Class 11 Conic Section

Q.1)	Find the equation of the ellipse whose vertices $(\pm 6,0)$ and foci $(\pm 4,0)$.
Sol.1)	Comparing foci $(\pm 4,0)$ with $(\pm ae, 0)$ We have, $ae = 4$ Comparing vertices $(\pm 6,0)$ with $(\pm a, 0)$ We have $a = 6$ Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{\frac{a^2-b^2}{a^2}}$ $\Rightarrow ae = \sqrt{a^2 - b^2}$ $\Rightarrow 4 = \sqrt{36 - b^2}$ Squaring $16 = 36 - b^2$ $b^2 = 20$ Now, equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\Rightarrow \frac{x^2}{36} + \frac{y^2}{16a} = 1$
Q.2)	Find equation of ellipse whose length of major axis is 26 and foci $(\pm 5,0)$.
Sol.2)	Comparing foci $(\pm 5,0)$ with $(\pm ae, a)$ We have $ae = 5$ And major axis with $2a$ $\Rightarrow 2a = 26$ $\Rightarrow a = 13$ Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{\frac{a^2-b^2}{a^2}}$ $\Rightarrow ae = \sqrt{a^2 - b^2}$ $\Rightarrow 5 = \sqrt{169 - b^2}$ $\Rightarrow 25 = 169 - b^2$ $\Rightarrow b^2 = 144$ Equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\Rightarrow \frac{x^2}{169} + \frac{y^2}{144} = 1$ ans.
Q.3)	Find the equation of the ellipse major axis on the y-axis and passes through the point $(3,2)$ and $(1,6)$.
Sol.3)	Let the equation of the ellipse is $\frac{x^2}{b^2} + \frac{y^2}{b^2} = 1$ $(3,2)$ lies on ellipse $\therefore \frac{9}{a^2} + \frac{4}{b^2} = 1$ $\Rightarrow 9b^2 + 4a^2 = a^2b^2$ .... (i) $(1,6)$ lies on ellipse $\frac{1}{a^2} + \frac{36}{b^2} = 1$ $\Rightarrow b^2 + 36a^2 = a^2b^2$ .... (ii) Solving (i) and (ii) $9b^2 + 4a^2 = a^2b^2$ $-(9b^2 + 324a^2) = -(9a^2b^2)$ $-320a^2 = -8a^2b^2$ $b^2 = 40$ put in equation (i)

	$360 + 4a^2 = 40a^2$ $\Rightarrow 360 = 36a^2$ $\Rightarrow a^2 = 10$ $\therefore$ equation of ellipse becomes $\frac{x^2}{10} + \frac{y^2}{40} = 1$ ans.
Q.4)	Find e, vertices, foci, LR, length of transverse axis, Conjugate axis and equation of directrix of given hyperbola $5y^2 - 9x^2 = 36$.
Sol.4)	We have , $5y^2 - 9x^2 = 36$ $\Rightarrow -9x^2 + 5y^2 = 36$ $\Rightarrow -\frac{9x^2}{36} + \frac{5y^2}{36} = 1$ $\Rightarrow -\frac{x^2}{4} + \frac{y^2}{\frac{36}{5}} = 1$ $\Rightarrow \frac{-x^2}{(2)^2} + \frac{y^2}{\left(\frac{6}{\sqrt{5}}\right)^2} = 1$ Compare with $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a = 2$ and $b = \frac{6}{\sqrt{5}}$ The given hyperbola is conjugate hyperbola (2nd) 1) Centre $= (0,0)$ 2) $e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{4}{\frac{36}{5}}} = \sqrt{1 + \frac{20}{36}} = \sqrt{1 + \frac{5}{9}}$ $e = \sqrt{\frac{14}{9}} = \frac{\sqrt{14}}{3}$ 3) Vertices $= (0, \pm b) = \left(0, \pm\frac{6}{\sqrt{5}}\right)$ 4) foci $= (0, \pm be) = \left(0 \pm, \frac{2\sqrt{14}}{\sqrt{5}}\right)$ 5) LR $= \frac{2a^2}{b} = \frac{2\times 4}{\frac{6}{\sqrt{5}}} = \frac{8\sqrt{5}}{6} = \frac{4\sqrt{5}}{3}$ 6) Length of transverse axis $= 2b = \frac{12}{\sqrt{5}}$ 7) Length of conjugate axis $= 2a = 4$
Q.5)	Find the equation of hyperbola with vertices $(\pm 2,0)$ and foci $(\pm 3,0)$.
Sol.5)	The given data is of 1st hyperbola (transverse) Compare vertices $(\pm 2,0)$ with $(\pm a, 0)$ We get, $a = 2$ Compare foci $(\pm 3,0)$ with $(\pm ae, 0)$ We get $we = 3$ Now, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{a^2+b^2}{a^2}}$ $\Rightarrow ae = \sqrt{a^2 + b^2}$ $\Rightarrow 3 = \sqrt{4 + b^2}$ $\Rightarrow 9 = 4 + b^2$ $\Rightarrow b^2 = 5$ $\therefore$ equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\Rightarrow \frac{x^2}{4} - \frac{y^2}{5} = 1$ ans.
Q.6)	Find the equation of hyperbola with foci $(0, \pm 13)$ and conjugate axis is of length 24.

