	COMPLEX NUMBERS ***Class XI***
Q.11)	Where does z lies, if $\left\|\frac{z-5i}{z+5i}\right\| = 1$?
Sol.11)	Let $z = x + iy$ We have, $\left\|\frac{z-5i}{z+5i}\right\| = 1$ $\Rightarrow \frac{\|z-5i\|}{\|z+5i\|} = 1$ $\Rightarrow \|z - 5i\| = \|z + 5i\|$ $\Rightarrow \|x + iy - 5i\| = \|x + iy + 5i\|$ $\Rightarrow \|x - i(y - 5)\| = \|x + i(y + 5)\|$ $\Rightarrow \sqrt{x^2 + (y-5)^2} = \sqrt{x^2 + (y+5)^2}$ Squaring $\Rightarrow x^2 + y^2 + 25 - 10y = x^2 + y^2 + 25 + 10y$ $\Rightarrow 20y = 0$ $\Rightarrow y = 0$ $\Rightarrow z = x$ lies on $x - axis$
Q.12)	Evaluate: $1 + i^2 + i^4 + i^6 + \cdots \ldots \ldots . i^{2n}$
Sol.12)	$= 1 - 1 + 1 - 1 + \cdots \ldots \ldots (-1)^n$ This cannot be evaluated unless value of n is known
Q.13)	If $\|z_1\| = \|z_2\|$, is it necessary that $z_1 = z_2$?
Sol.13)	No. Example $z_1 = 3 + 4i$ & $z_2 = 4 + 3i$ $\Rightarrow \|z_1\| = \sqrt{9 + 16} = 5$ $\Rightarrow \|z_2\| = \sqrt{16 + 9} = 5$ Clearly $\|z_1\| = \|z_2\|$ but $\|z_1\| \neq \|z_2\|$
Q.14)	What is the polar form of complex number $= (i^{25})^3$
Sol.14)	Let $z = (i^{25})^3$ $\Rightarrow z = (i^{-1})^3$ $\Rightarrow z = -i$ $\Rightarrow z = 0 - 1i$ Here, $a = 0, b = -1$ $\Rightarrow r = \sqrt{0 + 1} = 1$ $\Rightarrow \tan\alpha = \left\|\frac{-1}{0}\right\| = \left\|\frac{1}{0}\right\| = 0$ $\alpha = \frac{\pi}{2}$ since z is in 4th quadrant $\Rightarrow \theta = -\alpha \Rightarrow \theta = \frac{-\pi}{2}$ Polar form $\Rightarrow z = r(\cos\theta + i\sin\theta)$ $\Rightarrow z = 1\left(\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)\right)$ $\Rightarrow z = \cos\left(\frac{\pi}{2}\right) - i\sin\left(\frac{\pi}{2}\right)$ ans.
Q.15)	Find the complex number satisfying the equation $z + \sqrt{z}\|z + 1\| + i = 0$
Sol.15)	We have, $z + \sqrt{z}\|z + 1\| + i = 0$ Let $z = x + iy$

StudiesToday.com

	$\Rightarrow x+iy+\sqrt{2}\lvert x+iy+1\rvert+i=0$ $\Rightarrow x+iy+\sqrt{2}\lvert(x+1)+iy\rvert+i=0$ $\Rightarrow x+iy+\sqrt{2}\sqrt{(x+1)^2+y^2}+i=0$ $\Rightarrow x+\sqrt{2}\sqrt{(x+1)^2+y^2}+i(y+1)=0+0i$ Equating real & imaginary parts We get $y+1=0$ $\Rightarrow x+\sqrt{2}\sqrt{(x+1)^2+y^2}=0$ Put $y=-1$ $\Rightarrow x+\sqrt{2}\sqrt{x^2+1+2x+1}=0$ $\Rightarrow x+\sqrt{2}\sqrt{x^2+2x+2}=0$ $\Rightarrow \sqrt{2}\sqrt{x^2+2x+2}=-x$ Squaring, $2(x^2+2x+2)=x^2$ $\Rightarrow x^2+4x+4=0$ $\Rightarrow (x+2)^2=0$ $\Rightarrow x=-2$ $\Rightarrow z=x+iy=-2-i$ ans.	
Q.16)	If $\arg(z-1)=\arg(z+3i)$ then find $x-1:y$ if $z=x+iy$.	
Sol.16)	We have, $z=x+iy$ $\Rightarrow z-1=z=x+iy-1=(x-1)+iy$ And $z+3i=x+iy+3i=x+i(y+3)$ $\Rightarrow \arg(z-1)=\tan\alpha=\frac{b}{a}=\frac{y}{x-1}$ $\Rightarrow \arg(z+3i)=\tan\alpha=\frac{b}{a}=\frac{y+3}{x}$ Given $\arg(z-1)=\arg(z+3i)$ $\Rightarrow \frac{y}{x-1}=\frac{y+3}{x}$ $\Rightarrow xy=xy+3x-y-3$ $\Rightarrow y=3(x-1)$ $\Rightarrow \frac{x-1}{y}=\frac{1}{3}$ $\Rightarrow 1:3$ ans.	
Q.17)	If $a=\cos\theta+i\sin\theta$, find the value of $\frac{1+a}{1-a}$.	
Sol.17)	We have, $\frac{1+a}{1-a}=\frac{(1+\cos\theta)+i\sin\theta}{(1-\cos\theta)-i\sin\theta}$ $=\frac{2\cos^2\frac{\theta}{2}+i2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}-i2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}$ $=\frac{2\cos^2\left(\frac{\theta}{2}\right)\left[\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right]}{2\sin^2\left(\frac{\theta}{2}\right)\left[\sin\frac{\theta}{2}+i\cos\frac{\theta}{2}\right]}$ Rationalize $=\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}\cdot\left[\frac{1}{-1}\frac{\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)}\right]$ $=i\cot\left(\frac{\theta}{2}\right)$ ans.	

Q.18)	Evaluate: $i^n + i^{n+1} + i^{n+2} + i^{n+3}$	
Sol.18)	$i^n + i^n.i + i^n.i^2 + i^n i^3$ $= i^n\left(1 + i + (i^2 + i^3)\right)$ $= i^n(1 + i - 1 - i)$ $= i^n = 0$ $= 0$ ans.	