $x^2 + y^2 - 4x + 6y = 12$
(we have to control in to standard form)
$x^2 - 4x + y^2 + 6y = 12$
$\Rightarrow (x-2)^2 - 4 + (y+3)^2 - 9 = 12$
$\Rightarrow (x-2)^2 + (y+3)^2 = 25$
Compare with $(x-h)^2 + (y-k)^2 = r^2$
We have, $h =; k = -3$ and $r = 5$
$\therefore$ centre $(2, -3)$ and radius = 5
ii) $2x^2 + 2y^2 - 6y = 2$
$\Rightarrow x^2 + y^2 - 3y = 1$

	$x^2 + y^2 - 3y = 1$ $\Rightarrow x^2 + \left(y - \frac{3}{2}\right)^2 - \frac{9}{4} = 1$ $\Rightarrow x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{13}{4}$ Here centre is $\left(0, \frac{3}{2}\right)$ and $radius = \frac{\sqrt{13}}{2}$ ans.
Q.6)	Find the equation of the circle which passes through the points $(5, -8), (2, -9)$ and $(2,1)$.
Sol.6)	Let the equation of circle is $(x-h)^2 + (y-k)^2 = r^2$ $(5, -8)$ lies on circle $\therefore (5-5)^2 + (-8-k)^2 = r^2$ $\Rightarrow h^2 + 25 - 10h + 64 + k^2 + 16k = r^2$ $\Rightarrow h^2 + k^2 - 10h + 16k + 89 = r^2$ ……… (i) $(2, -9)$ lies on circle $\therefore (2-h)^2 + (-9-k)^2 = r^2$ $\Rightarrow 4 + h^2 - 4h + 81 + k^2 + 18k$ $\Rightarrow h^2 + k^2 - 4h + 18k + 85 = r^2$ ……….. (ii) $(2,1)$ lies on circle $\therefore (2-h)^2 + (1-k)^2 = r^2$ $\Rightarrow 4 + h^2 - 4h + 1 + k^2 - 2k = r^2$ $\Rightarrow h^2 + k^2 - 4h - 2k + 5 = r^2$ ……….. (iii) Equating eq. (i) and (ii) $h^2 + k^2 - 10h + 16k + 89 = h^2 + k^2 - 4h + 18k + 85$ ans.
Q.7)	Find all data of the following parabolas: i) $y^2 = -12x$ ii) $16y = -4x^2$
Sol.7)	i) $y^2 = -12x$ Compare with $y^2 = -4ax$ We have, $4a = 12$ $\Rightarrow 6\text{h} + 2\text{k} = 4$ Or $3h + k = 2$ ……… (iv) Equating equation (i) and (ii) $\Rightarrow h^2 + k^2 - 4h + 18k + 85 = h^2 + k^2 - 4h - 2k + 5$ $\Rightarrow 20k = -80$ $\Rightarrow k = -4$, put in eq. (iv) $\Rightarrow 3h - 4 = 2$ $\Rightarrow 3\text{h} = 6$ $\Rightarrow h = 2$ $\therefore$ centre is $(2, -4)$, put in equation (i) $\Rightarrow 4 + 16 - 20 - 64 + 89 = r^2$ $\Rightarrow 25 = r^2$ $\Rightarrow r = 5$ $\therefore$ equating circle is $(x-2)^2 + (y+4)^2 = 25$ $\Rightarrow x^2 + y^2 - 4x + 8y - 5 = 0$ ans.
Q.8)	Find the equation of parabola whose focus = $(6,0)$ and directrix $x = -6$.
Sol.8)	Since the focus $(6,0)$ lies on $X - axis$ and directrix $x = -6$ is on the left side of origin.

