	COMPLEX NUMBERS ***Class XI***
	Quadratic Equation:-
Q.1)	Find all non-zero complex numbers z satisfying $\|z\| = iz^2$
Sol.1)	Let $z = x + iy$ We have $\|z\| = iz^2$ $\Rightarrow x - iy = i(x+iy)^2$ $\Rightarrow x - iy = i(x^2 - y^2 + 2ixy)$ $\Rightarrow x - iy = ix^2 - iy^2 - 2xy$ $\Rightarrow x - iy = -2xy + i(x^2 - y^2)$ Equating real & imaginary parts $\Rightarrow x = -2xy$ and $-y = x^2 - y^2$ $\Rightarrow x + 2xy = 0$ and $x^2 - y^2 + y = 0$ Consider $x + 2xy = 0$ $\Rightarrow x(1+2y) = 0$ $\Rightarrow x = 0$ (or) $y = \frac{-1}{2}$ Case 1 , when $x = 0$ $\Rightarrow x^2 - y^2 + y = 0$ $\Rightarrow y(-y+1) = 0$ $\Rightarrow y = 0$ (or) $y = 1$ $\therefore x = 0 \;\&\; y = 0$ and $x = 0 \;\&\; y = 1$ $\Rightarrow z = 0 + 0i$ and $z = 0 + 1i$ (rejected) $\quad\therefore z = i$ Case 2, when $y = \frac{-1}{2}$ $\Rightarrow x^2 - y^2 + y = 0$ $\Rightarrow x^2 - \frac{1}{4} - \frac{1}{2} = 0$ $\Rightarrow x^2 = \frac{3}{4}$ $\Rightarrow x = \pm\frac{\sqrt{3}}{2}$ $\therefore x = \frac{\sqrt{3}}{2}, y = \frac{-1}{2}$ and $x = \frac{-\sqrt{3}}{2}, y = \frac{-1}{2}$ $\Rightarrow z = \frac{\sqrt{3}}{2} - \frac{1}{2}i$ and $z = \frac{-\sqrt{3}}{2} - \frac{1}{2}i$ $\therefore$ required complex numbers are $= i, \frac{\sqrt{3}}{2} - \frac{1}{2}i$ and $\frac{-\sqrt{3}}{2} - \frac{1}{2}i$ ans.
Q.2)	Solve the equation $z^2 + \|z\| = 0$.
Sol.2)	Let $z = x + iy$ We have $z^2 + \|z\| = 0$ $\Rightarrow (x+iy)^2 + \sqrt{x^2+y^2} = 0$ $\Rightarrow x^2 - y^2 + 2ixy + \sqrt{x^2+y^2} = 0$ $\Rightarrow \left(x^2 - y^2 + \sqrt{x^2+y^2}\right) + i2xy = 0 + 0i$ $\Rightarrow x^2 - y^2 + \sqrt{x^2+y^2} = 0$ and $2xy = 0$ Consider $2xy = 0$

	$x = 0$ or $y = 0$ Case 1, when $x = 0$ Then $x^2 - y^2 + \sqrt{x^2 + y^2} = 0$ $\Rightarrow -y^2 + \sqrt{y^2} = 0$ $\Rightarrow -y^2 + \lvert y \rvert = 0$ If $y > 0$ then $\lvert y \rvert = y$ $\Rightarrow -y^2 + y = 0$ $\Rightarrow y(-y + 1) = 0$ $y = 0$ or $y = 1$ If $y < 0$ then $\lvert y \rvert = -y$ $\Rightarrow -y^2 + y = 0$ $\Rightarrow -y(y + 1) = 0$ $y = 0$ or $y = -1$ $x = 0; y = 0$ or $x = 0; y = -1$ ………… (ii) Case 2, when $y = 0$ Then $x^2 - y^2 + \sqrt{x^2 - y^2} = 0$ $\Rightarrow x^2 + \sqrt{x^2} = 0$ $\Rightarrow x^2 + \lvert x \rvert = 0$ If $x > 0$ then $\lvert x \rvert = x$ $\Rightarrow x^2 + x = 0$ $\Rightarrow x(x + 1) = 0$ $x = 0$ or $x = -1$ (rejected since $x > 0$) $x = 0; y = 0$ …………… (iii) If $x < 0$ then $\lvert x \rvert = -x$ $\Rightarrow x^2 - x = 0$ $\Rightarrow x(x - 1) = 0$ $x = 0$ or $x = 1$ (rejected since $x < 0$) $x = 0; y = 0$ …………… (iv) From (i), (ii), (iii) & (iv) $z = 0 + oi;\ z = 0 + i; z = 0 - i$ ans.	