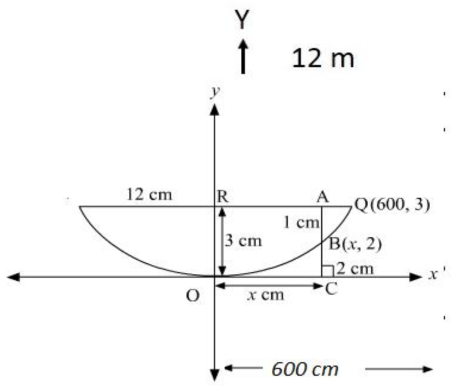
Sol.6)	The given is of 2nd hyperbola Compare foci $(0, \pm 13)$ with $(0, \pm be)$ $\Rightarrow be = 13$ Conjugate axis $= 2a = 24$ $\Rightarrow a = 12$ Now, $e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{\frac{b^2+a^2}{b^2}}$ $\Rightarrow be = \sqrt{b^2 + a^2}$ $\Rightarrow 13 = \sqrt{b^2 + 144}$ Squaring $\Rightarrow 169 = b^2 + 144$ $\Rightarrow b^2 = 25$ $\therefore$ equation of hyperbola is $\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\Rightarrow \frac{-x^2}{144} + \frac{y^2}{25} = 1$ ans.
Q.7)	Find the equation of hyperbola with foci $(\pm 3\sqrt{5}, 0)$ and latus rectum is of length 8.
Sol.7)	The given data is of 1st hyperbola Compare foci $(\pm 3\sqrt{5}, 0)$ with $(\pm ae, 0)$ We get $ae = 3\sqrt{5}$ LR $= 8$ $\Rightarrow \frac{2b^2}{a} = 8$ $\Rightarrow b^2 = 4a$ .... (i) Now, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{a^2+b^2}{a^2}}$ $\Rightarrow ae = \sqrt{a^2 + b^2}$ $\Rightarrow 3\sqrt{5} = \sqrt{a^2 + 4a}$ Squaring $\Rightarrow 45 = a^2 + 4a$ $\Rightarrow a^2 + 4a - 45 = 0$ $\Rightarrow (a + 9)(a - 5) = 0$ $\Rightarrow a = -9$ and $a = 5$ For $a = -9$; $b^2 = -36$ (from equation (i)) For $a = 5$; $b^2 = 20$ $\therefore$ equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\Rightarrow \frac{x^2}{25} - \frac{y^2}{12} = 1$ ans.
Q.8)	Find the equation hyperbola with foci $(0, \pm\sqrt{10})$; passing through $(2,3)$.
Sol.8)	Let equation of hyperbola is $\frac{-x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(2,3)$ lies on hyperbola $\Rightarrow \frac{-4}{a^2} + \frac{9}{b^2} = 1$ $\Rightarrow -4b^2 + 9a^2 = a^2b^2$ .... (i) Compare foci $(a, \pm\sqrt{10})$ with $(0, \pm be)$ $\Rightarrow be = \sqrt{10}$ Now, $e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{\frac{b^2+a^2}{b^2}}$

$\Rightarrow be = \sqrt{b^2 + a^2}$
$\Rightarrow \sqrt{10} = \sqrt{b^2 + a^2}$
Squaring
$\Rightarrow 10 = b^2 + a^2$
$\Rightarrow b^2 = 10 - a^2$ put in equation (i)
$\Rightarrow -4(10 - a^2) + 9a^2 = a^2(10 - a^2)$
$\Rightarrow -40 + 4a^2 + 9a^2 = 10a^2 - a^4$
$\Rightarrow a^4 + 3a^2 - 40 = 0$
$\Rightarrow a^4 + 8a^2 - 5a^2 - 40 = 0$
$\Rightarrow (a^2 + 8)(a^2 - 5) = 0$
$\Rightarrow a^2 = -8\,; a^2 = 5$ (rejected)
$\therefore b^2 = 10 - 5$
$\Rightarrow b^2 = 5$
$\therefore$ equation of hyperbola is $\frac{-x^2}{5} + \frac{y^2}{5} = 1$ ans.

➢ MISCELLANEOUS

Q.9) A beam is supported at its ends by supports which are 12 meters a part. Since the load is concentrated at the centre, there is a deflection of 3 cm at the centre and the deflected beam is in the shape of a parabola. How far from the centre is the deflection 1cm?

Sol.9) Let equation parabola is $x^2 = 4ay$
$A(600,3)$ lies on it
$\Rightarrow 3600 = 4a(3)$
$\Rightarrow 3600 = 12a$
$\Rightarrow a = 300$
$\therefore$ equation becomes
$x^2 = 1200y$



Now, $B(x, 2)$ lies on it
$x^2 = 2400$
$x = \sqrt{2400}$
$= 200\sqrt{6}cm$ or $2\sqrt{6}cm$
$\therefore$ required distance $= 2\sqrt{6}m$ ans.

Q.10) The cable of a uniformly loaded suspension bridge hangs on the form of a parabola. The roadway which is horizontal & 100 m long is supported by vertical wire attached to the cable, the longest wire being 30m and the shortest being 6m. Find the length of a supporting wire attached to the roadway 18m from the middle.

Sol.10) Let equation of parabola is $x^2 = 4ay$ ……….. (i)
$A(50,24)$ lies on parabola
$\Rightarrow 250 = 96a$

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$\Rightarrow a = \frac{2500}{96}$ put in equation (i)

We have $x^2 = 4\left(\frac{2500}{96}\right)y$

$\Rightarrow x^2 = \frac{2500}{24}y$

Now, $A(18, y)$ lies on it

$\therefore 324 = \frac{2500}{24}y$

$\Rightarrow \frac{324 \times 24}{2500} = y$

$\Rightarrow y = 3.11$

$\therefore$ required length $= 6 + y = 6 + 3.11$

$= y = 9311\ m$