	∴ parabola must be of the form $y^2 = 4ax$ Now compare focus with $(a, 0)$ and directrix with $x = -a$ We get $a = 6$ Put value of a in equation $y^2 = 4ax$ We get $y^2 = 4(6x) \Rightarrow y^2 = 24x$ ans.
Q.9)	Find the equation of parabola with vertex (0,0) and passing through (5,2) and symmetric w.r.t y-axis.
Sol.9)	Since parabola is symmetric w.r.t y-axis ∴ it may be either $x^2 = 4ay$ or $x^2 = -4ay$ but it passes through the point (5,2) which is in first quadrant ∴ parabola must be of the form $x^2 = 4ay$ Now, (5,2) lies on it $\therefore 25 = 4a(2)$ $\Rightarrow 25 = 8a$ $\Rightarrow a = \frac{25}{8}$ $\therefore x^2 = 4\left(\frac{25}{8}\right)y$ $\Rightarrow x^2 = \frac{25}{2}y$ ans.
Q.10)	Find vertex, foci, e, LR, Major axis and Minor axis $36x^2 + 4y^2 = 144$.
Sol.10)	We have, $36x^2 + 4y^2 = 144$ $\Rightarrow \frac{36}{144}x^2 + \frac{4y^2}{144} = 1$ $\Rightarrow \frac{x^2}{4} + \frac{y^2}{36} = 1$ $\Rightarrow \frac{x^2}{2^2} + \frac{y^2}{6^2} = 1$ Comparing with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a = 2$ and $b = 6$ here $b > a$ (2nd ellipse) i) $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{4}{36}} = \sqrt{1 - \frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$ ii) Vertices $(0, \pm be) = (0, \pm 6)$ iii) Foci $= (0, \pm be) = (0, \pm 4\sqrt{2})$ iv) LR $= \frac{2a^2}{b} = \frac{2\times 4}{6} = \frac{4}{3}$ v) Major axis $= 2b = 12$ vi) Minor axis $= 2a = 4$ ans.
➢	**SHIFTING HYPERBOLA**
Q.11)	Find centre, e, foci, vertices, LR, directrix, length of the axis (transverse & conjugate axis) of the hyperbola $9x^2 - 16y^2 - 18x + 32y - 151 = 0$.
Sol.11)	We have, $9x^2 - 16y^2 - 18x + 32y - 151 = 0$ $\Rightarrow 9x^2 - 18x - 16y^2 + 32y - 151 = 0$ $\Rightarrow 9(x^2 - 2x) - 16(y^2 - 2y) - 151 = 0$ $\Rightarrow 9[(x-1)^2 - 1] - 16[(y-1)^2 - 1] - 151 = 0$ $\Rightarrow 9(x-1)^2 - 9 - 16(y-1)^2 + 16 - 151 = 0$ $\Rightarrow 9(x-1)^2 - 16(y-1)^2 = 144$

$\Rightarrow \frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$

Let $x - 1 = X$ and $y - 1 = Y$

$\therefore x = X + 1$ and $y = Y + 1$

$\therefore$ equation becomes $\frac{X^2}{16} - \frac{Y^2}{9} = 1$

Clearly this is 1st (transverse hyperbola) with $a = 4$ and $b = 3$

i) eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{9}{16}}$

$$e = \sqrt{\frac{25}{10}} = \frac{5}{4}$$

ii) centre with respect to new axis $= (0,0)$

centre with respect to old axis $(0 + 1, 0 + 1) = (1,1)$

iii) vertices with respect to new axis $= (\pm a, 0) = (\pm 4, 0)$

vertices with respect to old axis $= (4 + 1, 0 + 1)$ and $(-4 + 1, 0 + 1)$

$= (5,1)$ and $(-3 + 1)$

iv) foci with respect to new axis $(\pm ae, ,0) = (\pm 5, 0)$

foci with respect to old axis $= (5 + 1, 0 + 1)$ and $(-5 + 1, 0 + 1)$

$= (6,1)$ and $(-4,1$

v) LR $= \frac{2b^2}{a} = 2\left(\frac{9}{4}\right) = \frac{9}{2}$

vi) directrix with respect to new axis $X = \pm \frac{a}{e}$

$\Rightarrow X = \pm \frac{16}{5}$

Directrix with respect to old axis $x = X + 1$

$\Rightarrow x = \frac{16}{5} + 1$ and $x = \frac{-16}{5} + 1$

$\Rightarrow x = \frac{21}{5}$ and $x = \frac{-11}{5}$

vii) length of transverse axis $2a = 8$

viii) length of conjugate axis $2b = 6$.