Q.3)	If $\lvert z_1 \rvert = \lvert z_2 \rvert = \lvert z_3 \rvert \ldots \ldots \ldots \ldots \lvert z_n \rvert = 1$ show that $\lvert z_1 + z_2 + z_3 + \cdots \ldots \ldots z_n \rvert = \left\lvert \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \cdots \ldots \ldots \frac{1}{z_n} \right\rvert$	
Sol.3)	We have, $\lvert z_1 + z_2 + z_3 + \cdots \ldots \ldots z_n \rvert$ $= \left\lvert \frac{z_1\overline{z_1}}{\overline{z_1}} + \frac{z_2\overline{z_2}}{\overline{z_2}} + \frac{z_3\overline{z_3}}{\overline{z_3}} + \cdots \ldots \ldots \frac{z_n\overline{z_n}}{\overline{z_n}} \right\rvert$ (multiply & divide) $= \frac{\lvert z_1 \rvert^2}{\overline{z_1}} + \frac{\lvert z_2 \rvert^2}{\overline{z_2}} + \frac{\lvert z_3 \rvert^2}{\overline{z_3}} + \cdots \ldots \ldots \frac{\lvert z_n \rvert^2}{\overline{z_n}}$ ……….. $\{z\bar{z} = \lvert z \rvert^2\}$ $= \left\lvert \frac{1}{\overline{z_1}} + \frac{1}{\overline{z_2}} + \frac{1}{\overline{z_3}} + \cdots \ldots \ldots \frac{1}{\overline{z_n}} \right\rvert$ ………….. $\{\lvert z_1 \rvert = \lvert z_2 \rvert = 1\}$ $= \left\lvert \overline{\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \cdots \ldots \ldots \frac{1}{z_n}} \right\rvert$ …………….. $\{\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}\}$ $= \left\lvert \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \cdots \ldots \ldots \frac{1}{z_n} \right\rvert$ …….. $\lvert \bar{z} \rvert = \lvert z \rvert$	
Q.4)	If $\lvert z^2 - 1 \rvert = \lvert z \rvert^2 + 1$, show that z lies on imaginary axis.	
Sol.4)	Let $z = x + iy$	

	Then $\|z^2-1\| = \|z\|^2+1$ $= \|(x+iy)^2-1\| = \sqrt{x^2+y^2}^2+1$ $= \|(x^2-y^2-1)+2ixy\| = (x^2+y^2)+1$ $= \sqrt{(x^2-y^2-1)+4x^2y^2} = x^2+y^2+1$ Squaring both sides $= (x^2-y^2-1)^2+4x^2y^2 = (x^2+y^2+1)^2$ $= x^4+y^4+1-2x^2y^2+2y^2-2x^2+4x^2y^2$ $= x^4+y^4+1+2x^2y^2+2y^2+2x^2+4x^2y^2$ $= 4x^2 = 0$ $= x^2 = 0 = x = 0$ $z = 0+iy$ Clearly z lies on $y-axis$	
Q.5)	If the imaginary part of $\frac{2z+1}{iz+1}$ is -2, then show that the locus of the point representing z in the argand plane is a straight line. LOCUS is a path traced by a moving point (x, y)	
Sol.5)	Let $z = x+iy$ Here, we have prove that equation containing x & y must be linear i.e., straight line Now $\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1}$ $= \frac{(2x+1)+i2y}{(1-y)+ix}$ Rationalize $= \frac{(2x+1)+i2y}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}$ $= \frac{(2x+1)(1-y)-ix(2x+1)+i2y(1-y)-i^2 2xy}{(1-y)^2-i^2x^2}$ $= \frac{2x-2xy+1-y-i(2x^2+x)+i(2y-2y^2)+2xy}{1+y^2-2y+x^2}$ $= \frac{(2x+1-y)}{x^2+y^2-2y+1} + i\frac{(2y-2y^2-2x^2-x)}{x^2+y^2-2y+1}$ Here $Im\left(\frac{2z+1}{iz+1}\right) = \frac{2y-2y^2-2x^2-x}{x^2+y^2-2y+1}$ Given $Im\left(\frac{2z+1}{iz+1}\right) = -2$ $= \frac{2y-2y^2-2x^2-x}{x^2+y^2-2y+1} = -2$ $\Rightarrow 2y-2y^2-2x^2-x = -2x^2-2y^2+4y-2$ $\Rightarrow x+2y-2 = 0$ Clearly this represents the equation of straight line	
Q.6)	Let z_1 & z_2 be two complex numbers such that $\|z_1+z_2\| = \|z_1\|+\|z_2\|$ then show that $\arg(z_1) - \arg(z_2) = 0$.	
