

CHAPTER - 8

BINOMIAL THEOREM

KEY POINTS

- $(a + b)^n = n_{C_0} a^n + n_{C_1} a^{n-1} b + n_{C_2} a^{n-2} b^2 + \dots + n_{C_n} b^n$

$$= \sum_{r=0}^n n_{C_r} a^{n-r} b^r, n \in \mathbb{N}$$

- $T_{r+1} = \text{General term}$

$$= n_{C_r} a^{n-r} b^r \quad 0 \leq r \leq n$$

- Total number of terms in $(a + b)^n$ is $(n + 1)$

- If n is even, then in the expansion of $(a + b)^n$, middle term is $\left(\frac{n}{2} + 1\right)^{\text{th}}$

term i.e. $\left(\frac{n+2}{2}\right)^{\text{th}}$ term.

- If n is odd, then in the expansion of $(a + b)^n$, middle terms are $\left(\frac{n+1}{2}\right)^{\text{th}}$ and $\left(\frac{n+3}{2}\right)^{\text{th}}$ terms

- In $(a + b)^n$, r^{th} term from the end is same as $(n - r + 2)^{\text{th}}$ term from the beginning.

- r^{th} term from the end in $(a + b)^n$
 $= r^{\text{th}}$ term from the beginning in $(b + a)^n$

- In $(1 + x)^n$, coefficient of x^r is n_{C_r}

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Compute $(98)^2$, using binomial theorem.
2. Expand $\left(x - \frac{1}{x}\right)^3$ using binomial theorem.
3. Write number of terms in the expansion of $(1 + 2x + x^2)^{10}$.
4. Write number of terms in $(2a - b)^{15}$
5. Simplify :

$$\frac{{}^nC_r}{{}^nC_{r-1}}$$

6. Write value of

$${}^{2n-1}C_5 + {}^{2n-1}C_6 + {}^{2n}C_7$$

[Hint : Use ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$]

7. In the expansion, $(1 + x)^{14}$, write the coefficient of x^{12}
8. Find the sum of the coefficients in $(x + y)^8$

[Hint : Put $x = 1, y = 1$]

9. If ${}^nC_{n-3} = 120$, find n .

[Hint : Express 720 as the product of 3 consecutive positive integers]

10. In $\left(\frac{x}{2} - \frac{2}{x}\right)^8$, write 5th term.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a , b and n .

12. In $\left(3x^2 - \frac{1}{x}\right)^{18}$, which term contains x^{12} ?

13. In $\left(2x - \frac{1}{x^2}\right)^{15}$, find the term independent of x .
14. Evaluate : $(\sqrt{2} + 1)^5 - (\sqrt{2} - 1)^5$ using binomial theorem.
15. Evaluate $(0.9)^4$ using binomial theorem.
16. Prove that if n is odd, then $a^n + b^n$ is divisible by $a + b$.
[Hint : $a^n = (a + b - b)^n$. Now use binomial theorem]
17. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of x^6 and x^4 .
18. In $\left(2x - \frac{3}{x}\right)^8$, find 7th term from end.
19. In $\left(2x^3 - \frac{1}{x^2}\right)^{12}$, find the coefficient of x^{11} .
20. Find the coefficient of x^4 in $(1 - x)^2 (2 + x)^5$ using binomial theorem.
21. Using binomial theorem, show that $3^{2n+2} - 8n - 9$ is divisible by 8.
[Hint : $3^{2n+2} = 9 \left(3^2\right)^n = 9 (1 + 8)^n$, Now use binomial theorem.]
22. Prove that,

$$\sum_{r=0}^{20} {}^{20}C_{20-r} (2 - t)^{20-r} (t - 1)^r = 1$$
23. Find the middle term(s) in $\left(x - \frac{1}{x}\right)^8$
24. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:3:5, then show that $n = 7$.
25. Show that the coefficient of middle term in the expansion of $(1 + x)^{20}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{19}$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

26. Show that the coefficient of x^5 in the expansion of product $(1 + 2x)^6 (1 - x)^7$ is 171.
27. If the 3rd, 4th and 5th terms in the expansion of $(x + a)^n$ are 84, 280 and 560 respectively then find the values of a , x and n
28. In the expansion of $(1 - x)^{2n-1}$, find the sum of coefficients of x^{r-1} and x^{2n-r}
29. If the coefficients of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$ are equal, then show that $ab = 1$

ANSWERS

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|---------------------------------|---|
| 1. 9604 | 2. $x^3 - \frac{1}{x^3} - 3x + \frac{3}{x}$ |
| 3. 21 | 4. 16 |
| 5. $\frac{n-r+1}{r}$ | 6. $^{2n+1}C_7$ |
| 7. 91 | 8. 256 |
| 9. $n = 10$ | 10. 70 |
| 11. $a = 3, b = 2, n = 3$ | 12. 9 th term |
| 13. $-2^{10} \times {}^{15}C_5$ | 14. 82 |
| 15. 0.6561 | 17. 28 |
| 18. $16128 x^4$ | 19. -101376 |
| 20. 10 | 23. 70 |
| 27. $a = 2, x = 1, n = 7$ | 28. 0 |