Sol.6)	Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ And $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ Here $\|z_1\| = r_1$ $\arg(z_1) = \theta_1$ $\Rightarrow \|z_2\| = r_2$ $\arg(z_2) = \theta_2$	

	We have, $\|z_1+z_2\| = \|z_1\|+\|z_2\|$ $\Rightarrow \|r_1(\cos\theta_1 + i\sin\theta_1) + r_2(\cos\theta_2 + i\sin\theta_2)\| = r_1 + r_2$ $\Rightarrow \|(r_1\cos\theta_1 + r_2\cos\theta_2) + i(r_1\sin\theta_1 + r_2\sin\theta_2)\| = r_1 + r_2$ $\Rightarrow \sqrt{(r_1\cos\theta_1 + r_2\cos\theta_2)^2 + (r_1\sin\theta_1 + r_2\sin\theta_2)^2} = r_1 + r_2$ Squaring both sides $\Rightarrow \quad {r_1}^2\cos^2\theta_1 + {r_2}^2\cos^2\theta_2 + 2\,r_1r_2\cos\theta_1\cos\theta_2 + {r_2}^2\sin^2\theta_2 + 2r_1r_2\ \sin\theta_1\sin\theta_2 = (r_1+r_2)^2$ $\Rightarrow {r_1}^2(\cos^2\theta_1 + \sin^2\theta_1) + {r_2}^2(\cos^2\theta_2 + \sin^2\theta_2) + 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) = {r_1}^2 + {r_2}^2 + 2r_1r_2$ $\Rightarrow {r_1}^2 + {r_2}^2 + 2r_1r_2\cos(\theta_1-\theta_2) = {r_1}^2 + {r_2}^2 + 2r_1r_2$ $\Rightarrow \cos(\theta_1-\theta_2) = 1$ $\Rightarrow \theta_1 - \theta_2 = 0$ .... $\{\cos\theta = 1\ then\ \theta = 0\}$ $\Rightarrow$arg(z_1) $-$arg$(z_2) = 0$	
Q.7)	What does this equation $\|z+1-i\| = \|z-1+i\|$ represents.	
Sol.7)	Let $\ z = x+iy$ Then $\|z+1-i\| = \|z-1+i\|$ $\Rightarrow \|x+iy+1-i\| = \|x+iy-1+i\|$ $\Rightarrow \|(x+1)i(y-1)\| = \|(x+1)i(y+1)\|$ $\Rightarrow \sqrt{(x+1)^2+(y-1)^2} = \sqrt{(x-1)^2+(y+1)^2}$ $\Rightarrow \sqrt{x^2+1+2x+y^2+1-2y} = \sqrt{x^2+1-2x+y^2+1-2y}$ Squaring $\Rightarrow x^2+y^2+2x-2y+2 = x^2+y^2+2y-2x+2$ $\Rightarrow 4x-4y = 0$ $\Rightarrow x-y = 0$ Clearly this represents the equation of a straight line.	
Q.8)	Evaluate: $1+i^2+i^4+i^6+\cdots\ldots\ldots.i^{20}$	
SOL.8)	$= 1-1+1-1+\cdots.+1$ $= 1$ ans.	
Q.9)	Find the value of $\frac{i^{4n+1}-i^{4n-1}}{2}$	
Sol.9)	$\Rightarrow \frac{i^{4n}.i-i^{4n}.i^{-1}}{2}$ $\Rightarrow \frac{1(i)-1\left(\frac{1}{i}\right)}{2}$ .... $\{i^{4n} = (i^4)^n = (1)^n = 1\}$ $\Rightarrow \frac{i-(i)}{2}$ .... $\frac{1}{i} = \frac{1}{i}\times\frac{i}{i} = -i$ $\Rightarrow \frac{2i}{2} = i$	
Q.10)	What is the smallest positive integer n for which $(1+i)^{2n} = (1-i)^{2n}$.	
Sol.10)	We have, $(1+i)^{2n} = (1-i)^{2n}$ $\Rightarrow \frac{(1+i)^{2n}}{(1-i)^{2n}} = 1$ $\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n} = 1$	